On the Spatial Dynamical Model of Vibratory Displacement

Victor S. Zviadauri, Merab A. Chelidze, George I. Tumanishvili

Abstract. Methods of drawing up of mathematical model of spatial movement of the three-mass oscillating system - an analogy of the vibratory technological machine, are given. Differential equations are interconnected non-linearly and technological process can be controlled with their help depending on dynamical and kinematical parameters of the whole system.

Some results of the numerical experiment are given, indicating dependence of speed of vibratory transportation on separate spatial vibrations of the vibratory machine working member.

Keywords: differential equations, model of the friable material, spatial vibrations, vibratory technologic machine, speed of vibratory displacement.

I. INTRODUCTION

Rise of non-working spatial vibrations in vibratory technologic machines, disturbing their normal operating conditions is often observed. In this connection a problem of elaboration of spatial dynamical and corresponding mathematical models of these machines with the purpose of study of influence of non-working (parasitic) spatial vibrations on the proceeding technologic process, arises.

Interconnected equations enable us to determine influence of separate, as well as of combinations of various vibrations on behavior of the technologic load. In the result, correlations of the system parameters can be revealed, favoring improvement of quality and quantity of the produce being processed.

II. SYSTEM DESCRIPTION

A vibratory technologic machine can be considered as a three-mass oscillating system consisting of the elements: active mass M_1 (working member); reactive mass M_2 (vibroexciter); material M_3 being processed or transported (Fig.1; 2).

The main distinction between the system in consideration and classical n-mass spatial system contains the following aspects: a) specificity of technologic mass M_3 (friable, lump and other materials) performing relative movement with respect to working member M_1; thereat masses M_1 and M_2 perform independent movement under action of the external force and mass M_3 – under action of mass M_1; b) specified initial disposition of masses M_1, M_2 and M_3 relative to each other (such a condition makes the generally used succession of drawing up of mathematical model of the system movement, asymmetric); c) peculiarity of interaction of masses M_1 and M_3, as connected with each other by conditional non-permanent elastic link existing only in the time of their contact.

To facilitate deduction of equations we represent a vibratory machine (Fig. 1) in the classical form of three-mass oscillating system (Fig. 2), taking into account above mentioned distinctive features.

For obtaining of general vector and then analytical expressions of kinetic energy we determine absolute speed of any material point of each masses Ai, B_i, C_i. They are related vectorially (R_j, Rji, r_{ji}) with the origins of their own coordinate axes, as well as with the origin of inertial coordinate system (Oξεζ); besides M_3 is related with the centroid (origin of the coordinate system $O_i x_i y_i z_i$) of mass M_1.

Vector expressions of speeds of points A_i, B_i and Ci have the form

$$\begin{aligned} \vec{V}_{Ai} &= \vec{V}_{O1} + \vec{\omega}_{O1} \times \vec{r}_{1i} \\ \vec{V}_{Ci} &= \vec{V}_{O2} + \vec{\omega}_{O2} \times \vec{r}_{2i} \\ \vec{V}_{Bi} &= \vec{V}_{O1} + \vec{\omega}_{O1} \times \vec{R}_{3i} + \vec{V}_{O3} + \vec{\omega}_{O3} \times \vec{r}_{3i} \end{aligned} \qquad (1)$$

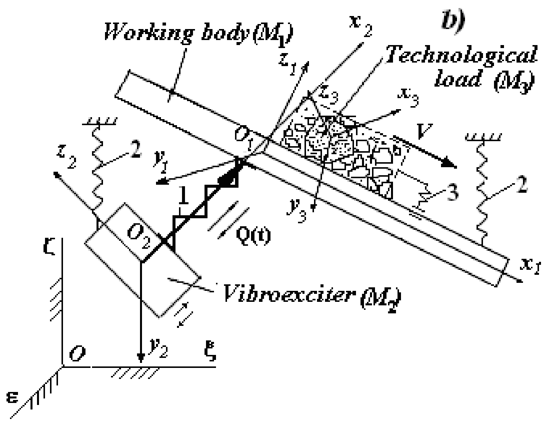

Fig. 1. Model of the vibratory transport technological machine

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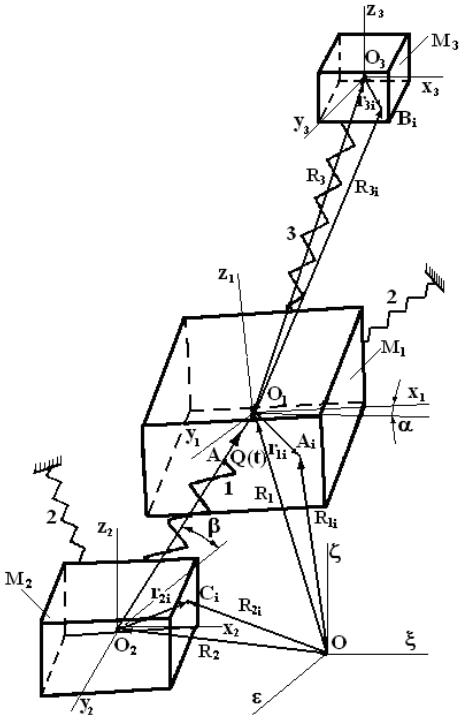

Fig. 2. Three-mass oscillating system - analogue of the vibratory transport technological machine

Correspondingly, expressions of kinetic energy of masses will be

$$T_j = \frac{1}{2}\sum_{i=1}^{(n)_j} (M_j)_i [V_{Ai,Di,Ci}]^2 T \,, \qquad (2)$$

where j=1,2,3; M_{1i}, M_{2i}, M_{3i} are masses of particles A_i, B_i, C_i; n_1, n_2, n_3 –number of corresponding particles. Rotary movements of masses M_1, M_2, M_3 will be described by directing cosines of Euler ship angles [4] (thereby small changes of Euler angles are ensured at small deflections of masses), considering angles of inclination of the working member vibrating surface (α) and exciting force (β).

To obtain an analytical expression of spatial movement of the mentioned interconnected oscillating system, it is necessary to determine coordinates of centroides of masses, points of fastening of elastic elements to the masses and to reduce them to one specified coordinate system. Since technologic load (M_3) is connected directly and interacts with working member (M_1) it is expedient to choose $O_1x_1y_1z_1$ as such a system.

Projections of coordinates of points are defined with the help of angular coefficients according to Table 1, where directing cosines of angles between axes of coordinate systems $O_1\, x_1\, y_1\, z_1$ and $O_1^{'} x_1^{'} y_1^{'} z_1^{'}$ are given. Expressions of coordinates of fastening of point *A* of the basic elastic system 1 (Fig. 2) in coordinate system $O_1x_1y_1z_1$, after dynamical displacement of masses, are shown here as an example

$$x_A = x_{O1_1} + x_{1A}\nu_{11} + y_{1A}\nu_{12} + z_{1A}\nu_{13};$$

$$y_A = y_{O1_1} + x_{1A}\nu_{21} + y_{1A}\nu_{22} + z_{1A}\nu_{23};...;z_A,$$

where $x_{O1_1}, y_{O1_1}, z_{O1}$ are coordinates of point O_1 (after displacement); x_{1A}, y_{1A}, z_{1A} - coordinates of point *A*; $\nu_{11},...,\nu_{23},...,\nu_{33}$ – directing cosines of angles between axes of coordinate systems $O_1x_1y_1z_1$ and $O_1^{'} x_1^{'} y_1^{'} z_1^{'}$, i.e. between initial and dynamical positions of mass M_1 (Fig.2; Table 1).

For illustration we give here analytical expression of kinetic energy of mass M_1:

$$\begin{aligned} T_1 = {} & \frac{1}{2}M_1\,(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2}[(J_{x1}\cos^2\alpha_1 + \\ & + J_{z1}\sin^2\alpha_1)\dot{\theta}_1^2 + (J_{x1}\sin^2\alpha_1 + \\ & + J_{z1}\cos^2\alpha_1)\dot{\phi}_1^2] + J_{y1}\dot{\psi}_1^2 + \dot{\theta}_1\,\dot{\phi}_1\,(J_{x1} - \\ & - J_{z1})\sin\alpha_1\cos\alpha_1, \end{aligned} \qquad (3)$$

where J_{x1}, J_{y1}, J_{z1} are moments of inertia of mass M_1 relative to axes of system $O_1x_1y_1z_1$.

Expansion of kinetic energy of mass M_3 has more complicated form; since it performs relative movement with respect to moving mass M_1 (we did not consider it necessary to give this expression here).

Table 1.Directing cosines of angles between initial and dynamical positions of mass M_1

	$x_1^{'}$	$y_1^{'}$	$z_1^{'}$
x_1	$(1-\psi_1^2/2-\varphi_1^2/2)\cos\alpha_1 -$ $-(\psi_1+\varphi_1\theta_1)\sin\alpha_1$	$-\varphi_1\cos\alpha_1+\theta_1\sin\alpha_1$	$\psi_1\cos\alpha_1-(\psi_1^2/2-\theta_1^2/2)\sin\alpha_1 -$
y_1	$-\varphi_1$	$-1-\varphi_1^2/2-\theta_1^2/2$	$-\theta_1$
z_1	$-(1-\psi_1^2/2-\varphi_1^2/2)\sin\alpha_1 -$ $-(\psi_1+\varphi_1\theta_1)\cos\alpha_1$	$\varphi_1\sin\alpha_1+\theta_1\cos\alpha_1$	$-\psi_1\sin\alpha_1+(1-\psi_1^2/2-\theta_1^2/2)\cos\alpha_1$

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To include technologic load (mass M_3) in common spatial system (Fig. 2) and to give it a generalized character, we represent it formally as a rigid body, connected with working member (M_1) by conventional elastic system 3, describing elastic features of friable material.

At fixed moment of time elastic system 3 (as well as 1 and 2; Fig. 1; 2) is decomposed into three components describing elastic features of material in space.

Peculiarity of elastic system 3 is in non-permanent character of its link with the working member (WM). Interaction between layers of material and between lower layer and WM are described with the help of elastic and damping elements. In contrast to existent models [1,2] the offered model considers all the degrees of freedom, i.e. it can be included in the model of general spatial system (Fig. 1; 2) and, depending on concrete tasks, it can be reduced to simpler forms (plane, linear).

Representation of TL by the rigid body (at drawing up of expression for kinetic energy) is stipulated by necessity to obtain an equation of movement of TL in more generalized form – not only for reciprocating (in this case TL would have been considered as a material point), but also for rotary movements.

Deformation of a layer of the friable TL is simulated by elastic elements with coefficients of elasticity k_{x3}, k_{y3}, k_{z3}, $\kappa_{\theta3}$, $\kappa_{\psi3}$, $\kappa_{\varphi3}$ (Fig. 3).

Dissipation of energy at deformation of a layer is considered by dampers with coefficients of resistance c_{x3}, c_{y3}, c_{z3}, $c_{\theta3}$, $c_{\psi3}$, $c_{\phi3}$ (not shown in the Figure). Therefore, direct contact between TL and WM is replaced by elastic and frictional links. In determining of potential forces of elastic systems of WM and TL, depending on the value of displacement, two approaches can be used. In one case (at small displacements) components of the elastic force along the coordinate axes are determined according to potential energy and Lagrange equation. In another case according to determined elastic force its components on the coordinate axes are found directly. Sizes of conventional elastic system 3 can be determined as a guide, depending on location of TL relative to surfaces of the concrete WM.

For deduction of equations of spatial movement of three –mass oscillating system (Fig.1; 2), Lagrange equation of the second kind is used in the following form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}=Q_q+Q_q^{'} \qquad (4)$$

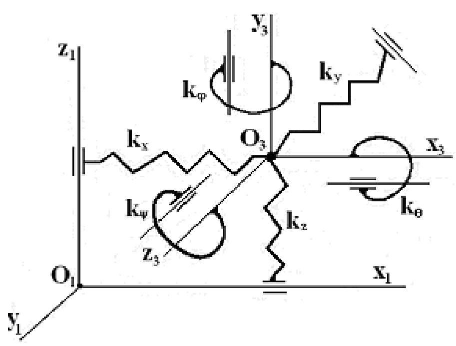

Fig. 3. Spatial model of the friable technological load

where T is a sum of kinetic energies of masses M_1, M_2, M_3 drawn up by analogy of equation (3) for each mass; q – generalized coordinate adopting magnitudes $x_1, y_1, z_1, \ldots, y_3, z_3, \theta_1, \psi_1, \varphi_1, \ldots, \psi_3, \varphi_3$; $Q_q^{'}$ - potential (elastic) forces and moments, stipulated by the machine elastic system; Q_q – forces, not related with deformation of an elastic system or with inertness of the oscillating system in consideration: external (exciting) forces; forces of gravity, forces of resistance of the kind of external friction (friction force between TL and WM).

On the basis of adopted assumptions about smallness of rotary displacements products of variables no higher than second order are considered in the equations.

In view of the fact that out of interconnected masses, interaction of M_1 and M_3 is more peculiar and M_3 can be of various internal structure, we give equations of movement of these masses only along coordinate axis x (methods of deduction of equations of movement of all the masses along other coordinate axes are similar and we do not give them here).

For mass M_1:

$$\begin{aligned}&(M_1+M_3)\ddot{x}_1+M_3[(\ddot{\psi}_1 z_3+2\dot{\psi}_1\dot{z}_3-\\&-\ddot{\phi}_1 y_3-2\dot{\phi}_1\dot{y}_3-\ddot{y}_3\phi_1+\ddot{z}_3\psi_1)\cos\alpha_1+\\&+\ddot{x}_3\cos\alpha_1+\ddot{z}_3\cos\alpha_1+(\ddot{\theta}_1 y_3-2\dot{\theta}_1\dot{y}_3-\\&-\ddot{\psi}_1 x_3-2\dot{x}_3\dot{\psi}_1+\ddot{y}_3\theta_1+\ddot{x}_3\psi_1)\sin\alpha_1]=\\&=Q(t)(\psi_1\sin\alpha_1+\cos\alpha_1)+\\&+f\,(k_q,\ q,\ q_iq_j,\ c_q,\ \dot{q},\ \dot{q}_i\dot{q}_j).\end{aligned} \qquad (5)$$

For mass M_3:

$$\begin{aligned}&M_3[(\ddot{x}_3+\ddot{x}_1-\ddot{z}_1\psi_1)\cos\alpha_1-(\ddot{z}_1+\\&+\ddot{x}_1\psi_1)\sin\alpha_1-(2\dot{\psi}_1\dot{z}_3-\ddot{\phi}_1 y_3+2\dot{\phi}_1\dot{y}_3-\\&-\ddot{y}_1\phi_1-\ddot{\psi}_3 z_3)]=f_xN_z sign(\dot{x}_3)+\\&+f^*(k_q^*,\ q,\ q_iq_j,\ c_q^*,\ \dot{q},\ \dot{q}_i\dot{q}_j),\end{aligned} \qquad (6)$$

where f, f^* are functions of coordinates and speeds and their products; $\alpha_1 = \alpha+\beta$; Q(t) -exciting force of the vibro-exciter; “sign” – non-linear function depending on sign of speeds $\dot{x}_3, \dot{y}_3, \dot{z}_3$: *sign* = 1 at $\dot{x}_3(\dot{y}_3, \dot{z}_3) < 0$, *sign* = -1 at $\dot{x}_3(\dot{y}_3, \dot{z}_3) > 0$, N_z – reaction of TL relative to WM.

Equations (5) and (6) and the rest, not given here describe movement of the three-mass system (Fig. 1; 2) and they are interconnected by non-linear items of potential and inertial character. As for the form of link, they are similar for both systems. The difference is in presence of the sum of masses M_1 and M_3 in equations (5), while only one mass M_3 is in equations (6). Character of vibratory displacement of one body relative to another is determined by such form of interconnection.

It should be noted, that equations (5) and (6) describe movement of mass M_3 relative to M_1 at constant interconnection (at constant contact), and potential field in the form of Q_q (in the right-hand part) is an elastic and damping characteristic of mass M_3 (in the case of friable material) and depending on its position, Q_q (and coefficients k_q^*, c_q^*) can vary. Besides, dynamical dependence between M_1 and M_3 can be uneven and conditions of throwing up of material (M_3) from the vibrating surface (M_1) are added to systems (5) and (6) in this case.

Some results of solution of equations (5) and (6) are given in Figures 4 and 5; namely, dependences of speeds of movement of the material V_x and V_y on variation of the amplitude of the WM rotary vibrations ψ and φ are shown (resonances were provoked in these directions). From these dependences is seen that speed varies significantly in the zone of resonance that indicates possibility of use of given factor for the purpose of intensification of the vibratory technologic process.

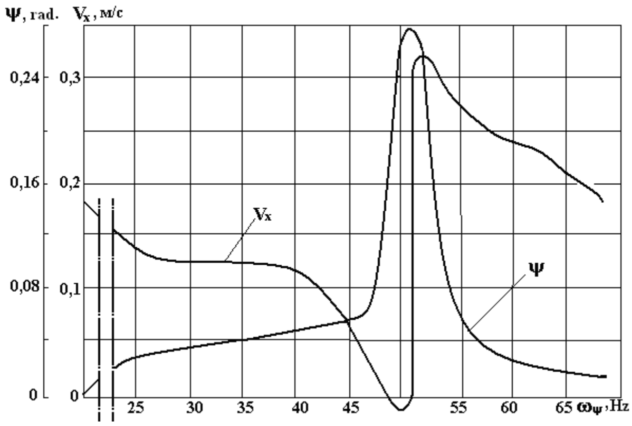

Fig. 4. Dependence of speed of movement of the material V_x on variation of the amplitude of the WM rotary vibrations ψ

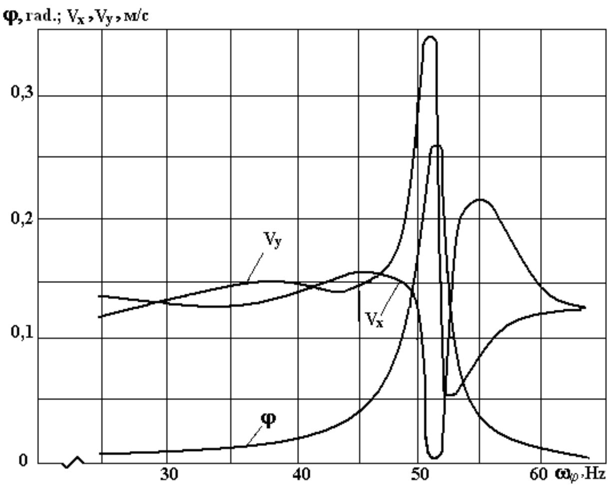

Fig. 5. Dependences of speeds of movement of the material V_x and V_y on variation of the amplitude of the WM rotary vibrations φ

III. CONCLUSIONS

Dynamical and corresponding mathematical models of three-mass oscillating system – an analogy of vibratory technologic machine are drawn up with the use of theory of relative movement. Methods of drawing up of equations of spatial movement of the vibro-machine working member and technological load of generalized form are offered. Influence of non-working spatial vibrations of the WM and other parameters of the vibro-machine on technologic process can be studied with the help of proposed mathematical model.

This will enable us to increase degree of their purposeful application relying upon the fact that combination of certain spatial vibrations and working vibrations increases intensity of the technologic process.

In future researches will be continued and constructions using combinations of working vibrations and certain spatial vibrations, ensuring improvement of the technologic process (For example, rise of speed of vibratory displacement) will be elaborated.

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