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Joint Accelerated Failure Mode Modeling of Degradation and Traumatic Failure Times

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***Abstract*—Many products have two kinds of failure modes, that is, degradation failure and traumatic failure. It is of great importance to study the competing failure of degradation failure and traumatic failure at use or accelerating conditions. In this paper, the problem of accelerated failure with competing causes of a degradation failure mode and multiple traumatic failure modes is treated. We discuss the scenario wherein the performance degradation process is described by Brownian motion process, traumatic failure-time follows exponential or Weibull distribution and the possibility of traumatic failure depends on the performance degradation. The accelerated failure model is proposed, method for estimating parameters is given and reliability model is deduced. As a nice application of the proposed model, we finally make the accelerated failure analysis of metallized film pulse capacitors.**

***Index Terms*—Degradation failure; Traumatic failure; Accelerated failure analysis; Brownian motion.**

I. Introduction

Many electronic products have two kinds of failure modes including degradation failure and traumatic failure. For example, the causes of a resistor’s failure may be dielectric breakdown, short circuit or degradation in resistance. Among them, dielectric breakdown and short circuit are traumatic failure while degradation in resistance is a degradation failure. Due to strong needs from consumers, most of electronic products are characterized as long life and highly-reliable. For highly-reliable products, it is not an easy task to assess their lifetime distribution under the use stress. For evaluating the reliability, accelerated tests are needed. However, the elevation of stress not only accelerates the performance degradation of products, but also increases the failure rate of traumatic failure modes. So we should combine information from both degradation failure and traumatic failure to evaluate the reliability of products in accelerated failure analysis.

The competing causes of degradation failure and traumatic failure have been studied by some authors in the literature. Yao [1] proposed a mathematical expression of mixed degradation when the two kinds of failure modes exist simultaneously, in which he combined the probability density function (PDF) of degradation and the probability of traumatic failure. When degradation failure and traumatic failure are assumed to be independent of each other, Huang [2], Huang and Askin [3] studied the competing failure problems, Zhao and Elsayed [4] studied the competing failure problems while stress is elevated, Li and Pham[5] discussed the reliability modeling of multiple failure modes and multiple states degradation system. Zhao [6] proposed a more general model involving both traumatic and degradation failures in which the dependence of traumatic failure intensities on degradation level is included. The inference methods of unknown parameters of the competing failure model are presented based on proportional hazard rate model and location-scale model.

However, few people has considered accelerated failure analysis with competing causes of degradation failure and traumatic failure. This paper proposes an accelerated failure model of combining degradation failure mode with multiple traumatic failure modes when the traumatic failures depend on degradation level. Furthermore, the methods for estimating parameters and inference of reliability model are presented. As a nice application of the proposed model, we finally make the accelerated failure analysis of metallized film pulse capacitors.

The paper is organized as follows. Section 1 introduces the background. Section 2 deals with the basic assumptions and modeling procedure. Section 3 proposes the reliability model of competing failure. To illustrate the proposed method, we present an example in Section 4, the accelerated failure analysis of metallized film capacitor. A comparison is conducted between the assessment results of degradation failure only and those of combination of degradation failure and traumatic failure. In Section 5, conclusions are drawn and some possible directions for future work are discussed as well.

II. Accelerated Failure Model with Competing Failure

A. Assumption

1) The product has just one degradation failure mode. The performance parameter, denoted as $X(t)$ at time t, degrades monotonically (either increasing or decreasing). Once $X(t)$ reaches a certain threshold C, the product fails.

2) The product may fail due to the traumatic failure. Assume that the product has P traumatic failure modes which are statistically independent of each other, and the

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failure of product only is caused by one of the traumatic failure modes.

3) The traumatic failure probability depends on degradation level. The more the performance parameter degrades, the higher the product's traumatic failure probability will be. But given a certain degradation value $x(t)$, the traumatic failure modes are independent of each other.

4) Under testing stress S_k, $k=1,\cdots,K$, the failure time of the pth failure mode follows a probability distribution function, such as exponential or Weibull distribution.

Based on the assumptions above, an accelerated failure model with competing causes of a degradation failure and traumatic failures is discussed.

B. Accelerated failure model

At accelerating levels of S_k, suppose that n_k products are tested, l_{ki} measurements of the i th product are observed up to the termination time τ_k, which results in degradation measurements x_{kij} ($x_{kil_{ki}} < C$) at corresponding time t_{kij}. The traumatic failure time and traumatic failure mode of the products are observed while the performance parameter is measured. Suppose that the pth traumatic failure mode results in the i th product failure at corresponding time t_{ki}, marked as $c_{ki}=p$, where $k=1,\cdots,K$, $i=1,\cdots,n_k$, $j=1,\cdots,l_{ki}$, $p=1,\cdots,P$.

Suppose that the initial degradation level of the product is X_0, we have the following cumulative damage of the product at time t, where $\{D_u : 0\le u\le t\}$ is assumed to be a Brownian motion process with positive drift coefficient α and diffusion coefficient β:

$$X_t - X_0 = D_t = \alpha t + \beta B(t),$$

we assume here that α is dependent on the acceleration variable S, $\alpha_k = A_s(S_k)$, but β is not.

Set

$$\Delta d_{kij} = x_{kij+1} - x_{kij},\ \Delta t_{kij} = t_{kij+1} - t_{kij}.$$

Then, by the independent increment property of the Brownian motion process, we have independent but not identical random variables

$$\Delta d_{kij} \sim N\left(\alpha_k \Delta t_{kij}, \beta^2 \Delta t_{kij}\right).$$

The likelihood function can be expressed as

$$L_s = \prod_{k=1}^{K}\prod_{i=1}^{n_k}\prod_{j=1}^{l_{ki}} \frac{1}{\sqrt{2\pi\Delta t_{kij}}\,\beta}\cdot \exp\left(-\frac{\left(\Delta d_{kij}-\alpha_k\Delta t_{kij}\right)^2}{2\beta^2\Delta t_{kij}}\right).$$

So

$$\ln(L_s) = -\sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{l_{ki}}\left(\frac{\left(\Delta d_{kij}-\alpha_k\Delta t_{kij}\right)^2}{2\beta^2\Delta t_{kij}} + \ln\sqrt{2\pi\Delta t_{kij}}\,\beta\right). \quad (1)$$

For the P traumatic failure modes, at accelerating level S_k, if the failure time of the test product follows exponential distribution, the failure rate because of pth traumatic failure mode can be denoted as

$$\lambda_k^{(p)} = A_h^{(p)}(x_k(t)). \quad (2)$$

Similarly, if the failure time of the test product follows Weibull distribution, the failure rate because of pth traumatic failure mode can be denoted as

$$\lambda_k^{(p)} = m^{(p)} t^{m^{(p)}-1} \Big/ \left(\eta_k^{(p)}\right)^{m^{(p)}},$$

where scale parameter $\eta_k^{(p)}$ depends on the pth traumatic failure mode and degradation level $x_k(t)$ at accelerating level S_k, and for shape parameter $m_k^{(p)}$, $m_1^{(p)} = \cdots = m_K^{(p)} = m^{(p)}$, suppose

$$\eta_k^{(p)} = A_h^{(p)}(x_k(t)). \quad (3)$$

In (2) and (3), the selection of $A_h^{(p)}(\cdot)$ was discussed in reference [7].

So, the failure rate of the test product because of any traumatic failure mode can be written as

$$\lambda_k = \sum_{p=1}^{P} \lambda_k^{(p)}.$$

The incomplete probability density of the failure time by pth traumatic failure mode denotes as

$$f_k^{(p)}(t) = \lambda_k^{(p)} \exp\left(-\int_0^t \lambda_k\, ds\right).$$

So the likelihood function of a sample's observation (t_{ki}, c_{ki}) [8] is

$$\prod_{p=1}^{P}\left[I(t_{ki})\lambda_k^{(p)}\exp\left(-\int_0^{t_{ki}}\lambda_k ds\right) + \bar{I}(t_{ki})\exp\left(-\int_0^{\tau_k}\lambda_k ds\right)\right]^{\delta_p(c_{ki})},$$

where

$$\delta_p(c_{ki}) = \begin{cases} 1 & c_{ki} = p \\ & \text{or there is no failure products due to} \\ & \text{traumatic failure mode} \\ 0 & c_{ki} \ne p \end{cases},$$

$$I(t_{ki}) = \begin{cases} 1 & t_{ki} \le \tau_k \\ 0 & t_{ki} > \tau_k \end{cases}.$$

For all tested samples, the likelihood function denotes as

$$L_h = \prod_{k=1}^{K}\prod_{i=1}^{n_k}\prod_{p=1}^{P}\left[I(t_{ki})\lambda_k^{(p)}\exp\left(-\int_0^{t_{ki}}\lambda_k ds\right) + \bar{I}(t_{ki})\exp\left(-\int_0^{\tau_k}\lambda_k ds\right)\right]^{\delta_p(c_{ki})}.$$

So

$$\ln(L_h) = \sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{p=1}^{P}\left(\delta_p(c_{ki})\left[I(t_{ki})\left(\ln\lambda_k^{(p)} - \int_0^{t_{ki}}\lambda_k ds\right) + \bar{I}(t_{ki})\left(-\int_0^{\tau_k}\lambda_k ds\right)\right]\right). \quad (4)$$

Because of the correlation of traumatic failure and degradation failure, we can't multiply the likelihood functions L_s and L_h and evaluate the parameters of model by maximum likelihood evaluation (MLE). So the process is divided into two steps. We make use of the degradation measurements data to evaluate the parameters of L_s by MLE at first; then, we estimate the parameters of L_h.

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III. Reliability Evaluation for Competing Failure

We assume that the degradation failure time of the product is T_s and the traumatic failure time of the product is $T_h^{(p)}$ due to pth traumatic failure mode, where $p = 1, \cdots, P$. So, the failure time of the product is

$$T = \min\{T_s, T_h^{(1)}, \cdots, T_h^{(P)}\}.$$

The reliability of the product at time t is

$$R(t) = P(T > t) = P(T_s > t, T_h^{(1)} > t, \cdots, T_h^{(P)} > t).$$

The degradation measurement and the failure rate of $T_h^{(p)}$ are assumed to $x(t)$ and $\lambda^{(p)}(x(t))$ respectively, the reliability function is

$$R_h^{(p)}(t|x) = P\{T_h^{(p)} > t|x\} = \exp\left(-\int_0^t \lambda^{(p)}(x(s))ds\right).$$

So, the probability of no products failed due to traumatic failures can be expressed as

$$R_h(t|x) = P\{T_h^{(1)} > t, \cdots, T_h^{(P)} > t|x\}$$
$$= \prod_{p=1}^{P} R_h^{(p)}(t|x) \quad ,$$
$$= \exp\left(-\sum_{p=1}^{P}\int_0^t \lambda^{(p)}(x(s))ds\right)$$

set $\Lambda(x(t)) = \sum_{p=1}^{P}\int_0^t \lambda^{(p)}(x(s))ds$.

Since the cumulative damage is assumed to be Brownian motion process, the first passage time T_s follows inverse Gaussian distribution with a failure threshold C,

$$F_s(t) = P(T_s \le t)$$
$$= \Phi\left[\sqrt{\frac{1}{\beta^2 t}}(\alpha t - C)\right] + \exp\left\{\frac{2\alpha C}{\beta^2}\right\}\Phi\left[-\sqrt{\frac{1}{\beta^2 t}}(\alpha t + C)\right].$$

So

$$R_s(C;t) = 1 - F_s(t)$$
$$= 1 - \Phi\left[\sqrt{\frac{1}{\beta^2 t}}(\alpha t - C)\right] - \exp\left\{\frac{2\alpha C}{\beta^2}\right\}\Phi\left[-\sqrt{\frac{1}{\beta^2 t}}(\alpha t + C)\right],$$

and

$$R(t) = \int_{X_0}^{C} R_h(t|x) dR_s(x;t) = \int_{X_0}^{C} e^{-\Lambda(x(t))} dR_s(x;t) \quad . \tag{5}$$

In terms of the information of degradation and traumatic failures, we can estimate the parameters in (1) and (4). Furthermore, the reliability of the product can also be evaluated.

IV. Accelerated Failure Analysis of Metallized Film Pulse Capacitors

Metallized film pulse capacitors are typical products with long life and high reliability which are used in the energy module of SG-III, an ICF facility in China. Most capacitors fails due to degradation. However, when subjected to high voltage, dielectric breakdown occurs in some defects which results in failures of the capacitors. So the competing failure of degradation failure and traumatic failure occurs in the use condition of the capacitor, one is the degradation of capacity (degradation failure), the other is dielectric breakdown (traumatic failure). The use voltage of the metallized film pulse capacitors which are used in the energy module is $V_0 = 23KV$, the threshold is regarded to be 5% of the initial capacity as $C = 2.8\mu F$. The accelerating level are $V_1 = 30KV$ and $V_2 = 35KV$ respectively. Suppose that 5 capacitors are tested at accelerating level $V_1 = 30KV$, one of them fails due to traumatic failure and 6 capacitors are tested at accelerating level $V_2 = 35KV$, two of them fail. The simulation data is illustrated in Fig. 1.

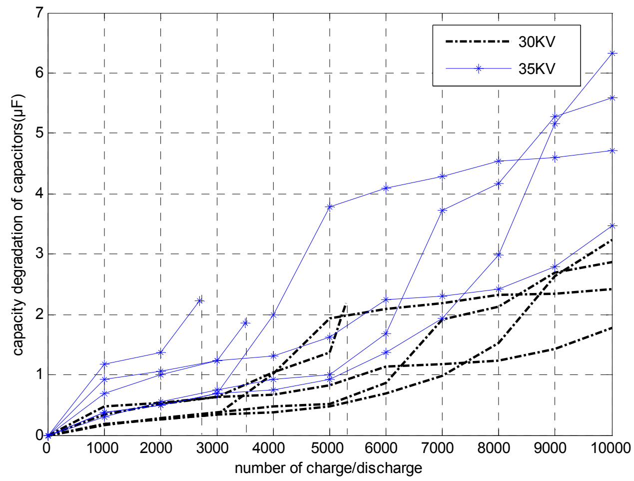

Fig. 1 the capacity accelerated degradation data of capacitors

From Fig. 1, the capacity of the capacitors is measured every 1000 shots during the test process. Define the relation of α and V as

$$\alpha = A(V) = aV^b.$$

From (1), where $K = 2$, $n_1 = 5$, $n_2 = 6$, $l_{11} = l_{12} = l_{13} = l_{14} = 10$, $l_{15} = 5$, $l_{21} = l_{22} = l_{23} = l_{24} = 10$, $l_{25} = 2$, $l_{26} = 3$, the estimated values of the relevant parameters are listed as follows:

$$(a, b, \beta) = (0.9639 \times 10^{-10}, 4.3507, 0.013558). \tag{6}$$

Because only one traumatic failure mode is considered, if the failure time follows exponential distribution, we define the relation of failure rate λ_h and degradation of capacity $x(t)$ as

$$\lambda_h = A_h(x(t)) = a_h(x(t))^{b_h} = a_h(\alpha t)^{b_h},$$

and if the failure time follows Weibull distribution, we define the relation of scale parameter η_h and degradation of capacity $x(t)$ as

$$\eta_h = A_h(x(t)) = a_{h\eta}(x(t))^{b_{h\eta}} = a_{h\eta}(\alpha t)^{b_{h\eta}}.$$

From (4) and (6), where $K = 2$, $n_1 = 5$, $n_2 = 6$, $P = 1$, we can get the estimated value of relevant parameters of traumatic failure as Table 1.

Table 1 Parameter estimates of traumatic failure

exponential distribution		*Weibull distribution*		
a_h	b_h	$a_{h\eta}$	$b_{h\eta}$	m
3.0721e-5	0.2261	1.3612e+5	-2.0229	0.3802

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Now we can obtain the reliability function of metallized film pulse capacitors as follows.

exponential distribution:

$$R_e(t)=\exp\left(-3.6508\times10^{-6}\cdot t^{1.2261}\right)\cdot\left(1-\Phi\left(0.0060\sqrt{t}-\frac{206.5201}{\sqrt{t}}\right)-11.7957\cdot\Phi\left[-\left(0.0060\sqrt{t}+\frac{206.5201}{\sqrt{t}}\right)\right]\right),$$

Weibull distribution:

$$R_w(t)=\exp\left(-7.9668\times10^{-6}\cdot t^{1.1493}\right)\cdot\left(1-\Phi\left(0.0060\sqrt{t}-\frac{206.5201}{\sqrt{t}}\right)-11.7957\cdot\Phi\left[-\left(0.0060\sqrt{t}+\frac{206.5201}{\sqrt{t}}\right)\right]\right).$$

If we assume that degradation failure occurs only during the test process, the reliability function of metallized film pulse capacitors can be obtained as

$$R_s(t)=1-\Phi\left(0.0060\sqrt{t}-\frac{206.5201}{\sqrt{t}}\right)-11.7957\cdot\Phi\left[-\left(0.0060\sqrt{t}+\frac{206.5201}{\sqrt{t}}\right)\right].$$

Furthermore, we can get the failure PDF of metallized film pulse capacitors as follows:

$$f_e(t)=-\frac{dR_e(t)}{dt},\ f_w(t)=-\frac{dR_w(t)}{dt},\ f_s(t)=-\frac{dR_s(t)}{dt}.$$

Fig. 2 and Fig. 3 compare the reliability and the failure PDF of the capacitors with competing failure and with degradation failure only respectively.

From the figures we can see that both exponential and Weibull distributions can model the traumatic failure data very well. And the traumatic failure influences the reliability of products greatly. So, failure modes of products should be analyzed before evaluating their reliability, so that the lifetime of the products can be estimated more accurately.

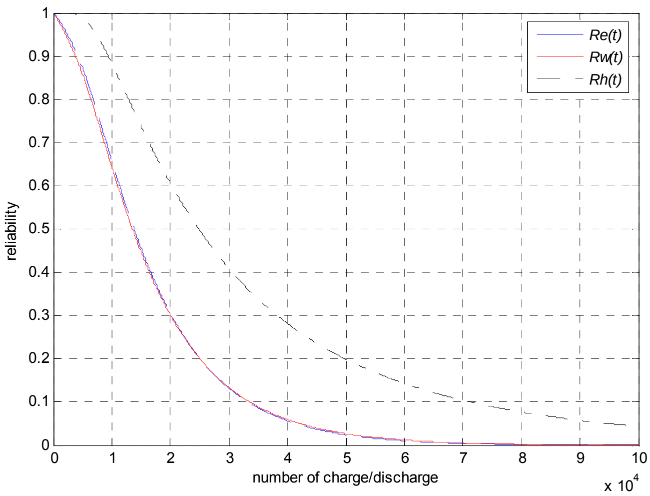

Fig. 2 the comparison of reliability: *Re(t)* ,*Rw(t)* and *Rs(t)*

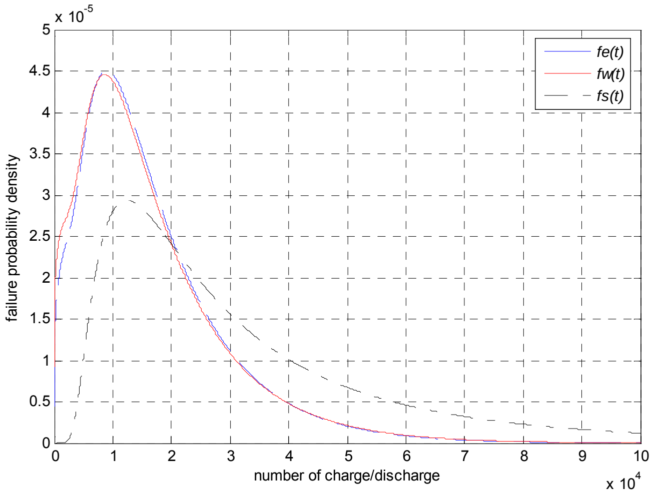

Fig. 3 the comparison of failure PDF: *fe(t), fw(t)* and *fs(t)*

V. Conclusion

Most of the electronic products have two kinds of failure modes, degradation failure and traumatic failure. It is of great importance to study the competing failure of degradation failure and traumatic failure at use or accelerating conditions. The accelerated failure analysis with competing causes of degradation failure and traumatic failure is discussed in this paper. We use exponential and Weibull distributions to model traumatic failure data respectively and propose the models and methods of parameters evaluation. At last, the accelerated failure analysis of metallized film pulse capacitors is presented to validate the models and we also analyze the results contractively.

As a preliminary research on the topic, we believe that much work remains to be done. Some directions for future work are as follows:

1) Other theories and methods of statistics inference, such as Bayesian method, should also be considered to see if they can be used here.

2) How to design the accelerated degradation test of products with degradation failure mode and traumatic failure mode should be studied.

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