Eigenvalue Approach to Thermoelastic Interactions in an Unbounded Body with a Spherical Cavity

A.Lahiri *, B. Das† and S. Sarkar‡

Abstract—**The problem of a thermoelastic interaction without energy dissipation of homogeneous and isotropic infinite medium with a spherical cavity is considered. Exact expressions for temperature distribution, stress and displacement components are obtained in Laplace transform domain for three different cases- (i) surface is stress free and subjected a thermal shock acting only for a finite period of time L. (ii) surface is stress free and subjected to a ramp-type increase in heating and (iii) surface is assumed to maintain the constant reference temperature T_0 and subjected to a ramp-type increase in boundary load. A numerical approach is implemented for the inversion of Laplace transform in order to obtain the solution in physical domain. Finally numerical computations of the stress, temperature and displacement have been made and presented graphically.**

Keywords: Eigenvalue Approach, Generalized Thermoelasticity, Isotropic, Laplace transform and Vector-matrix differential Equation.

1 Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena not consistent with physical observations. The equation of heat conduction of this theory - (i) does not contain any elastic terms, but the fact, the elastic change produce heat effects and (ii) heat conduction equation is of parabolic type predicting infinite speeds of propagation for heat waves. This amounts to say that classical thermoelasticity predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances which are coupled together. To eliminate this paradox, in 1967, Lord and Shulman [1] formulated generalized thermoelsticity with one relaxation time parameter for special case of isotropic body, using non Fourier thermal wave model which is also known as L-S model. This theory was extended by Dhaliwal and Sherief[2] to include the anisotropic case.In this theory a modification law of heat conduction on both the heat flux and its time derivative replaces the conventional Fourier law. Uniqueness of the solution for this theory was proved under different conditions by Ignaczak [3], [4], Sherief and Dhaliwal [5], [2] and Sherief [6].

After five years of L-S Model, Green and Lindsay(G-L theory) [7] introduced another generalized thermoelasticity theory (with relaxation time parameter) known as temperature rate dependent thermoelasticity from another physical point of view . This theory was initiated by Muller [8], it was further extended by Green and Laws [9].

Green and Nagdhi [10] formulated a model of thermoelasticity based on without energy dissipation of thermal energy. This G-N theory is known as Thermoelasticity Without Energy Dissipation Theory(TEWOEDT). In the development of this theory the thermal displacement gradient is considered as a constitutive variable, where as in the conventional thermoelasticity the temperature gradient is taken as a constitutive variable. The linearized version of uniqueness theorem has been given by Green and Nagdhi[11] and Chandrasekharaiah [12] independently. Recently Chandrasekharaiah [13], Chandrasekharaiah and Srinath [14], [15], Mukhopadhyay [16], [17],Honig and Dhaliwal [18], Sharma and Chauhan [19], Lahiri and Das [20] worked on thermoelastic interaction without energy dissipation.

Most of the problems of thermoelasticity have been solved by using thermoelastic potential function by Nowacki [21] and Sherief [22]. Bahar and Hetnarski [23], [24] discussed the limitations of using potential function approach, introduced the state-space approach to solve the problem of coupled thermoelasticity not containing any heat source. Many problems solved by Das and Bhakta [25], [26], Sherief and Anwar[27], [28]. Recently Sherief[29] extended the results in Sherief and Anwar [28]to problems with heat sources.

In this paper we consider thermoelastic infinite isotropic medium with a spherical cavity within the context of the theory of thermoelasticity without energy dissipation. Laplace transform have been used in the basic equations of thermoelasticity and finally the resulting equations are written in the form of a Vector-matrix differential equation which is then solved by eigenvalue approach for three different cases (i) surface is stress free and subjected to a thermal shock acting only for a finite period of time L, (ii) surface is stress free and subjected to a ramp-type in-

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crease in heating and (iii) surface is assumed to maintain the constant reference temperature T_0 and subjected to a ramp-type increase in boundary load. Finally numerical computations of the stresses, temperature and displacement have been made and presented graphically.

Nomenclature

λ, μ= Lamè constants. u=Displacement components. T= Absolute temperature. T_0 = Reference temperature chosen such that $|\frac{T-T_0}{T_0}| < 1$. ρ = Mass density. c_v = Specific heat per unit mass at constant volume. C_e = Specific heat at constant strain. t = Time variable. K = Coefficient of thermal conductivity. $\gamma = (3\lambda + 2\mu)\beta^*$. β^* = Coefficient of volume expansion. β= Coefficient of stress temperature. k^* = Material constants characteristic of the theory. $H(t)$ = Heaviside unit step function.

2 Basic Equations and Formulation of the Problem

We now consider a homogeneous,isotropic and thermoelastic infinite medium with a spherical cavity of radius a in the absence of body force or heat source. Due to spherical symmetry, the displacement components have the form

$$u_r = u(r,t), \quad u_\theta = u_\phi = 0 \tag{1}$$

The three principal stresses in the radial,cross radial and transverse directions are σ_{rr},$\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ respectively. The equation of motion in radial direction and the heat conduction equation are as follows vide [30], [31] :

$$(\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2u}{r^2}\right) - \gamma\frac{\partial T}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \tag{2}$$

$$k^*\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) = c_v\frac{\partial^2 T}{\partial t^2} + \gamma T_0\frac{\partial^2}{\partial t^2}(\frac{\partial u}{\partial r} + \frac{2u}{r}) \tag{3}$$

And the constitutive stress-components are given by

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + 2\lambda\frac{u}{r} - \gamma T \tag{4}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \lambda\frac{\partial u}{\partial r} + 2(\lambda + \mu)\frac{u}{r} - \gamma T \tag{5}$$

$$\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0 \tag{6}$$

Introducing the non-dimensional notations such as

$$R = \frac{r}{a}, U = \frac{(\lambda + 2\mu)u}{\gamma T_0 a}, \eta = \frac{v}{a}t, Z = \frac{T}{T_0},$$
$$\sigma_{RR} = \frac{1}{\gamma T_0}\sigma_{rr}, \sigma_{\phi\phi} = \frac{1}{\gamma T_0}\sigma_{\phi\phi}, \rho v^2 = \lambda + 2\mu \tag{7}$$

Equations (2)-(5) becomes

$$\left(\frac{\partial^2 U}{\partial R^2} + \frac{2}{R}\frac{\partial U}{\partial R} - \frac{2U}{R^2}\right) - \frac{\partial Z}{\partial R} = \frac{\partial^2 U}{\partial \eta^2} \tag{8}$$

$$\frac{\partial^2 Z}{\partial R^2} + \frac{2}{R}\frac{\partial Z}{\partial R} = \frac{1}{C_T^2}\frac{\partial^2 Z}{\partial \eta^2} + \frac{\varepsilon}{C_T^2}\frac{\partial^2}{\partial \eta^2}\left(\frac{\partial U}{\partial R} + \frac{2U}{R}\right) \tag{9}$$

And the non-dimensional stress-components are

$$\sigma_{RR} = \frac{\partial U}{\partial R} + 2\bar{\lambda}\frac{U}{R} - Z \tag{10}$$

$$\sigma_{\phi\phi} = \bar{\lambda}\frac{\partial U}{\partial R} + (\bar{\lambda} + 1)\frac{U}{R} - Z \tag{11}$$

where

$$\varepsilon = \frac{\gamma^2 T_0}{(\lambda + 2\mu)c_v}, C_T^2 = \frac{k^*}{c_v v^2}, \bar{\lambda} = \frac{\lambda}{\lambda + 2\mu} \tag{12}$$

3 Solution Procedure

Formulation of the Vector-matrix Differential Equation

We now apply the Laplace transform defined by

$$[\bar{U}(R,p), \bar{Z}(R,p)] = \int_0^\infty [U(R,t), Z(R,t)]exp(-pt)dt \tag{13}$$

to the equation (8)-(11),we get

$$\left(\frac{d^2\bar{U}}{dR^2} + \frac{2}{R}\frac{d\bar{U}}{dR} - \frac{2\bar{U}}{R^2}\right) - \frac{d\bar{Z}}{dR} = p^2\bar{U} \tag{14}$$

$$\frac{d^2\bar{Z}}{dR^2} + \frac{2}{R}\frac{d\bar{Z}}{dR} = \frac{p^2}{C_T^2}\bar{Z} + \frac{p^2\varepsilon}{C_T^2}\left(\frac{d\bar{U}}{dR} + \frac{2\bar{U}}{R}\right) \tag{15}$$

$$\bar{\sigma}_{RR} = \frac{d\bar{U}}{dR} + 2\bar{\lambda}\frac{\bar{U}}{R} - \bar{Z} \tag{16}$$

$$\bar{\sigma}_{\phi\phi} = \bar{\lambda}\frac{d\bar{U}}{dR} + (\bar{\lambda} + 1)\frac{\bar{U}}{R} - \bar{Z} \tag{17}$$

Since at time t=0, the body is at rest and in an undeformed and unstressed state i.e.initially the displacement component along with their derivative with respect to t are zero and maintained at the reference temperature T_0, so the following initial conditions hold.

$$u(r,0) = \frac{\partial u(r,0)}{\partial t} = 0\ ;\quad T(r,0) = T_0\ ;\quad \frac{\partial T(r,0)}{\partial t} = 0 \tag{18}$$

Using the equation (7),we get-

$$U(R,0) = \frac{\partial U(R,0)}{\partial \eta} = 0\ ;$$
$$Z(R,0) = \frac{\partial Z(R,0)}{\partial \eta} = 0\ ;\quad Z(R,0) = 1 \tag{19}$$

Introducing the operator $L \equiv \frac{d^2}{dR^2} + \frac{2}{R}\frac{d}{dR} - \frac{2}{R^2}$ and as in Das and Bhakta [26] equations (14) and (15) can be written as

$$L\underline{V} = \underline{A}\ \underline{V} \tag{20}$$

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Where

$$\underline{V} = \begin{bmatrix} \overline{U} & \frac{d\bar{Z}}{dR} \end{bmatrix}^T \tag{21}$$

And

$$\underline{A} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{22}$$

$$C_{=}p^2, C_{12} = 1, C_{21} = \frac{p^4\bar{\varepsilon}}{c_T^2}, C_{22} = \frac{p^2(1+\varepsilon)}{c_T^2} \tag{23}$$

Solution of the Vector-matrix Differential Equation

For the solution of the Vector-matrix differential equation(20), we apply the method of eigenvalue approach as in Das and Bhakta [26]
We substitute

$$\underline{V}(R) = \underline{X}(\lambda)\omega(R,\gamma) \tag{24}$$

Where λ is a scalar, $\underline{X}$ is a vector independent of R and $\omega(R,\gamma)$ is a non-trivial solution of the scalar differential equation:

$$L\omega = 0 \tag{25}$$

The solution of the above equation can be written as

$$\omega = \frac{1}{R^2}e^{-\gamma R} + \frac{\gamma}{R^2}e^{-\gamma R} \tag{26}$$

Using equations (24) and (25) in (20), we obtain

$$\underline{A}\ \underline{X}(\lambda) = \lambda \underline{X}(\lambda) : \lambda = \gamma^2 \tag{27}$$

The characteristic equation corresponding to the matrix $\underline{A}$ can be written as

$$\gamma^4 - \gamma^2(C_{11} + C_{22}) + (C_{11}C_{22} - C_{12}C_{21}) = 0 \tag{28}$$

The root(eigenvalues) of the characteristic equation (28) are of the form $\gamma = \gamma_1^2$ and $\gamma = \gamma_2^2$ where

$$\gamma_1^2 + \gamma_2^2 = C_{11} + C_{22} \quad and \quad \gamma_1^2\gamma_2^2 = C_{11}C_{22} - C_{12}C_{21} \tag{29}$$

The eigenvectors are $\underline{X}(\gamma_j^2)$ corresponding to the eigenvalues γ_j^2 can be calculated as where $j = 1, 2$.

$$\underline{X}_j(\gamma_j^2) = \begin{bmatrix} X_1(\gamma_j^2) \\ X_2(\gamma_j^2) \end{bmatrix} = \begin{bmatrix} -C_{12} \\ C_{11} - \gamma_j^2 \end{bmatrix}_{j=1,2} \tag{30}$$

The solution of equation (20) as in Das and Bhakta [26] can be written as

$$\begin{aligned} \underline{V}(R,p) &= A\underline{X}(\gamma_1^2)(\frac{1}{R^2}e^{-\gamma_1 R} + \frac{\gamma_1}{R^2}e^{-\gamma_1 R}) \\ &+ B\underline{X}(\gamma_2^2)(\frac{1}{R^2}e^{-\gamma_2 R} + \frac{\gamma_2}{R^2}e^{-\gamma_2 R}) \end{aligned} \tag{31}$$

The components of the space vector $\underline{V}(R,p)$ in (20) can be written as

$$\begin{aligned} \overline{U}(R,p) &= -AC_{12}(\frac{1}{R^2}e^{-\gamma_1 R} + \frac{\gamma_1}{R^2}e^{-\gamma_1 R}) \\ &- BC_{12}(\frac{1}{R^2}e^{-\gamma_2 R} + \frac{\gamma_2}{R^2}e^{-\gamma_2 R}) \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{d\overline{Z}}{dR} &= A(C_{11} - \gamma_1^2)(\frac{1}{R^2}e^{-\gamma_1 R} + \frac{\gamma_1}{R^2}e^{-\gamma_1 R}) \\ &+ B(C_{11} - \gamma_1^2)(\frac{1}{R^2}e^{-\gamma_2 R} + \frac{\gamma_2}{R^2}e^{-\gamma_2 R}) \end{aligned} \tag{33}$$

$$\overline{Z} = -A(C_{11} - \gamma_1^2)\frac{e^{-\gamma_1 R}}{R} - B(C_{11} - \gamma_1^2)\frac{e^{-\gamma_2 R}}{R} \tag{34}$$

Where A and B are constants which are to be determined from the boundary conditions.
Using (32) and (34) to the equations (16) and (17), we now get

$$\begin{aligned} \overline{\sigma}_{RR} = A[&\frac{C_{12}}{R}(2e^{-\gamma_1 R} + \frac{\gamma_1}{R}e^{-\gamma_1 R} + 2\gamma_1 e^{-\gamma_1 R} + \frac{\gamma_1^2}{R}e^{-\gamma_1 R}) \\ &- \frac{2\overline{\lambda}C_{12}}{R^3}(e^{-\gamma_1 R} + \gamma_1 e^{-\gamma_1 R}) + (C_{11} - \gamma_1^2)\frac{e^{-\gamma_1 R}}{R}] \\ +B[&\frac{C_{12}}{R}(2e^{-\gamma_2 R} + \frac{\gamma_2}{R}e^{-\gamma_2 R} + 2\gamma_2 e^{-\gamma_2 R} + \frac{\gamma_2^2}{R}e^{-\gamma_2 R}) \\ &- \frac{2\overline{\lambda}C_{12}}{R^3}(e^{-\gamma_2 R} + \gamma_2 e^{-\gamma_2 R}) + (C_{11} - \gamma_1^2)\frac{e^{-\gamma_2 R}}{R}] \end{aligned}$$

$$\begin{aligned} \overline{\sigma}_{\phi\phi} = A[&\frac{\overline{\lambda}C_{12}}{R}(2e^{-\gamma_1 R} + \frac{\gamma_1}{R}e^{-\gamma_1 R} + 2\gamma_1 e^{-\gamma_1 R} + \frac{\gamma_1^2}{R}e^{-\gamma_1 R}) \\ &- \frac{(\overline{\lambda}+1)C_{12}}{R^3}(e^{-\gamma_1 R} + \gamma_1 e^{-\gamma_1 R}) + (C_{11} - \gamma_1^2)\frac{e^{-\gamma_1 R}}{R} \\ +B[&\frac{\overline{\lambda}C_{12}}{R}(2e^{-\gamma_2 R} + \frac{\gamma_2}{R}e^{-\gamma_2 R} + 2\gamma_2 e^{-\gamma_2 R} + \frac{\gamma_2^2}{R}e^{-\gamma_2 R}) \\ &- \frac{(\overline{\lambda}+1)C_{12}}{R^3}(e^{-\gamma_2 R} + \gamma_2 e^{-\gamma_2 R}) + (C_{11} - \gamma_1^2)\frac{e^{-\gamma_2 R}}{R}] \end{aligned}$$

4 Boundary Conditions

The constants A and B are to be determined from three different cases of boundary conditions.
Case - I
Considering the thermoelastic interactions when the surface of the cavity is stress-free and kept at a temperature $F(t)$, then the boundary condition takes the form

$$\begin{aligned} &(a)\sigma_{rr}(r,t) = 0, \quad at \quad r = a \\ &(b)T(r,t) = F(t), \quad at \quad r = a \end{aligned} \tag{35}$$

$$where \quad F(t) = T_0 h_L(t) \tag{36}$$

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where $h_L(t) = H(t) - H(t - L)$.Thus,F(t) is a thermal shock acting only for a finite period of time equal to L.
Case - II
Now considering the surface of the cavity is assumed to be stress-free and subjected to a ramp-type increase in heating, then

$$\begin{aligned}(a)\sigma_{rr}(r,t) = 0, \quad at \quad r = a \\ (b)T(r,t) = T_0 h(t), \quad at \quad r = a\end{aligned} \qquad (37)$$

$$where \quad h(t) = \begin{cases} 0 & ,t \le 0 \\ \frac{t}{t_0} & ,0 < t \le \eta_0 \\ 1 & ,t \ge \eta_0 \end{cases} \qquad (38)$$

Where $\eta_0 \ge 0$ is a fixed moment of time to raise the steady temperature T_0.
Case - III
In this case the cavity surface is assumed to maintain the constant reference temperature T_0 and is subjected to a ramp-type increase in boundary load, i.e.

$$\begin{aligned}(a) \quad \sigma_{rr}(r,t) = -\sigma_0 h(t), \quad at \quad r = a \\ (b) \quad T(r,t) = 0, \quad at \quad r = a\end{aligned} \qquad (39)$$

where σ_0 is a constant load.
Using equation (7) and (13), we get transformed boundary conditions as follows-
From equation (35): (Case - I)

$$\begin{aligned}\overline{\sigma}_{RR}(R,p) = 0, \quad at \quad R = 1 \\ \overline{Z}(R,p) = \overline{F}(p), \quad at \quad R = 1\end{aligned} \qquad (40)$$

$$where \quad \overline{F}(p) = \frac{T_0(1 - e^{-p^L})}{p} \qquad (41)$$

From equation (37) : (Case - II)

$$\begin{aligned}\overline{\sigma}_{RR}(R,p) = 0, \quad at \quad R = 1 \\ \overline{Z}(R,p) = T_0\overline{h}(p), \quad at \quad R = 1\end{aligned} \qquad (42)$$

$$where \quad \overline{h}(p) = \frac{T_0(1 - e^{-p\eta_0})}{\eta_0 p^2} \qquad (43)$$

From equation (39) : (Case - III)

$$\begin{aligned}\overline{\sigma}_{RR}(R,p) = -\sigma_0\overline{h}(p), \quad at \quad R = 1 \\ \overline{Z}(R,p) = 0, \quad at \quad R = 1\end{aligned} \qquad (44)$$

Using the equations (34), $\overline{\sigma}_{RR}$ and (40),(42),(44) we get the arbitrary constants A and B as follows -

With the help of the boundary condition (40), the constants A and B can be determined as -

$$A = -\frac{GK_2}{(K_2K_3 - K_1K_4)} \; and \; B = -\frac{GK_1}{(-K_2K_3 + K_1K_4)}$$

By the boundary condition (42) -

$$A = -\frac{MK_2}{(K_2K_3 - K_1K_4)} \; and \; B = -\frac{MK_1}{(-K_2K_3 + K_1K_4)}$$

By the boundary condition (44) -

$$A = -\frac{QK_4}{(-K_2K_3 + K_1K_4)} \quad and \quad B = -\frac{QK_3}{(K_2K_3 - K_1K_4)}$$

Where,

$$G = \frac{T_0(1 - e^{-p^L})}{p}, M = \frac{T_0(1 - e^{-p\eta_0})}{\eta_0 p^2}, Q = \frac{(1 - e^{-p\eta_0})}{\eta_0 p^2}$$
$$\begin{aligned}K_1 = C_{12}[(2e^{-\gamma_1} + \gamma_1 e^{-\gamma_1} + 2\gamma_1 e^{-\gamma_1} + \gamma_1^2 e^{-\gamma_1}) \\ -2\overline{\lambda}(e^{-\gamma_1} + \gamma_1 e^{-\gamma_1})] + (C_{11} - \gamma_1^2)e^{-\gamma_1} \\ K_2 = C_{12}[(2e^{-\gamma_2} + \gamma_2 e^{-\gamma_2} + 2\gamma_2 e^{-\gamma_2} + \gamma_2^2 e^{-\gamma_2})] \\ -2\overline{\lambda}(e^{-\gamma_2} + \gamma_2 e^{-\gamma_2}) + (C_{11} - \gamma_1^2)e^{-\gamma_2} \\ K_3 = (C_{11} - \gamma_1^2)e^{-\gamma_1} \\ K_4 = (C_{11} - \gamma_1^2)e^{-\gamma_2}\end{aligned} \qquad (45)$$

5 Numerical Solution

The Laplace inversion of the expressions for the displacement, temperature and stresses in space-time domain are very complex and we prefer to develop an efficient computer programme for the inversion of these integral transforms. For this inversion of Laplace transform we follow the method of Bellman, Kalaba and Lockett [32] and choose seven values of the time $t = t_i : i = 1,2,3,4,5,6,7$ as the time range at which the displacement, temperature and stresses are to be determined where t_i are the roots of the Legendre polynomial of degree seven.
With an aim to illustrate the problem, we will present some numerical results. For this purpose, numerical computation is carried out for the material Aluminum-epoxy composite, which has the following data given as [33], [34]
$\varepsilon = 0.073$
$\lambda = 7.59 \times 10^{11} \; DyneCm^{-2}$
$\mu = 1.89 \times 10^{11} \; DyneCm^{-2}$
$\rho = 2.19 \; GmCm^{-2}$
$C_e = 0.23 \; Cal/^0C$
$K = 0.6 \times 10^{-2} \; Cal/CmS^0C$
$C_T = 0.5$
$p_0 = 1$
$T_0 = 298^0C$

Concluding Remarks

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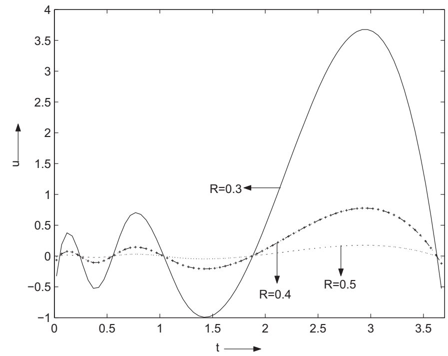

Figure 1: CaseI:Distribution of displacement vs.time

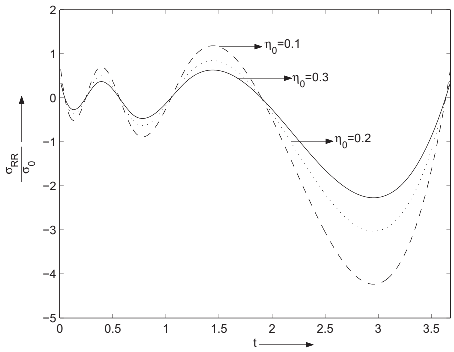

Figure 2: CaseII:Distribution of stress vs. time

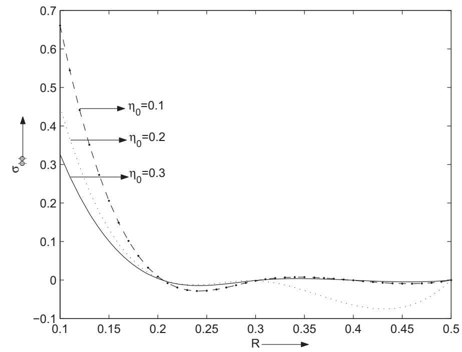

Figure 3: CaseIII :Distribution of stress vs.R

In order to study the characteristic of displacement, temperature and stresses, we have drawn several graphs for different values of the space variable and at times $t_1 = 0.025775, t_2 = 0.138382, t_3 = 0.352509, t_4 = 0.693147, t_5 = 1.21376, t_6 = 2.04612, t_7 = 3.67119$.
Case - I
1. Fig.(1)exhibits the variation of displacement with time for fixed values of R, we observe that-
(i) the absolute values of displacement, stresses and temperature gradually
increase with greater wave length as t increases.
(ii) for fixed values of time t the absolute values of displacement, stresses and temperature decrease as R increase.
Case - II
2. Fig.(2) exhibits the variation of stress σ_{RR} with time for fixed value of R=0.2, we observe that -
(i) the absolute values of displacement, stresses and temperature decrease
with an increase in η_0.
(ii) the absolute values of displacement, stresses and temperature are
maximum at t=3.0.
(iii) from fig.(8), it is clear that radial stress is compressive for $t > 2$.
Case - III
3. Fig.(3)exhibits the variation of stress $\sigma_{\phi\phi}$ with R for fixed values of times, we observe that -
(i) the absolute values of displacement, stresses and temperature
decreases with an increase in η_0.
(ii) the absolute values of displacement, stresses and temperature
decreases with radial co-ordinate and vanishes for all times and η_0.
(iii) the radial stress is compressive.

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