Flow and Heat Transfer In a Power-law Fluid with Variable Conductivity over a Stretching Sheet

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Abstract—**The steady, laminar incompressible MHD stagnation-point flows and heat transfer with variable conductivity of a Non-Newtonian Fluid over a stretching sheets are analyzed for three cases of heating conditions, namely, (i)the sheet with the constant surface temperature; (ii) the sheet with the prescribed surface temperature; (iii) surface temperature with the prescribed surface heat flux. The governing system of partial differential equations is first transformed into a system of dimensionless ordinary differential equations. The numerical solutions are presented to illustrate the influence of the various values of the ratio of free stream velocity and stretching velocity, the magnetic field parameter, Prandtl number, the wall temperature exponent and the power-law index. These effects of the different parameters on the velocity and temperature as well as the skin friction and wall heat transfer are presented in tables and graphically. The results are found to be in good agrement with those of earlier investigations reported in existing scientific literatures.**

***Keywords:** Non-Newtonian power-law fluid, Stagnation point, Stretching sheet, Surface heat flux*

1 Introduction

Flow of an incompressible viscous fluid and heat transfer phenomena over a stretching sheet have received great attention during the last decades owing to the abundance of practical applications in chemical and manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous casting of metals, wire drawing and glass blowing. Since the pioneering work of Sakiadis[1], various aspects of the problem have been investigated by many authors. Crane[2] studied a steady flow past a stretching sheet and presented a closed form solution to it. Following them, Gupta[3] examined the heat and mass transfer using a similarity transformation for the boundary layer flow over a stretching sheet subject to suction or blowing while Mahapatra and Gupta[4]studied the heat transfer in stagnation-point flow towards a stretching sheet. Very recently, Layek et al.[5]investigated the stagnation-point flow of an incompressible viscous fluid towards a porous stretching surface embedded in a porous medium subject to suction/blowing with internal heat generation or absorption. However, above researches were restricted to flows of Newtonian fluids.

It's worth mentions here that a number of industrially important fluids such as molten plastics, polymers, pulps, foods and slurries exhibit Non-Newtonian fluid behavior. Because of the growing use of the Non-Newtonian fluid in various manufacturing and processing industries, the study of non-Newtonian liquid films are important. The theoretical analysis of an external boundary layer flow of a non-Newtonian fluid was first performed by Schowalter[6]. Zheng et al.[7] investigated the flow in a power-law fluid over a flat plate moving at constant speed in the direction and opposite to the direction of the main stream. The heat transfer aspect of such problems had been considered recently by Chen[8]. Andersson et al.[9] had further investigated the magnetohydrodynamic flow over a stretching sheet of an electrically conducting incompressible fluid obeying the power-law model. Recently, Liao[10]–[11] obtained an accurate analytic solution of unsteady magnetohydrodynamic flows of non-Newtonian fluids caused by an impulsively stretching plate.

In this paper, we investigate the similarity solutions for the steady laminar incompressible MHD stagnation-point flows and heat transfer with variable conductivity of a Non-Newtonian Fluid subject to a transverse uniform magnetic field over a stretching sheets for three cases of heating conditions, namely, (i)the sheet with the constant surface temperature; (ii) the sheet with the prescribed surface temperature and (iii) surface temperature with the prescribed surface heat flux. The governing equations are transformed into nonlinear ordinary differential equations using appropriate transformations, and then solved by the numerical method. Numerical results for the dimensionless velocity profiles, the temperature profiles, the local friction coefficient and the local Nusselt number are presented for the various values of the ratio of free stream velocity and stretching velocity, the magnetic

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field parameter, Prandtl number, the wall temperature exponent and the power-law index.

2 Flow analysis

Consider the steady MHD flow of a non-Newtonian power-law fluid near the stagnation point of a flat sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. x and y are the Cartesian coordinates with the origin at the stagnation point along and normal to the plate, respectively. Two equal and opposite forces are applied along the x-axis so that the local tangential velocity is $u_w = bx$, where b is a positive constant. It is also assumed that the ambient fluid is moved with a velocity $u_e = cx$, where $c > 0$ is a constant. In this coordinate system, the steady appropriate boundary-layer for two-dimensional MHD flow of a power-law fluid is described by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial y}\right) + \frac{\sigma B_0^2}{\rho}(u_e - u), \tag{2}$$

$$u(x,0) = bx, \quad v(x,0) = 0, \quad at \quad y = 0, \tag{3}$$

$$u(x,\infty) = u_e = cx, \quad as \quad y \to \infty. \tag{4}$$

Where u and v are the velocity components along the x-axes and y-axes, respectively. ρ is the density of the fluid, and τ_{xy} is the shear stress. σ is the electric conductivity, B_0 is the uniform magnetic field along the y-axis. The non-Newtonian fluid model used in this study is the power law model with the parameters defined by

$$\tau_{xy} = k|\frac{\partial u}{\partial y}|^{n-1}\frac{\partial u}{\partial y}. \tag{5}$$

Where k is called consistency coefficient and $v = \frac{k}{\rho}|\frac{\partial u}{\partial y}|^{n-1}$is the kinematic viscosity. n is the power-law index. Near the sheet, we assume that the flow field is given by the stream function and similarity variable t as:

$$\Psi = x^{\frac{2n}{n+1}}f(t), t = x^{\frac{1-n}{1+n}}y, u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x}. \tag{6}$$

Further, introducing the following dimensionless quantities and transformations

$$\eta = tb^{\frac{2-n}{n+1}}(n\nu_0)^{\frac{-1}{n+1}}, \quad g(\eta) = \frac{\sqrt[n+1]{b^{1-2n}}}{\sqrt[n+1]{n\nu_0}}f(t). \tag{7}$$

Eq.(2) reduces to

$$|g''|^{n-1}g''' + \frac{2n}{n+1}gg'' - (g')^2 - M^2\left(g' - \frac{c}{b}\right) + \left(\frac{c}{b}\right)^2 = 0. \tag{8}$$

The boundary conditions (3–4) may be expressed in dimensionless form as

$$g(0) = 0, \quad g'(0) = 1, \quad g'(+\infty) = \frac{c}{b}. \tag{9}$$

Where $M^2 = \frac{\sigma B_0^2}{b\rho}$ is the Hartman number, $d = \frac{c}{b}$ is velocity ratio parameter.

3 Heat transfer analysis

The energy equation for the above two-dimensional flow may be written

$$\rho c_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \frac{\partial}{\partial y}(\lambda(x)\frac{\partial T}{\partial y}). \tag{10}$$

Where c_p is the specific heat capacity and $\lambda(x)$ is the thermal conductivity which is assumed to be variable here.

3.1 Constant surface temperature

In this circumstance, the boundary conditions are

$$T(x,0) = T_w, \quad T(x,\infty) = T_\infty, \tag{11}$$

where T_w is the wall temperature, T_∞ is the temperature of the fluid far from the sheet.

Introducing the following dimensionless quantities

$$\theta_1 = \frac{T - T_\infty}{T_w - T_\infty}. \tag{12}$$

On using (6-7) and (12), the governing problem (10) is transformed to

$$\lambda(x)\theta_1'' x^{\frac{2-2n}{1+n}} b^{\frac{3-3n}{n+1}} n^{\frac{-2}{n+1}} \nu_0^{\frac{-2}{n+1}} + \frac{2n}{n+1}g\theta_1' = 0. \tag{13}$$

If $\lambda(x) = k_0 x^{\frac{2n-2}{1+n}}$, Eq.(13) can be transformed to the ordinary differential equation, where k_0 is constant. For conciseness, define the variable

$$\lambda(x) = \lambda_0 x^{\frac{2n-2}{1+n}} b^{\frac{3n-3}{n+1}} n^{\frac{2}{n+1}} \nu_0^{\frac{1-n}{n+1}} \rho c_p,$$

Eq.(13) reduces to

$$\theta_1'' + Pr\frac{2n}{n+1}g\theta_1' = 0, \tag{14}$$

with boundary conditions

$$\theta_1(0) = 1, \quad \theta_1(+\infty) = 0, \tag{15}$$

where $Pr = \nu_0/\lambda_0$.

3.2 Prescribed surface temperature

Here, the boundary conditions are

$$T(x,0) = T_w = T_\infty + G(x), \quad T(x,\infty) = T_\infty. \tag{16}$$

Where T_w is the wall temperature which is assumed to be variable .

Introducing the following dimensionless quantities

$$\theta_2 = \frac{T - T_\infty}{T_w - T_\infty}. \tag{17}$$

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On using (6-7) and (17), Eq.(10) can be written

$$\begin{aligned}&\lambda(x)G(x)\theta_2''x^{\frac{2-2n}{1+n}}b^{\frac{4-2n}{n+1}}n^{\frac{-2}{n+1}}\nu_0^{\frac{-2}{n+1}}\\&-b\rho c_p\left(xg'\theta_2G'(x)-\tfrac{2n}{n+1}g\theta_2'G(x)\right)=0.\end{aligned}\tag{18}$$

This equation cannot be transformed to the ordinary differential equation for arbitrary η unless $\lambda(x)G(x)x^{\frac{2-2n}{1+n}}/xG'(x) = const$, $xG'(x)/G(x) = const$, $\lambda(x)G(x)x^{\frac{2-2n}{1+n}}/G(x) = const$. Hence the variable $G(x) = Cx^s$ and $\lambda(x) = k_0x^{\frac{2n-2}{1+n}}$, where the wall temperature exponent s are constants.

For conciseness, define the variables

$$\lambda(x) = \lambda_0 x^{\frac{2n-2}{1+n}}b^{\frac{n-2}{n+1}}n^{\frac{2}{n+1}}\nu_0^{\frac{1-n}{1+n}}\rho c_p,$$

Eq.(18) reduces to

$$\theta_2'' + \frac{2n}{n+1}Prg\theta_2' - sPr\theta_2 g' = 0 \tag{19}$$

with boundary conditions

$$\theta_2(0) = 1, \quad \theta_2(+\infty) = 0. \tag{20}$$

3.3 Prescribed heat flow

In the PHF case, the boundary conditions are

$$q_w = -\lambda(x)\frac{\partial T}{\partial y} = Dx^m, \quad T(x,\infty) = T_\infty. \tag{21}$$

Where D and the surface heat flux exponent m are constants, q_w is the wall heat flux.

For the prescribed surface heat flux case, the dimensionless temperature is defined as

$$T - T_\infty = D\frac{Pr}{\rho c_p \sqrt[n+1]{n\nu_0}}\sqrt[n+1]{x^{m+1+mn-n}}\theta_3(\eta). \tag{22}$$

On using (6-7) and (22), Eq.(10) can be written

$$\begin{aligned}&\lambda(x)\theta_3''x^{\frac{2-2n}{1+n}}b^{\frac{4-2n}{n+1}}n^{\frac{-2}{n+1}}\nu_0^{\frac{-2}{n+1}}\\&-b\rho c_p\left(\tfrac{m-n+1+mn}{1+n}xg'\theta_3-\tfrac{2n}{n+1}g\theta_3'\right)=0\end{aligned}\tag{23}$$

if $\lambda(x) = k_0x^{\frac{2n-2}{1+n}}$, Eq.(23) can be transformed to the ordinary differential equation, where k_0 is constant. For conciseness, define the variables

$$\lambda(x) = \lambda_0 x^{\frac{2n-2}{1+n}}b^{\frac{n-2}{n+1}}n^{\frac{2}{n+1}}\nu_0^{\frac{1-n}{1+n}}\rho c_p.$$

Eq.(30) and the boundary conditions of (21) are transformed to

$$\theta_3'' + \frac{2n}{n+1}Prg\theta_3' - \frac{m-n+1+mn}{1+n}Pr\theta_3 g' = 0, \tag{24}$$

$$\theta_3'(0) = 1, \quad \theta_3(+\infty) = 0. \tag{25}$$

Table 1: Comparison of $f''(0)$ for several values of d with $n = 1$ and $M = 0.0$

d	Layek et al.[5]	Gupta[4]	Present study
0.1	-0.9601	-0.9694	-0.96944
0.2		-0.9181	-0.91812
0.5	-0.6499	-0.6673	-0.66727
0.8			-0.29939
2.0	1.9991	2.0175	2.01747
3.0	4.5011	4.7293	4.72923
5.0			11.75177

Table 2: Values of $f''(0)$ for several values of d, n and M

n	M	$d = 0.1$	$d = 0.8$	$d = 1.5$	$d = 3.0$
0.5	0.0	-0.72022	-0.15456	1.04129	6.22805
0.5	0.5	-0.83211	-0.16668	1.06949	6.42112
0.5	1.0	-1.13807	-0.20151	1.14846	7.00117
0.5	2.0	-2.09708	-0.31897	1.19967	9.10889
0.8	0.0	-0.88311	-0.24210	0.83329	5.21930
0.8	0.5	-0.98919	-0.25663	0.86846	5.34991
0.8	1.0	-1.26500	-0.29815	0.96791	5.72804
1.5	0.0	-1.11815	-0.43016	0.68395	3.87743
1.5	0.5	-1.19727	-0.44701	0.71989	3.94097
1.5	1.0	-1.39935	-0.49276	0.82776	4.12362

4 Results and discussion

The systems of ordinary differential equations (8),(14),(19) and (24) with the aforementioned boundary conditions equations are solved numerically using means of the fourth–order Runge–Kutta method and shooting techniques until the free stream conditions are identically satisfied. In order to verify the accuracy of our present method, we have compared our results with those of Layek et al.[5] and Gupta[4]. The comparisons in all the above cases are found to be in good agreement, as shown in Table 1.

Figs.1-2 reveal the influence of the parameter d, n on the horizontal velocity profiles $g'(\eta)$ for flows of the Non-Newtonian fluids. Fig.1 shows the velocity profiles $g'(\eta)$ for different values of the power-law index n with $d = 1.5$ and $M = 0.5$. It is observed that the velocity profiles $g'(\eta)$ change dramatically as n is varied. Fig.2 gives the effects of d on the velocity profiles $g'(\eta)$ with $M = 1.0, n = 0.5$. It is noted that the variation of $g'(\eta)$ depends on the ratio $d = c/b$ of the velocity of the stretching surface to that of the frictionless potential flow in the neighborhood of the stagnation point and increases with increasing of parameter $d = c/b$. It is also observed that the flow has a boundary layer structure when $c/b > 1$ while an inverted boundary layer is formed for $c/b < 1$. Further the thickness of the boundary layer decreases with increasing of c/b.

Figs.3-6 have been made in order to see the effects of

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Pr, n and s, m on the dimensionless temperature distributions $\theta_1(\eta), \theta_2(\eta)$ and $\theta_3(\eta)$. In the CST case we plot the dimensionless temperature profile $\theta_1(\eta)$ with $M = 0.0, n = 0.5, d = 1.5$, as shown in Fig.3 for the various values of Pr. It can be clearly seen that the variation of $\theta_1(\eta)$ decreases as Pr increases. As anticipated, the thermal boundary layer thickness decreases with increasing Prandtl number. In the PST case we plot the dimensionless temperature profile $\theta_2(\eta)$, as shown in Figs.4-5. It is noted that the variations of $\theta_2(\eta)$ are decreasing as s and n, respectively, increases. Fig.6 shows the fluid wall temperature $\theta_3(\eta)$increases with increasing Prandtl number.

Finally, we compute the dimensionless shear stress at the wall and the wall heat flux. The computed variation of $f''(0)$ with n, M and d has been summarized in Tables 2. It shows that the dimensionless shear stress $f''(0)$ increases as d increases with all other parameter fixed. On the other hand, the variation of $|f''(0)|$ increases with increasing the magnetic field parameter. Also, the wall temperature $|\theta_2'(0)|$ in PST case is increased as a result of the applied magnetic field as shown in Table.3. The effect of the power-law index is found to decrease both the dimensionless shear stress $f''(0)$ and the wall temperature $\theta_2(0)$ in CST case.

5 Conclusions

In this paper, we investigate the similarity solutions for the steady laminar incompressible MHD stagnation-point flows and heat transfer with variable conductivity of a Non–Newtonian Fluid subject to a transverse uniform magnetic field over a stretching sheets for three cases of heating conditions, namely, (i)the sheet with the constant surface temperature; (ii) the sheet with the prescribed surface temperature; (iii) surface temperature with the prescribed surface heat flux. The following observations have been made from the present analysis.

(1) The flows of Non-Newtonian fluids have a boundary layer structure when $c/b > 1$ while an inverted boundary layer is formed for $c/b < 1$. As expected, boundary layer thickness decreases by increasing c/b.
(2) The horizontal velocity profile $g'(\eta)$ increases with increasing of parameter $d = c/b, n, M$.
(3) The temperature profiles $\theta_1(\eta)$ for the CST case and $\theta_2(\eta)$ for the PST case decrease as Pr increases. However, the temperature profiles $\theta_3(\eta)$ for the PHF case increases with increasing Prandtl number.
(4) The variation of $\theta_2(\eta)$ is decreasing for Pr, s and n.
(5) The magnitude of the wall shear stress $f''(0)$ increases as d increases. The variations of $|f''(0)|$ and $|\theta_2'(0)|$ in CST case increase with increasing the magnetic field parameter. The effect of the power-law index is found to decrease both the dimensionless shear stress $f''(0)$ and the wall temperature $\theta_2'(0)$ for PST case. Also, the fluid wall temperature $\theta_3'(0)$ increases with increasing Prandtl number in PHF case.

Table 3: Values of $\theta_2'(0)$ for several values of Pr, n and M with $d = 1.5, s = 0.5$

n	M	$Pr = 0.1$	$Pr = 0.7$	$Pr = 3.5$	$d = 6.7$
0.5	0.0	-0.34209	-0.86511	-1.86734	-2.55372
0.5	0.5	-0.34274	-0.86567	-1.87056	-2.56021
0.5	1.0	-0.34579	-0.87337	-1.88359	-2.56892
1.5	0.0	-0.41073	-1.04001	-2.22557	-3.03578
1.5	0.5	-0.41097	-1.04107	-2.23129	-3.03985
1.5	1.0	-0.41157	-1.04473	-2.23821	-3.04842

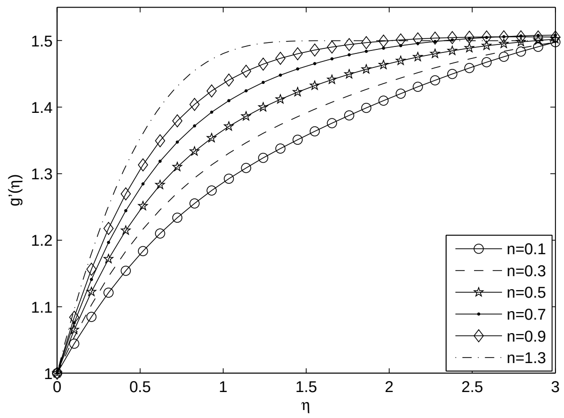

Fig.1 Effect of power-law index for variation of $g'(\eta)$ with $d = 1.5$ and $M = 0.5$.

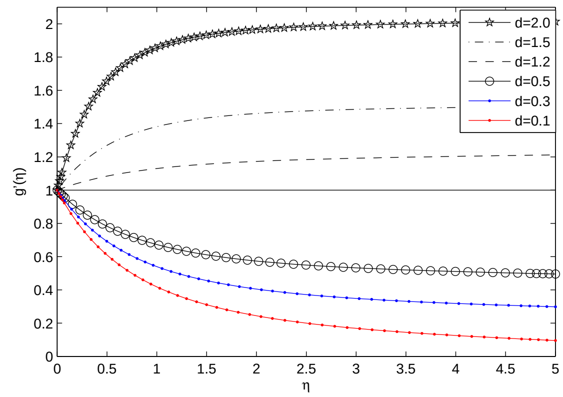

Fig.2 Velocity profiles $g'(\eta)$ of flow for different values of d with $M = 1.0$ and $n = 0.5$.

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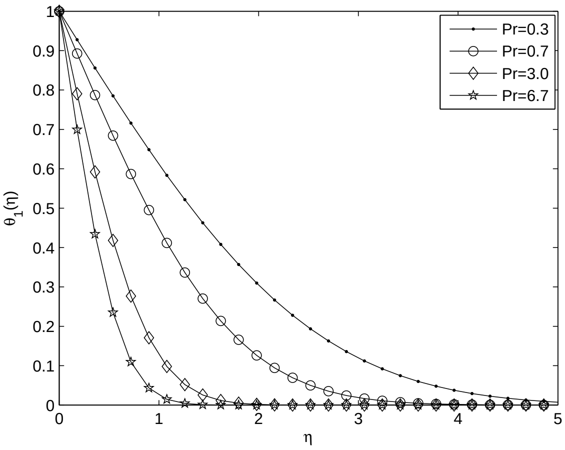

Fig.3 Effect of Prandtl number for variation of $\theta_1(\eta)$ with $M = 0.0$ and $n = 0.5$,$d = 1.5$.

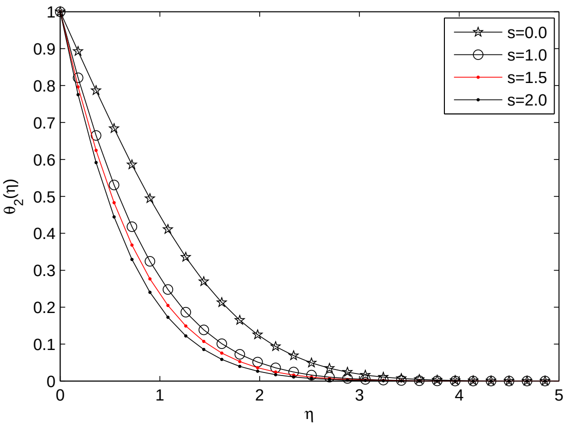

Fig.4 Effect of the wall temperature exponent for variation of $\theta_2(\eta)$ with $M = 0.0, Pr = 0.7$ and $n = 0.5$,$d = 1.5$.

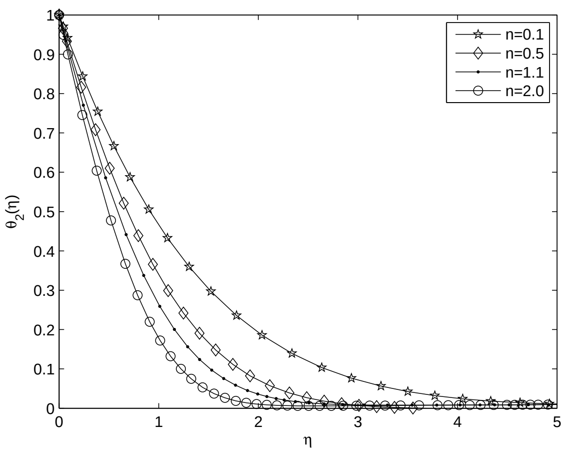

Fig.5 Effect of power-law index for variation of $\theta_2(\eta)$ with $M = 0.5, Pr = 0.7$ and $s = 0.5$,$d = 1.5$.

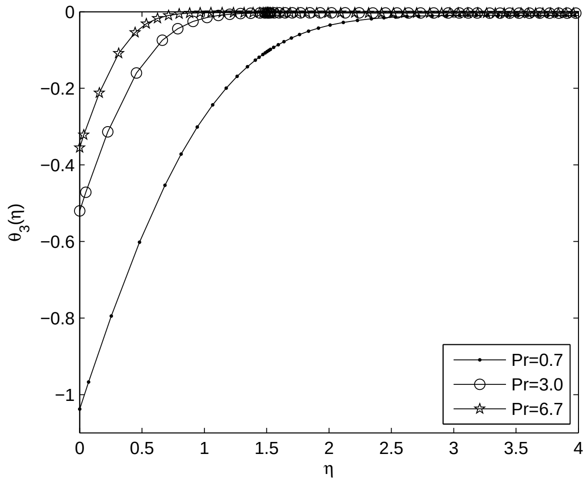

Fig.6 Effect of Prandtl number for variation of $\theta_3(\eta)$ with $M = 0.5$, $m = 0.5$,$d = 1.5$, and $n = 0.5$.

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