An Iteration Process For Two Finite Families Of Mappings

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Abstract—**An iteration process is introduced and a necesary and sufficient condition is given to approximate common fixed points of two finite families of continuous pseudocontractive mappings defined on a nonempty closed convex subset of a real Banach space. Also, strong and weak convergence theorems for two finite families of strictly pseudocontractive mappings of Browder-Petryshyn type are obtained. The results presented extend and improve the corresponding results in literature.**

Keywords: ***Continuous pseudocontractive mappings, Strictly pseudocontractive mappings, Iteration process, Strong and weak convergence, Common fixed points***

1 Introduction and preliminaries

Let E be a real Banach space and J denote the normalized duality mapping from E into 2^{E^*} given by, for $\forall x \in E$,

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\| \|f\|, \|x\| = \|f\|\},$$

where E^* denotes the dual space of E and $\langle \cdot, \cdot\rangle$ the generalized duality pairing. If E^* is strictly convex, then J is single-valued. In the sequel, we denote the single-valued duality mapping by j, the set of fixed points of a mapping T by $F(T) = \{x \in E : Tx = x\}$, the identity mapping by I and the set of natural numbers by $\mathbb{N}$.

Definition 1 [2, 7] *(i) A mapping T with domain $D(T)$ and range $R(T)$ in E is called strongly pseudocontractive, if for all $x, y \in D(T)$, there exists $k \in (0,1)$ and $j(x-y) \in J(x-y)$ such that*

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2. \quad (1)$$

If $k = 1$, then T is called pseudocontractive.

(ii) A mapping T with domain $D(T)$ and range $R(T)$ in E is called strictly pseudocontractive in the terminology of Browder-Petryshyn, if there exists $k \in [0,1)$ such that

$$\begin{aligned}&\langle Tx - Ty, j(x-y)\rangle \\ \leq\ & \|x-y\|^2 - k\|(x-y) - (Tx-Ty)\|^2 \quad (2)\end{aligned}$$

for all $x, y \in D(T)$ and some $j(x-y) \in J(x-y)$. It is well known that is equivalent to the following:

$$\begin{aligned}&\langle (I-T)x - (I-T)y,\ j(x-y)\rangle \\ \geq\ & k\|(I-T)x - (I-T)y\|^2 \quad (3)\end{aligned}$$

for all $k \in [0,1)$ and all $x, y \in D(T)$.

From the above definitions, every strictly pseudocontractive in the terminology of Browder-Petryshyn is a pseudocontractive mapping. However, the converse is not true, in general [12]. It is easy to see that every strictly pseudocontractive map is Lipschitzian and continuous.

The class of strictly pseudocontractive mappings has been studied by several authors (see for example, [1, 5, 7, 9, 10, 11, 12]). In 2007, G. Marino and H.K. Xu [8] studied Mann's process for fixed points of strict pseudocontractions in Hilbert space, and proved weak convergence theorem.

In 2001, H.K. Xu and R.G. Ori [13] introduced the following implicit iteration process for common fixed points of a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in Hilbert spaces:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n,\ n \in \mathbb{N} \quad (4)$$

where $T_n = T_{n \bmod N}$, and proved weak convergence theorems. Osilike [9] extended the results of Xu and Ori [13] from the class of nonexpansive mappings to the more general class of strictly pseudocontractive mappings and proved some strong and weak convergence theorems. Afterwards, Chen et al.[4] further extended the results of Osilike [9] from Hilbert spaces to Banach spaces. They investigated the process (4) for common fixed points of a finite family of continuous pseudocontractive mappings $\{T_i\}_{i=1}^N$ in Banach spaces through weak and strong convergence theorems.

Recently, Z. Li, S. He and J. Zhao [6] introduced the following iteration process:

$$x_n = \alpha_n x_{n-1} + \beta_n T_n x_{n-1} + \gamma_n T_n x_n,\ n \in \mathbb{N} \quad (5)$$

where $T_n = T_{n \bmod N}$, for common fixed points of a finite family of strictly pseudocontractive mappings $\{T_i\}_{i=1}^N$ in Banach spaces, and they proved some weak and strong

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convergence theorems. They extended a strong convergence theorem (Theorem 2.5 [4]) using semicompactness of one of the mappings and the weak convergence theorem (Theorem 2.6 [4]). Their process constitutes a generalization of (4) if $\beta_n = 0$. However, they use an extra condition $\sum_{n=1}^{\infty} \beta_n < +\infty$ on thier way to generalize the above mentioned two reults of [4]. All other theorems of [4] were left unattended as they involved continuous pseudocontractive mappings.

Now, we introduce the following iteration process for two finite families of continuous pseudocontractive mappings $\{T_i\}_{i=1}^{N}$, $\{S_i\}_{i=1}^{N}$ as follows:

$$x_n = \alpha_n x_{n-1} + \beta_n S_n x_n + \gamma_n T_n x_n,\ n \in \mathbb{N}, \tag{6}$$

where $T_n = T_{n \bmod N}$ and $S_n = S_{n \bmod N}$, $N \in \mathbb{N}$. Note that (6) reduces to (4) when $S_n = T_n$ or $S_n = I$ or $T_n = I$ for all $n = 1, 2, ..., N$.

This process is well-defined. In fact, let K be a nonempty convex subset of E and T and S two continuous pseudocontractive mappings. For every $u \in K$, define an operator $W : K \to K$ by $Wx = \alpha u + \beta Sx + \gamma Tx$, $\alpha, \beta, \gamma \in (0, 1)$. Then it is easy to show that $\forall x, y \in K$, $\exists j(x - y) \in J(x - y)$ such that $\langle Wx - Wy, j(x - y)\rangle \leq (1 - \alpha)\|x - y\|^2$. Thus W is strongly pseudocontractive. Since W is also continuous, so W has unique fixed point $x^* \in K$ (see [4]), i.e., $x^* = \alpha u + \beta Sx^* + \gamma Tx^*$. Let $\{T_i\}_{i=1}^{N}$ and $\{S_i\}_{i=1}^{N}$ be two finite families of continuous pseudocontractive self-mappings of K. Thus, if $\alpha_n > 0$, the implicit iteration process (6) can be employed for the approximation of common fixed points of two finite families of continuous pseudocontractive mappings.

In this paper, by using iteration process (6), we not only extend the results of [4] (Lemma 2.1 and Theorem 2.3) left over by Li et al. [6] but also further extend the results of [6] to the case of two families of mappings. Our results also show that the condition $\sum_{n=1}^{\infty} \beta_n < +\infty$ used by Li et al. [6] is superfluous. The results of [1, 2, 3] will also be extended and improved,in turn.

Recall the following definitions. Let K be a closed subset of a real Banach space E. A mapping $T : K \to K$ is said to be semicompact, if for any bounded sequence $\{x_n\}$ in K such that $\|x_n - Tx_n\| \to 0\ (n \to \infty)$, there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ and some $x^* \in K$ such that $x_{n_i} \to x^*\ (i \to \infty)$. A Banach space E is said to satisfy Opial's condition, if whenever $\{x_n\}$ is a sequence in E which converges weakly to x, as $n \to \infty$, then $\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|$, $\forall y \in E$, $y \neq x$. A Banach space E is said to be q-uniformly smooth $(q > 1)$, if exists a constant $c > 0$, such that $\rho_E(t) \leq ct^q$, where $\rho_E(t)$ is modulus of smoothness of E defined by

$$\rho_E(t) = \sup\left\{ \begin{array}{c} \frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \\ \|x\| = 1, \|y\| = t \end{array} \right\},\ t > 0.$$

Theorem 1[11] *Let E be a real q-uniformly smooth Banach space which is also uniformly convex. Let K be a nonempty closed convex subset of E and $T : K \to K$ a strictly pseudocontractive mapping in the terminology of Browder-Petryshyn. Then $(I - T)$ is demiclosed at zero, i.e., $\{x_n\} \subset D(T)$ such that $\{x_n\}$ converges weakly to $x \in D(T)$ and $\{(I - T)x_n\}$ converges strongly to 0, then $x - Tx = 0$.*

Lemma 1 *If $J : E \to 2^{E^*}$ is a normalized duality mapping, then for all $x, y \in E$, $\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle$, $\forall j(x + y) \in J(x + y)$.*

2 Main results

In all the results to follow, we will not use the condition $\sum_{n=1}^{\infty} \beta_n < +\infty$.

Lemma 2 *Let E be a real Banach space and let K be a nonempty closed convex subset of E. Let $T_i, S_i : K \to K$, $i = 1, 2, ..., N$ be two finite families of continuous pseudocontractive mappings such that $F = \left(\cap_{i=1}^{N} F(T_i)\right) \cap \left(\cap_{i=1}^{N} F(S_i)\right) \neq \emptyset$, and $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\gamma_n\}_{n=1}^{\infty}$ be three real sequences in $(0, 1)$ satisfying $\alpha_n + \beta_n + \gamma_n = 1$. Let $x_0 \in K$ and let $\{x_n\}$ be defined by (6). Then (i) $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F$, (ii) $\lim_{n\to\infty} d(x_n, F)$ exists, where $d(x_n, F) = \inf_{p\in F} \|x_n - p\|$.*

Proof. (i) Let $p \in F$, $\forall n \in \mathbb{N}$, $\exists j(x_n - p) \in J(x_n - p)$. It follows from the definition a pseudocontractive mapping that

$$\begin{aligned} \|x_n - p\|^2 &\leq \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle \\ &\quad + \beta_n \langle S_n x_n - p, j(x_n - p)\rangle \\ &\quad + \gamma_n \langle T_n x_n - p, j(x_n - p)\rangle \\ &\leq \alpha_n \|x_{n-1} - p\| \|x_n - p\| \\ &\quad + (1 - \alpha_n)\|x_n - p\|^2 \end{aligned}$$

Since $\alpha_n > 0$ for all $n \in \mathbb{N}$, then we have

$$\|x_n - p\| \leq \|x_{n-1} - p\|. \tag{7}$$

Hence $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F$.

(ii) Taking infimum over all $p \in F$ in (7), we have $d(x_n, F) \leq d(x_{n-1}, F)$ so that $\lim_{n\to\infty} d(x_n, F)$ exists.

Theorem 2 *Let E, K, T_i, S_i, F, $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$, $\{\gamma_n\}_{n=1}^{\infty}$ and $\{x_n\}$ be as in Lemma 2. Then $\{x_n\}$ converges strongly to a point of F if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$.*

Proof. The necessity is obvious. For sufficiency, let $\liminf_{n\to\infty} d(x_n, F) = 0$. Then from (ii) in Lemma 2, $\lim_{n\to\infty} d(x_n, F) = 0$. From (7), we have $\|x_{n+m} - x_n\| \leq \|x_{n+m} - p\| + \|x_n - p\| \leq 2\|x_n - p\|$, $\forall p \in F$. Thus $\|x_{n+m} - x_n\| \leq 2d(x_n, F) \to 0\ (n \to \infty)$. Therefore $\{x_n\}$ is Cauchy sequence. Since E is complete,

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$\{x_n\}$ is convergent. Suppose $\lim_{n\to\infty} x_n = q$. Since K is closed, we get $q \in K$. Now, we prove that $q \in F$. Since $\lim_{n\to\infty} x_n = q$ and $\lim_{n\to\infty} d(x_n, F) = 0$, we have $d(q, F) = 0$. Thus $q \in F$.

Remark 1 *Theorem 2 gives a necesary and sufficient condition to approximate common fixed points of two finite families of continuous pseudocontractive mappings. It is also an extension of Theorem 2.3 of [4] obtained by putting $S_n = T_n$ or $S_n = I$ or $T_n = I$ for all $n = 1, 2, ..., N$.*

The following is an extension of Corollary 2.4 of [4] and Corollary 2.1 of [6] to the case of two finite families of mappings and without using $\sum_{n=1}^{\infty} \beta_n < +\infty$.

Corollary 1 *Let E, K, T_i, S_i, F, $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$, $\{\gamma_n\}_{n=1}^{\infty}$ and $\{x_n\}$ be as in Lemma 2. Then $\{x_n\}$ converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^{N}$ and $\{S_i\}_{i=1}^{N}$ if and only if $\{x_n\}$ has a subsequence which converges to some $q \in F$.*

We now prove some strong and weak convergence theorems for two finite families of strictly pseudocontractive mappings of Browder-Petryshyn type.

Theorem 3 *Let E be a real Banach space and let K be a nonempty closed convex subset of E. Let $T_i, S_i : K \to K$, $i = 1, 2, ..., N$ be two finite families of strictly pseudocontractive mappings in the terminology of Browder-Peryshyn such that $F = \left(\cap_{i=1}^{N} F(T_i)\right) \cap \left(\cap_{i=1}^{N} F(S_i)\right) \neq \emptyset$. Let $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\gamma_n\}_{n=1}^{\infty}$ be three real sequences in $(0,1)$ satisfying $\alpha_n+\beta_n+\gamma_n = 1$, $0 < a \le \alpha_n, \beta_n, \gamma_n \le b < 1$ where a,b are some constants. Let $x_0 \in K$ and let $\{x_n\}$ be defined by (6). Then, for $\forall l = 1, 2, ..., N$*

$$\lim_{n\to\infty} \|x_n - T_l x_n\| = \lim_{n\to\infty} \|x_n - S_l x_n\| = 0.$$

Proof. Since $T_i, S_i : K \to K$, $i = 1, 2, ..., N$ are strictly pseudocontractive mappings, $\forall x, y \in K$, $\exists\ j(x-y) \in J(x-y)$ and $k = \min_{1\le i\le N} \{k_i\} \in [0, 1)$ such that

$$\begin{aligned} &\langle (I-T_i)x - (I-T_i)y, j(x-y)\rangle \\ \ge\ & k \|(I-T_i)x - (I-T_i)y\|^2 \end{aligned} \tag{8}$$

and

$$\begin{aligned} &\langle (I-S_i)x - (I-S_i)y, j(x-y)\rangle \\ \ge\ & k \|(I-S_i)x - (I-S_i)y\|^2 . \end{aligned} \tag{9}$$

From (6), it follows that

$$\begin{aligned} x_n - x_{n-1} \ \le\ & \frac{\beta_n}{\alpha_n}(S_n x_n - x_n) \\ & + \frac{\gamma_n}{\alpha_n}(T_n x_n - x_n) \end{aligned} \tag{10}$$

and

$$\begin{aligned} & \langle x_n - x_{n-1}, j(x_n - p)\rangle \\ =\ & \frac{\beta_n}{\alpha_n}\langle S_n x_n - x_n, j(x_n - p)\rangle \\ & + \frac{\gamma_n}{\alpha_n}\langle T_n x_n - x_n, j(x_n - p)\rangle . \end{aligned} \tag{11}$$

Now from $(8) - (11)$, $\forall p \in F$, $\forall n \in \mathbb{N}$, $\exists j(x_n - p) \in J(x_n - p)$ such that

$$\begin{aligned} \|x_n - p\|^2 \ \le\ & \|x_{n-1} - p\|^2 - \frac{2\beta_n k}{\alpha_n}\|x_n - S_n x_n\|^2 \\ & - \frac{2\gamma_n k}{\alpha_n}\|x_n - T_n x_n\|^2 . \end{aligned} \tag{12}$$

Thus, from (12) and the conditions $0 < a \le \alpha_n, \beta_n, \gamma_n \le b < 1$, we obtain $\frac{2\beta_n k}{\alpha_n}\|x_n - S_n x_n\|^2 + \frac{2\gamma_n k}{\alpha_n}\|x_n - T_n x_n\|^2 \le \|x_{n-1} - p\|^2 - \|x_n - p\|^2$. Since $\lim_{n\to\infty} \|x_n - p\|$ exists, and $0 < a \le \alpha_n, \beta_n, \gamma_n \le b < 1$ imply $\frac{a}{b} \le \frac{\gamma_n}{\alpha_n}$ and $\frac{a}{b} \le \frac{\beta_n}{\alpha_n}$, therefore we have

$$\begin{aligned} \lim_{n\to\infty} \|x_n - S_n x_n\| &= 0, \\ \lim_{n\to\infty} \|x_n - T_n x_n\| &= 0. \end{aligned} \tag{13}$$

It now follows from (6) that $\lim_{n\to\infty} \|x_n - x_{n-1}\| = 0$. Thus, for any $i = 1, 2, ..., N$, $\lim_{n\to\infty} \|x_n - x_{n+i}\| = 0$. Since every strictly pseudocontractive mapping is Lipschitzian, if we choose $L = \max_{1\le i\le N} \{L_i\}$, then $\|x_n - T_{n+i} x_n\| \le (1+L)\|x_{n+i} - x_n\| + \|x_{n+i} - T_{n+i} x_{n+i}\| \to 0 \quad (n \to \infty)$ and $\lim_{n\to\infty} \|x_n - T_{n+i} x_n\| = 0$, $\forall i = 1, 2, ..., N$. Because $T_n = T_{n \bmod N}$, for any $l = 1, 2, ..., N$,

$$\lim_{n\to\infty} \|x_n - T_l x_n\| = 0. \tag{14}$$

Similarly,

$$\lim_{n\to\infty} \|x_n - S_l x_n\| = 0. \tag{15}$$

Theorem 4 *Let E, K, T_i, S_i, F, $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$, $\{\gamma_n\}_{n=1}^{\infty}$ and $\{x_n\}$ be as in Theorem 2. Suppose one of the mappings in $\{T_i\}_{i=1}^{N}$ and $\{S_i\}_{i=1}^{N}$ is semicompact. Then $\{x_n\}$ converges strongly to a common fixed point of $\{T_i\}_{i=1}^{N}$ and $\{S_i\}_{i=1}^{N}$.*

Proof. By Lemma 2 $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F$. From the semicompactness of T_l and S_l, there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to a $q \in K$. By using (14) and (15), we have $\lim_{j\to\infty} \|x_{n_j} - T_l x_{n_j}\| = \|q - T_l q\| = 0$ and $\lim_{j\to\infty} \|x_{n_j} - S_l x_{n_j}\| = \|q - S_l q\| = 0$, for all $l = 1, 2, ..., N$. This implies that $q \in F$. Since $\lim_{n\to\infty} \|x_n - q\|$ exists,therefore we obtain that $\lim_{n\to\infty} x_n = q \in F$.

Remark 2 *Theorem 2 an extension of the Theorem 2.1 of [6] to the case of two families of mappings and without*

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using $\sum_{n=1}^{\infty}\beta_n < +\infty$. *Moreover, letting* $S_n = T_n$ *or* $S_n = I$ *or* $T_n = I$ *for all* $n = 1, 2, ..., N$ *in the proof of Theorem* 2, *we obtain Theorem* 2.5 *of [4].*

We now turn our attention to weak convergence theorems.

Theorem 5 *Let* E *be a real* q*-uniformly smooth Banach space which is also uniformly convex and satisfies Opial's condition. Let* K *be a nonempty closed convex subset of* E *and* $T_i, S_i : K \to K$, $i = 1, 2, ..., N$, *be two strictly pseudocontractive mappings in the terminology of Browder-Peryshyn such that* $F = \left(\cap_{i=1}^{N} F(T_i)\right) \cap \left(\cap_{i=1}^{N} F(S_i)\right) \neq \emptyset$. *Let* $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\gamma_n\}_{n=1}^{\infty}$ *be three real sequences in* $(0, 1)$ *satisfying* $\alpha_n + \beta_n + \gamma_n = 1$, $0 < a \leq \alpha_n, \beta_n, \gamma_n \leq b < 1$ *where a,b are some constants. Let* $x_0 \in K$ *and let* $\{x_n\}$ *be defined by* (6). *Then* $\{x_n\}$ *converges weakly to a common fixed point of the mappings* $\{T_i\}_{i=1}^{N}$ *and* $\{S_i\}_{i=1}^{N}$.

Proof. From (i) in the Lemma 2, we know that $\lim_{n\to\infty}\|x_n - p\|$ exists. Thus $\{x_n\}$ is bounded. Since E is uniformly convex, $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ which converges weakly to some $q \in K$, and hence $\lim_{n\to\infty}\|x_{n_k} - T_l x_{n_k}\| = \lim_{n\to\infty}\|x_{n_k} - S_l x_{n_k}\| = 0$ by Theorem 2. It now follows from Theorem 1 that $q \in F(T_l)$ and $q \in F(S_l)$ for all $l = 1, 2, ..., N$. Hence $q \in F$. We will show that $\{x_n\}$ converges weakly to q. Suppose that $\{x_n\}$ does not converge weakly to q, then there exists another subsequence $\{x_{n_j}\}$ of $\{x_n\}$ which is weakly convergent to some $q \neq x^*$, $x^* \in K$. We can also prove in the same manner as above that $x^* \in F$. Because E satisfies Opial's condition, we get a contradiction.Thus $\{x_n\}$ converges weakly to $q \in F$.

Remark 3 T*heerem* 2 *an extension of the Theorem* 2.2 *of [6] to the case of two families of mappings and without using* $\sum_{n=1}^{\infty}\beta_n < +\infty$. *Moreover, letting* $S_n = T_n$ *or* $S_n = I$ *or* $T_n = I$ *for all* $n = 1, 2, ..., N$ *in the proof of Theorem* 2, *we obtain Theorem* 2.6 *of [4].*

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