Proceedings of the World Congress on Engineering 2010 Vol III
WCE 2010, June 30 - July 2, 2010, London, U.K.

Some Oscillation Flows of a Generalized Oldroyd-B Fluid in Uniform Magnetic Field

Yaqing Liu, Liancun Zheng, Xinxin Zhang, Fenglei Zong

***Abstract*—This paper presents an analysis for magnetohydrodynamic (MHD) flow of an incompressible generalized Oldroyd-B fluid with fractional derivative. The fractional calculus approach is introduced to establish the constitutive relationship of a viscoelastic fluid. The exact solutions for the velocity and shear stress are obtained by using the Laplace transform technique for the fractional calculus. Moreover, we analyze the characteristics of the velocity field by using the analytical solutions.**

***Index Terms*—Oldroyd-B fluid, oscillation, Laplace transform, Fox H-function.**

I. INTRODUCTION

The interest for motion problems of non-Newtonian fluids has considerably grown because of the wide range of their applications. These fluids have been modeled in a number of diverse manners with their constitutive equations varying greatly in complexity. Among them the Oldroyd-B fluid as a special viscolesatic non-Newtonian fluid has had some success in describing polymeric liquids, it being more amenable to analysis and more importantly experimental.

Recently, the fractional derivatives [1] are found to be quite flexible for describing the behaviors of viscoelastic fluids. Many researchers have studied different problems related to such fluids. In their works, the constitutive equations for generalized non-Newtonian fluids are modified from the well known fluid models by replacing the time derivative of an integer order by the so-called Riemann-Liouville fractional calculus operators. Haitao and Xu [2] investigated the Stokes' problem for a viscoelastic fluid with a generalized Oldroyd-B model. Khan and Hyder *et al.* [3-4] considered some fluid with generalized Oldroyd-B model. Hyder [5] discussed the flows of generalized Oldroyd-B fluid between two side walls perpendicular to the plate. Fetecau *et al.* [6-9] investigated some accelerated flows of a generalized Oldroyd-B fluid. Hayat *et al.* [10-11] studied the flow of a Maxwell fluid between two side walls. Khan [12]studied the MHD flow of a generalized Oldroyd-B fluid in a circular pipe.

In this paper, we consider the MHD flow of an incompressible generalized Oldroyd-B fluid. The exact solutions for the velocity and shear stress fields are obtained by using the discrete Laplace transform technique for the fractional calculus. The important aspect of the study is that the solutions for generalized Oldroyd-B fluid, fractional second grade fluid and fractional Maxwell fluid are recovered by the current analysis.

II. GOVERNING EQUATIONS

The constitutive equation of an incompressible and unsteady Oldroyd-B fluid is written in the form [2]:

$$\mathbf{T}=-p\mathbf{I}+\mathbf{S}\,,\ \mathbf{S}+\lambda\frac{\mathrm{D}^{\alpha}\mathbf{S}}{\mathrm{D}t^{\alpha}}=\mu(1+\theta\frac{\mathrm{D}^{\beta}}{\mathrm{D}t^{\beta}})\mathbf{A}\,. \quad (1)$$

where $\mathbf{T}$ is the Cauchy stress tensor, $-p\mathbf{I}$ denotes the indeterminate spherical stress, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{A}=\mathbf{L}+\mathbf{L}^{\mathrm{T}}$ is the first Rivlin-Ericksen tensor, $\mathbf{L}$ is the velocity gradient, μ,λ,θ are material constants, known as the viscosity coefficient, the relaxation and retardation times, respectively , and

$$\begin{aligned}\frac{\mathrm{D}^{\alpha}\mathbf{S}}{\mathrm{D}t^{\alpha}}&=\mathrm{D}_t^{\alpha}\mathbf{S}+\mathbf{V}\cdot\nabla\mathbf{S}-\mathbf{LS}-\mathbf{SL}^{\mathrm{T}},\\ \frac{\mathrm{D}^{\beta}\mathbf{A}}{\mathbf{D}t^{\beta}}&=\mathrm{D}_t^{\beta}\mathbf{A}+\mathbf{V}\cdot\nabla\mathbf{A}-\mathbf{LA}-\mathbf{AL}^{\mathrm{T}}.\end{aligned} \quad (2)$$

In the above relations $\mathbf{V}$ is the velocity, ∇ is the gradient operator, D_t^{α} and D_t^{β} are based on Riemann- Liouville's definition is defined as [1]:

$$\mathrm{D}_t^{p}f(t)=\frac{1}{\Gamma(1-p)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t\frac{f(\tau)}{(t-\tau)^{p}}\mathrm{d}\tau\,,0\le p<1\,, \quad (3)$$

where $\Gamma(\cdot)$ is the Gamma function.

We consider the MHD flow of an incompressible generalized Oldroyd-B fluid due to an infinite accelerating plate. The fluid occupies the space $y>0$ and the motion is produced by the infinite plate. Initially, the system is at rest and at time $t=0^{+}$ the plate starts to oscillate according to $\mathrm{V}\cos(\omega\mathrm{t})$ or $\mathrm{V}\sin(\omega\mathrm{t})$. Assuming the velocity field and stress of the form

$$\mathbf{V}=u(y,t)\mathbf{i}\,,\ \mathbf{S}=S(y,t)\,. \quad (4)$$

Where u is the velocity and $\mathbf{i}$ is the unit vectors in the x -direction. Substituting Eq.(4) into Eq.(1) and taking account of the initial condition

Manuscript received January 15, 2010. This work was supported by the National Natural Science Foundations of China (No. 50936003), the Open Project of Institute of Rheological Mechanics & Material Engineering（Central South University of Forestry and Technology, No.09RM04）and The Research Foundation of Engineering Research Institute, University of Science and Technology Beijing.

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$$S(y,0)=0\,,\ y>0\,, \tag{5}$$

the fluid being at rest up to the time $t=0$, we get

$$(1+\lambda \mathrm{D}_t^{\alpha})S_{xy}=\mu(1+\theta \mathrm{D}_t^{\beta})\partial_y u(y,t)\,, \tag{6}$$

$S_{yy}=S_{zz}=S_{xz}=S_{yz}=0$, $S_{xy}=S_{yx}$. Consider that the conducting fluid is permeated by an imposed magnetic field B_0 which acts in the positive y-coordinate. In the low-magnetic Reynolds number approximation, the magnetic body force is represented $\sigma B_0^2 u$. Then, in the absence of a pressure gradient in the x-direction, the equation of motion yields the following scalar equations:

$$\rho\frac{\partial u}{\partial t}=\frac{\partial}{\partial y}S_{xy}-\sigma B_0^2 u\,, \tag{7}$$

where ρ is the constant density of the fluid. Eliminating S_{xy} between Eq.(6) and Eq.(7), we arrive at the following fractional differential equation

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u(y,t)}{\partial t}=\nu(1+\theta \mathrm{D}_t^{\beta})\frac{\partial^2 u(y,t)}{\partial y^2}-N(1+\lambda \mathrm{D}_t^{\alpha})u(y,t)\ , \tag{8}$$

where $\nu=\mu/\rho$ is the kinematic viscosity and $N=\sigma B_0^2/\rho$. The associate initial and boundary conditions as follows:

Initial condition: $\quad u(y,0)=\dfrac{\partial u(y,0)}{\partial t}=0\,,\ \ y>0\,.$ (9)

Boundary conditions: $\quad u(0,t)=\mathrm{V}\sin(\omega \mathrm{t})$ or

$$u(0,t)=\mathrm{V}\cos(\omega \mathrm{t})\,,\ t>0\,, \tag{10}$$

$$u(y,t)\,,\ \frac{\partial u(y,t)}{\partial y}\to 0\ \text{ as } y\to\infty\,,\ t>0\,. \tag{11}$$

where u is velocity in the x-coordinate direction.

III. Velocity field shear stress

Employing the non-dimensional quantities

$$u^*=\frac{u}{\mathrm{V}},y^*=\frac{y\mathrm{V}\rho}{\mu},t^*=\frac{t\mathrm{V}^2\rho}{\mu},$$
$$\lambda^*=\lambda\left(\frac{\mathrm{V}^2\rho}{\mu}\right)^{\alpha},\theta^*=\theta\left(\frac{\mathrm{V}^2\rho}{\mu}\right)^{\beta},N^*=\frac{N\mu}{A^2\rho}. \tag{12}$$

We obtain the dimensionless motion equation as follows (for brevity the dimensionless mark "*" is omitted here)

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u(y,t)}{\partial t}=(1+\theta \mathrm{D}_t^{\beta})\frac{\partial^2 u(y,t)}{\partial y^2}-N(1+\lambda \mathrm{D}_t^{\alpha})u(y,t)\,. \tag{13}$$

Boundary conditions:

$$u(0,t)=\sin(\omega \mathrm{t})\ \text{ or }\ u(0,t)=\cos(\omega \mathrm{t})\,,\ t>0\,. \tag{14}$$

In order to obtain an exact solution of the above equation, we use Laplace transforms principle of sequential fractional derivatives, yields

$$\frac{\partial^2 U(y,p)}{\partial y^2}-\frac{(p+N)(1+\lambda p^{\alpha})}{(1+\theta p^{\beta})}U(y,p)=0\,. \tag{15}$$

Boundary conditions become：

$$U(0,p)=\frac{\omega}{p^2+\omega^2}\ \text{ or }\ U(0,p)=\frac{p}{p^2+\omega^2}\,. \tag{16}$$

$$U(y,p)\,,\ \partial_y U(y,p)\to 0\ \text{ as } y\to\infty\,. \tag{17}$$

where $U(y,p)$ is the image function of $u(y,t)$ and p is the transform parameter. Solving Eqs.(15)- (17), we obtain

$$U(y,p)=\frac{\omega}{p^2+\omega^2}\exp[-(\frac{(p+N)(1+\lambda p^{\alpha})}{1+\theta p^{\beta}})^{\frac{1}{2}}y]\,. \tag{18}$$

$$U(y,p)=\frac{p}{p^2+\omega^2}\exp[-(\frac{(p+N)(1+\lambda p^{\alpha})}{1+\theta p^{\beta}})^{\frac{1}{2}}y]\,. \tag{19}$$

The shear stress can be calculated from Eq.(6), taking the Laplace transform and introducing Eq.(17)-(18), we get

$$T(y,p)=-(p+N)^{\frac{1}{2}}\left(\frac{1+\theta p^{\beta}}{1+\lambda p^{\alpha}}\right)^{\frac{1}{2}}\frac{\omega}{p^2+\omega^2}\exp[-(\frac{(p+N)(1+\lambda p^{\alpha})}{1+\theta p^{\beta}})^{\frac{1}{2}}y] \tag{20}$$

$$T(y,p)=-(p+N)^{\frac{1}{2}}\left(\frac{1+\theta p^{\beta}}{1+\lambda p^{\alpha}}\right)^{\frac{1}{2}}\frac{p}{p^2+\omega^2}\exp[-(\frac{(p+N)(1+\lambda p^{\alpha})}{1+\theta p^{\beta}})^{\frac{1}{2}}y] \tag{21}$$

where $T(y,p)=S_{xy}/A^2\rho$.

In order to avoid the burdensome calculations of residues and contour integrals, we apply discrete inverse Laplace transform to get to the velocity and the stress fields. Firstly, write Eqs.(18)-(19) as series forms, and apply inverse Laplace transform. In terms of Fox H-function, we write the solution as the simple form:

$$u(y,t)=$$
$$\begin{cases} \sin(\omega t)+\displaystyle\sum_{h=0}^{\infty}\frac{(-1)^h}{\omega^{2h+1}}\sum_{k=1}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}(-1)^m\frac{1}{m!\lambda^m} \\ \times t^{\frac{k}{2}(\beta-\alpha-1)-2h+\alpha m+l-1}H_{3,5}^{1,3}\left[\dfrac{t^{\beta}}{\theta}\middle|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\ (0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),(\frac{k}{2}(\alpha-\beta+1)+2h-\alpha m-l+1,\beta)\end{matrix}\right], \\ \hfill if\ |p|<|\omega|; \\ \sin(\omega t)+\displaystyle\sum_{h=0}^{\infty}(-1)^h\omega^{2h+1}\sum_{k=1}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}(-1)^m\frac{1}{m!\lambda^m} \\ \times t^{\frac{k}{2}(\beta-\alpha-1)+2h+\alpha m+l+1}H_{3,5}^{1,3}\left[\dfrac{t^{\beta}}{\theta}\middle|\begin{matrix}(1-l+\frac{k}{2}),(1-m+\frac{k}{2}),(1-\frac{k}{2},1)\\ (0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),(\frac{k}{2}(\alpha-\beta+1)-2h-\alpha m-l-1,\beta)\end{matrix}\right], \\ \hfill if\ |p|>|\omega|. \end{cases} \tag{22}$$

And

$$u(y,t)=$$
$$\begin{cases} \sin(\omega t)+\displaystyle\sum_{h=0}^{\infty}\frac{(-1)^h}{\omega^{2(h+1)}}\sum_{k=1}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}(-1)^m\frac{1}{m!\lambda^m} \\ \times t^{\frac{k}{2}(\beta-\alpha-1)+\alpha m+l-2h-2}H_{3,5}^{1,3}\left[\dfrac{t^{\beta}}{\theta}\middle|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\ (0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),(\frac{k}{2}(\alpha-\beta+1)-\alpha m-l+2h+1,\beta)\end{matrix}\right], \\ \hfill if\ |p|<|\omega|; \\ \sin(\omega t)+\displaystyle\sum_{h=0}^{\infty}(-1)^h\omega^{2h}\sum_{k=1}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}(-1)^m\frac{1}{m!\lambda^m} \\ \times t^{\frac{k}{2}(\beta-\alpha-1)+\alpha m+l+2h}H_{3,5}^{1,3}\left[\dfrac{t^{\beta}}{\theta}\middle|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\ (0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),(\frac{k}{2}(\alpha-\beta+1)-\alpha m-l+2h,\beta)\end{matrix}\right], \\ \hfill if\ |p|>|\omega|. \end{cases} \tag{23}$$

Where the property of the Fox H-function is

$$\sum_{n=0}^{\infty}\frac{(-z)^n\Pi_{j=1}^{p}\Gamma(a_j+A_jn)}{n!\Pi_{j=1}^{q}\Gamma(b_j+B_jn)}=H^{1,p}_{p,q+1}\left[z\left|^{(1-a_1,A_1),\cdots(1-a_p,A_p)}_{(0,1),(1-b_1,B_1),\cdots(1-b_q,B_q)}\right.\right] \quad (24)$$

Adopting a similar procedure, we obtain

$$T(y,t)=\begin{cases}-\sum_{h=0}^{\infty}\frac{(-1)^h}{\omega^{2h+1}}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k-1}{2}}\sum_{m=0}^{\infty}t^{\frac{k-1}{2}(\beta-\alpha-1)+\alpha m+l-2h-2} \\ \times\frac{(-1)^m}{m!\lambda^m}H^{1,3}_{3,5}\left[\frac{t^\beta}{\theta}\left|^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),(\frac{k-1}{2}(\alpha-\beta+1)-\alpha m-l+2h,\beta)}\right.\right], \\ \qquad if\ |p|<|\omega|; \\ -\sum_{h=0}^{\infty}(-1)^h\omega^{2h+1}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k-1}{2}}\sum_{m=0}^{\infty}t^{\frac{k-1}{2}(\beta-\alpha-1)+\alpha m++l+2h} \\ \times\frac{(-1)^m}{m!\lambda^m}H^{1,3}_{3,5}\left[\frac{t^\beta}{\theta}\left|^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),(\frac{k-1}{2}(\alpha-\beta+1)-\alpha m-l-2h,\beta)}\right.\right], \\ \qquad if\ |p|>|\omega|. \end{cases} \quad (25)$$

And

$$T(y,t)=\begin{cases}-\sum_{h=0}^{\infty}\frac{(-1)^h}{\omega^{2(h+1)}}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k-1}{2}}\sum_{m=0}^{\infty}t^{\frac{k-1}{2}(\beta-\alpha-1)+\alpha m+l-2h-3} \\ \times\frac{(-1)^m}{m!\lambda^m}H^{1,3}_{3,5}\left[\frac{t^\beta}{\theta}\left|^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),(\frac{k-1}{2}(\alpha-\beta+1)-\alpha m-l+2h+3,\beta)}\right.\right], \\ \qquad if\ |p|<|\omega|; \\ -\sum_{h=0}^{\infty}(-1)^h\omega^{2h}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{l=0}^{\infty}(-N)^l\frac{1}{l!}\left(\frac{\lambda}{\theta}\right)^{\frac{k-1}{2}}\sum_{m=0}^{\infty}t^{\frac{k-1}{2}(\beta-\alpha-1)+\alpha m+l+2h-1} \\ \times\frac{(-1)^m}{m!\lambda^m}H^{1,3}_{3,5}\left[\frac{t^\beta}{\theta}\left|^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),(\frac{k-1}{2}(\alpha-\beta+1)-\alpha m-l-2h,\beta)}\right.\right], \\ \qquad if\ |p|>|\omega|. \end{cases} \quad (26)$$

IV. LIMITING CASES

(1) When $N=0$, the Oldroyd-B fluid isn't influenced by magnetic field.

(2) If $N=0$ and $\alpha\neq 0$, $\lambda\to 0$, then the non-dimensional motion equation and the associated boundary conditions are

$$\frac{\partial u(y,t)}{\partial t}=(1+\theta D_t^{\beta})\frac{\partial^2 u(y,t)}{\partial y^2}. \quad (27)$$

Boundary conditions:

$$u(0,t)=\sin(\omega t)\ \text{ or }\ u(0,t)=\cos(\omega t),\ t>0. \quad (28)$$

Which represent the velocity field for a generalized second grade fluid. Further, Eqs.(27)- (28) are similar with ref. [13].

(3) If $N=0$ and $\beta\neq 0$, $\theta\to 0$. The solutions for a generalized Maxwell fluid are recovered.

V. RESULTS AND DISCUSSION

In this paper, we have presented some oscillating flow of a generalized Oldroyd-B fluid. The fractional calculus

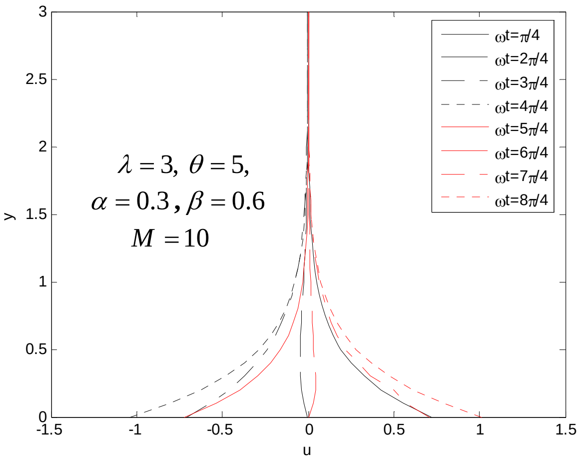

Fig.1 Profiles of the velocity at different times and M for $u(0,t)=\cos(\omega t)$.

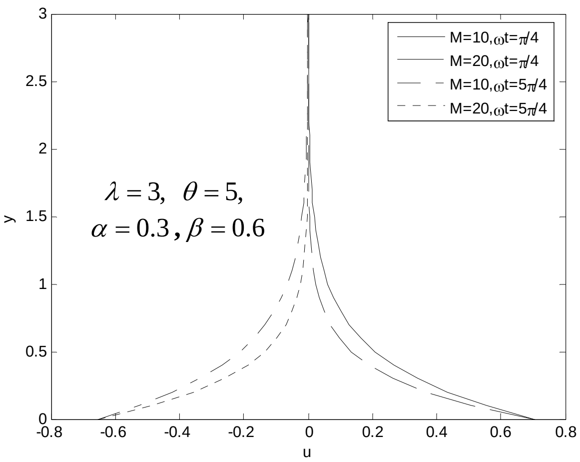

Fig.2 Profiles of the velocity at different times and M for $u(0,t)=\sin(\omega t)$.

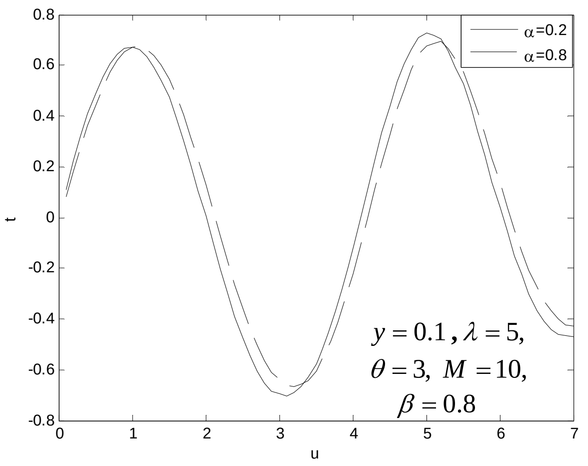

Fig.3 Profiles of the velocity with different values of α for $u(0,t)=\sin(\omega t)$.

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

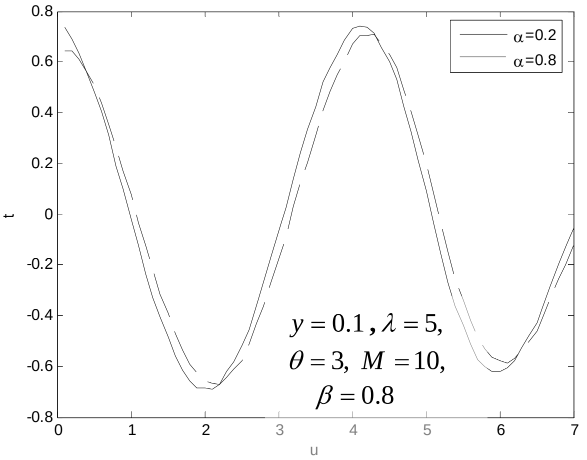

Fig.4 Profiles of the velocity with different values of α for $u(0,t) = \cos(\omega t)$.

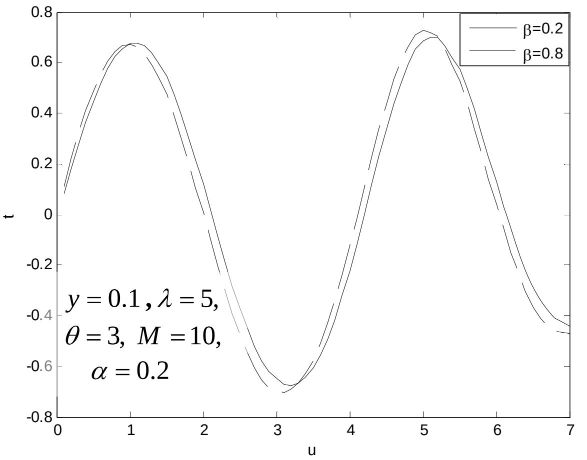

Fig.5 Profiles of the velocity with different values of β for $u(0,t) = \sin(\omega t)$.

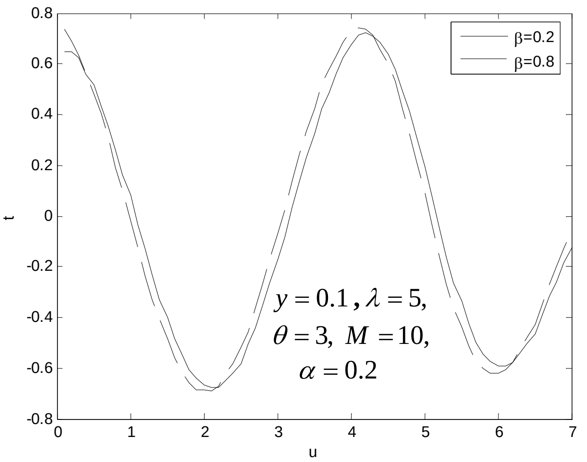

Fig.6 Profiles of the velocity with different values of β for $u(0,t) = \cos(\omega t)$.

approach is introduced to establish the constitutive relationship of a viscoelastic fluid. The Laplace transform and Fox H-function are used to establish analytical solutions. In the limiting cases, the generalized second grade fluid and generalized Maxwell fluid can be recovered from the present solutions. In there, we analyze the characteristics of velocity field by using the analytical solutions Eqs.(22)-(26) obtained in section 3.

The motion of the fluid was due to the oscillation of the plate parallel x direction, with angular frequency ω. The velocity profiles are displayed for different times $\omega t = k\pi/4$ $(k = 1,2,3,\ 4,5,6,7,8)$ with $\omega = 1.5$ in fig.1. Fig.2 shows the velocity in the case of magnetohydrodynamic fluid is more steady than hydrodynamic. And the magnetic body force is favorable to decay of the velocity. Fig.3-5 demonstrate the velocity changes with the fractional parameters α and β. We can see that their effects on both motions are opposite. The non-Newtonian effects are stronger at large values of α. The smaller the values of α, the more steady of the velocity field.

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ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)