

# Information Modeling Of Urban Traffic System Using Fuzzy Stochastic Approach

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**Abstract**— Information modeling is one of the most important techniques to represent dynamic behavior of real time systems. The previous models developed lacks dynamic binding of the information model in real time environment. This paper presents a mathematical model “CEFP” (Conditional Expectation Fuzzy petrinets) which links the stochastic movement of traffic and fuzzy control of Urban Traffic Systems.

**Index Terms**— Conditional Expectation, Fuzzy Petrinets, Urban Traffic System.

## I. INTRODUCTION

Urban Traffic Information model are generally described by Object diagram and State-charts, but these diagrams lacks the ability to present the behavioral aspect of Traffic Information flow. Other problem with these diagrams is the lack of formalization which leads to uncertainty in Information modeling[1,2,3,5].

In order to overcome the above discrepancies we convert the urban Traffic Information flow model into Mathematical model which provides a strong formal foundation to these models. It consists of combination of fuzzy set theory and stochastic algebra.

A discrete non-linear stochastic system is applied for traffic flow and unpredictable queue length. The control part is derived from the output received by stochastic traffic movement The prediction of real-time queue overflows are linked with Fuzzy Control circuit which helps in smoothing of traffic system by changes in Variable Message Sign Boards (VMS) and the necessary increase/decrease in traffic signal timing.

## II. LITERATURE REVIEW

We are inspired by the work of Those and Keen (1990) in which image processing of traffic movement is represented by the help of Fast Fourier Transform. Later on Cremer an Heminger (1993) developed a model for Urban Traffic

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movement using Payne's equation. The focus was to capture macroscopic traffic movement on freeways.

Antony Stathopoulos and Loukas Dimitrio further modified the work by developing a model known as FRBS (Fuzzy Rule Based System). This hybrid FRBS and ANN provides an online real world urban signalized arterial demonstration. We have extended their work to develop a mathematical model based on FRBS.

## III. STOCHASTIC PATTERNS TO MODEL URBAN TRAFFIC SYSTEM

Traffic movement follows certain basic patterns, which are closely related to the conditional expectation. The conditional expectation provides conditioning of an event which are linked with conditioning of an event. The combination of the "Conditional Expectation" and "Fuzzy Petrinet" help us to simulate the urban traffic movement close to real traffic movement. We have termed the combination as "CEFP" Model.

Applying CEFP Model to various patterns of traffic modeling we get:

### A. Pattern I:

It is a simple Cause - Result pattern describing an event relationship. It shows how an event "e1" can cause "e2" within a time period [I1-I2]. The conditional expectation of e2 can be defined as

$$\left[ E (e_2/e_1) = \frac{1}{P(B)} \int_{I_1}^{I_2} e_2 dP \right]$$

Amongst Universal set of "e" events, event e1 and e2 has occurred which is denoted as FE as shown in the figure 1.

$$F_E = \mu_{e_1} \cdot \mu_{e_2} (x)$$

$$F_E = \mu[\mu_{e_1} (x), \mu_{e_2} (x) ]$$

$$F_E = u \left[ 0, \frac{1}{P(B)} \int_{I_1}^{I_2} \mu_{e_2} (x) dx \right]$$

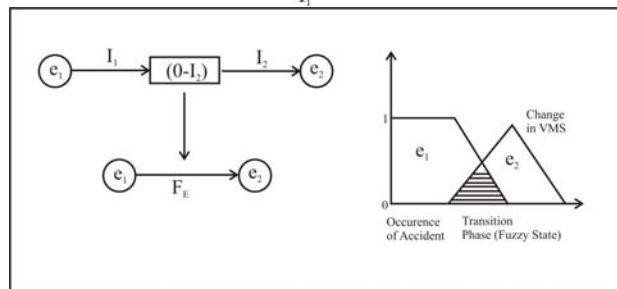


Figure 1 : CEFP MODEL OF EVENT CAPTURING

There is some fuzzy set condition whether the accident occurred will be cleared within time frame T1 or it requires a longer time than T1 to clear-up, hence "A Diversion" in Variable Message Sign (VMS) Board should be displayed.

**B. Pattern II:**

When multiple events occur on temporal conditions, a particular road section is jam with the result traffic is being diverted to multiple path leading to calming of traffic flow.

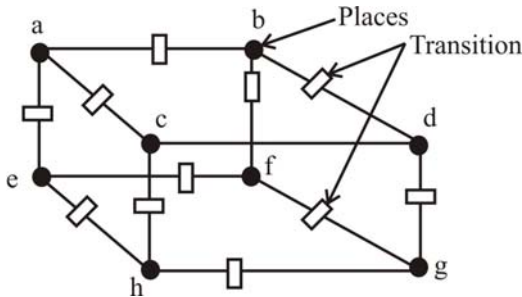


Figure 2 : Orthogonal Representation of Petri-nets

To model multiple occurrences of an event we extend our representation to orthogonal modeling specification. Each of these events and places are interlinked with each other forming a discrete random set of activity generation. We will be using Discrete Random Fuzzy Homomorphism property of our CEFP Model[4,6,8] which helps in finding out the real time simulation of traffic with respect to cascading of events. Under some situations the traffic diversion of every road junction may not be required for this situation we have to find out the conditional expectation given as:

$$E(e | x)(w) = E(e | \{\eta = e_n\}) \text{ if } \eta(w) = e_n$$

where n = a, b, c, d, e, .....h

Applying Fuzzy constraints to the above discrete equation we obtain.

$$E = \sum(E(e|\{\eta = e\}). E(e|\{\eta = e_b\})..... E(e|\{\eta = e_h\})).$$

The branching of events is given as :

$$H(E_a, E_b, ..... E_h) = H(PE_a, PE_b) + p EA ($$

$$H(E_a, E_b, .... E_h) = H(p_{Ea}, p_{Eb}) + p_{EA} \left( \frac{p_{Ea}}{e_a} + \frac{p_b}{e_b} + \dots \frac{p_h}{e_h} \right) + ..$$

$$p_{Eb} \left( \frac{p_a}{e_a} + \frac{p_b}{e_b} + \dots \right) + ..$$

$$+ p_{Eh} \left( \frac{p_a}{e_a} + \frac{p_b}{e_b} + \dots \right)$$

Where H represents a continuous function in all arguments of p<sub>a</sub>, p<sub>b</sub>, p<sub>c</sub> ..... p<sub>h</sub>.

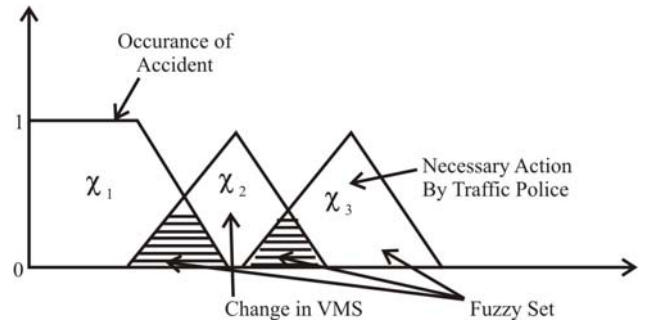


Figure 3. Fuzzy Membership of Activities

We can represent the above situation in cascade of Activity given as  $\chi_1, \chi_2$  and  $\chi_3$  representing non-interactive fuzzy sets modeled mathematically as:

Let set A represents overall sequence of Activities which can be represented in Cartesian space as  $X = \chi_1 \times \chi_2 \times \chi_3$

Set 'A' is separable into three Non-Fuzzy set called as orthogonal projections:

$$A = OP_{\chi_1}(A) \chi OP_{\chi_2}(A) \chi OP_{\chi_3}(A)$$

where :

$$\mu_{Pr \chi_1}(A)(\chi_1) = \max_{\chi_3 \in \chi_3} MA(\chi_1, \chi_2, \chi_3) \forall \chi_3 \in \chi_3$$

$$\mu_{Pr \chi_2}(A)(\chi_2) = \max_{\chi_3 \in \chi_3} MA(\chi_1, \chi_2, \chi_3) \forall \chi_3 \in \chi_3$$

The Equations represents membership functions for the orthographic projections of A on universe  $\chi_1, \chi_2$  and  $\chi_3$ .

$$\tilde{A} = OP_{\chi_1}(\tilde{A}) \chi OP_{\chi_2}(\tilde{A}) \chi OP_{\chi_3}(\tilde{A})$$

Where :

$$\mu_{Pr \chi_1}(A)(\chi_1) = \max_{\chi_3 \in \chi_3} \mu_A(\chi_1, \chi_2, \chi_3) \forall \chi_2 \in \chi_2$$

$$\mu_{Pr \chi_2}(A)(\chi_2) = \max_{\chi_1 \in \chi_1} \mu_A(\chi_1, \chi_2, \chi_3) \forall \chi_2 \in \chi_2$$

$$\mu_{Pr \chi_3}(A)(\chi_3) = \max_{\chi_2 \in \chi_2} \mu_A(\chi_1, \chi_2, \chi_3) \forall \chi_2 \in \chi_2$$

Combing the orthogonal Stochastic Networks and Fuzzy Cycle using Multi attribute measurement factors[9,10]we get a single linear equation as :

$$Y = A_1 \chi_1 + A_2 \chi_2 + ..... A_i \chi_i$$

The above equation helps in generating triangular Fuzzy Numbers which are helpful in generating probabilistic regression analysis given by a matrix "χ":

$$\chi = \begin{bmatrix} pE_{aa} & pE_{ab} & pE_{ac} & ..... & pE_{ah} \\ pE_{ba} & ..... & ..... & ..... & pE_{bh} \\ pE_{ha} & ..... & ..... & ..... & pE_{hh} \end{bmatrix}$$

Finally, we are able to derive a composite structure of stochastic petrinets and fuzzy UTS controls given by probabilistic regression equation as:

$$\left[ P \left\{ t_i | M \right\} = \frac{w_i}{W_i(M)} \right]$$

#### IV. CONCLUSION

The paper is designed to stimulate the mathematical model of Urban Traffic System, which is close to real time traffic modeling. The basic purpose of combining Fuzzy and Stochastic Algebra is to derive a mathematical foundation of uncertain behavior of Urban Traffic Information System. The limitations of standard modeling language are overcome by using stochastic approach supporting dynamic binding of Traffic Information System.

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