Coordinated Control of Waste Water Treatment Process

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Abstract—This paper develops coordinated control methods for the regulation policies of high-strength waste water treatment process. A dynamic model for the activated sludge process with wastewaters is presented and linearized for control studies. The control strategy regulates the feed rate to maintain a constant optimal substrate concentration in the reactor, which in turn minimizes the reaction time. The coordinated control method consists of three components: two components have independent direct effects on the behavior of the aerated basin and the settling tank and the third component coordinates the overall operation. Simulation results show the effectiveness of the developed methods.

Index Terms—Coordinated control, Linear optimal control, Waste water treatment process.

I. INTRODUCTION

The use of the activated sludge process (ASP) for biological nitrogen removal has increased in many countries. This is a result of stricter effluent demands. To remove the nitrogen from the wastewater in an ASP, the biological processes, nitrification and de-nitrification, are needed. In aerobic compartments, ammonium may be converted into nitrate (nitrification) and in anoxic compartments nitrate may be converted into gaseous nitrogen (de-nitrification). For the operaation to work well, a sufficiently high concentration of dissolved oxygen (DO) is needed together with a sufficiently large aeration volume [1, 2]. A comprehensive overview of different approaches to DO control is given in [3].

It is known that activated sludge systems are affected by several dynamic variables which have influence over the output concentration of operation parameters. Fluctuation of temperature, flow and organics modifies the performance of the process and makes steady-state models inefficient for explaining the normal perturbations in wastewater treatment plants. Under this situation, dynamic models are clearly in advantage.

The progressive deterioration of the water resources and the great quantity of polluted water produced in the industrial companies, give to the waste water treatment a great importance in the safeguarding of water's quality. So the monitoring of this kind of process has become an important

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task. The heart of this process is composed of two basins: the aerated basin and the settling tank Fig 1.

II. MATHEMATICAL MODEL

The fundamental phase of mathematical modeling consists in determining the reaction rates of the macroscopic variables of the system. The objective of the activated sludge process is to achieve, at a minimum cost, a sufficiently low concentration of biodegradable matter in the effluent, together with minimal sludge production. There is no reaction in the settling tank which delivers purified water after the decantation of sludge. A part of this later is recycled in the aerated basin. The fundamental phase of the mathematical modeling consists in determining the reaction rates of the macroscopic variables of the system to know the rate of biomass growth, substrate degradation and dissolved oxygen uptake. The second stage makes it possible to determine the system equations whose states variables are the concentrations in micro-organisms, substrate, recycled biomass and dissolved oxygen. These variables as well as the inputs and the outputs are gathered in mathematical expressions thus constituting the process model [4]. The mathematical model for the activated sludge process (aerated basin and settling tank) is based on the equations, resulting from mass balance considerations, carried out on each of the reactant of the process. The initial system is composed of nine states, four inputs and six outputs:

A. Aerated basin model

$$\begin{aligned} \frac{dS_{1}}{dt} &= \frac{Q_{in}}{V_{r}} (S_{1,in} - S_{1}), \\ \frac{dS_{s}}{dt} &= \frac{Q_{in}}{V_{r}} (S_{3,in} - S_{3}) - \frac{1}{Y_{H}} \rho_{1} + \rho_{3} \\ \frac{dX_{1}}{dt} &= \frac{Q_{in}}{V_{r}} (X_{1,in} - X_{1}) + \frac{Q_{r}}{V_{r}} (X_{1,in} - X_{1}) + f_{X_{1}} \rho_{2} \\ \frac{dX_{s}}{dt} &= \frac{Q_{in}}{V_{r}} (X_{3,in} - X_{3}) + \frac{Q_{r}}{V_{r}} (X_{3,nc} - X_{H}) + (1 - f_{X_{1}}) \rho_{2} - \rho_{3} \\ \frac{dX_{H}}{dt} &= \frac{Q_{in}}{V_{r}} (X_{H,in} - X_{H}) + \frac{Q_{r}}{V_{r}} (X_{H,nc} - X_{H}) + \rho_{1} - \rho_{2} \\ \frac{dS_{o}}{dt} &= \frac{Q_{in}}{V_{r}} (S_{o,in} - S_{o}) + Q_{L} (C_{3} - C_{o}) + \frac{(1 - Y_{H})}{Y_{H}} \rho_{1} \end{aligned}$$
(1)

B. Settling tank

$$\frac{dX_{H,rec}}{dt} = \frac{Q_{in} + Q_r}{V_{dec}} X_H - \frac{Q_r + Q_w}{V_{dec}} X_{H,rec},$$

$$\frac{dX_{I,rec}}{dt} = \frac{Q_{in} + Q_r}{V_{dec}} X_I - \frac{Q_r + Q_w}{V_{dec}} X_{I,rec}$$

$$\frac{dX_{s,rec}}{dt} = \frac{Q_{in} + Q_r}{V_{dec}} X_s - \frac{Q_r + Q_w}{V_{dec}} X_{s,rec} \quad (2)$$

Where the parameters ρ_1 , ρ_2 and ρ_3 are given by

$$\rho_{1} = \mu_{\max} \frac{S_{s}}{(K_{s} + S_{s})} \frac{S_{o}}{(K_{o} + S_{o})}, \ \rho_{2} = b_{H} X_{H},$$
$$\rho_{3} = K_{h} \frac{X_{s} X_{H}}{(K_{s} X_{H} + X_{s})} \frac{S_{o}}{(K_{o} + S_{o})}$$

From a control engineering standpoint, handling model (1)-(2) is quite hard [4] and therefore we direct attention to an alternative approaches. In terms of $x \in \mathbb{R}^{9}$, $u \in \mathbb{R}^{4}$ and

 $y \in \mathbb{R}^{6}$ we cast the model equations (1)-(2) into the format

$$\frac{dx}{dt} = \dot{x} = f(x, u), \quad y = Cx \tag{3}$$

Letting the system equilibrium point (x_e, u_e) be defined

by $\frac{dx}{dt}\Big|_{x=xe, u=ue}$. Performing a linearization of system (3)

around the equilibrium point yields the linearized model

$$\frac{dz}{dt} = \dot{z} = Az + Bv, \qquad w = Cz \qquad (4)$$

$$A = \frac{1}{\partial x}, \quad B = \frac{1}{\partial u}$$
$$z = x - x_e, \quad v = u - u_e, \quad w = y - y_e \quad (5)$$

into consideration the two subsystems (aerated basin and settling tank), we express model (4)-(5) in the form

$$\frac{d}{dt}z_1 = A_1 z_1 + B_1 v_1 + g_1 \tag{6}$$

$$g_1 = A_2 z_2$$
 (7)

$$\frac{d}{dt}z_2 = A_2 z_2 + B_2 v_2 + g_2 \tag{8}$$

$$g_2 = A_3 z_1 \tag{9}$$

where

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$
$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The numerical values of the respective matrices are given by

$$A_{1} = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 & 0 \\ 0 & -14.5 & 0 & 166 & -150 & 0 \\ 0 & 0 & -29 & 0 & 1060 & 0 \\ 0 & 0 & 0 & -29 & 1120 & 0 \\ 0 & -14.5 & 0 & 0 & -29 & 0 \\ 0 & -14.5 & 0 & 0 & 0 & -12.5 \end{bmatrix},$$
$$A_{2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 68.03 & 0 \\ 0 & 0 & 68.03 \\ 14.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_{3} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1136 & 0 \\ 0 & 0 & -1136 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 3 & 0 \end{bmatrix} \mathbf{A}_{4} = \begin{bmatrix} 83.77 & 0 & 0 \\ 0 & 83.77 & 0 \\ 0 & 0 & 83.77 \end{bmatrix}$$

$$\mathbf{B}_{1} = \begin{bmatrix} -5 & 0 \\ 5 & 0 \\ 0 & -540 \\ 8 & -314 \\ -13 & 0 \\ 0 & -620 \end{bmatrix}, \mathbf{B}_{2} = \begin{bmatrix} -98 & -583 \\ 0 & -339 \\ 0 & -670 \end{bmatrix},$$

$$\mathbf{C}_{1} = \mathbf{I}_{6}, \mathbf{C}_{2} = \mathbf{I}_{3}$$

We recall that the linearized model (6)-(9) represents an interconnected system with linear coupling pattern [6]. In the next section, we present two distinct decentralized control methods to regulate the dynamic behavior of the waste water treatment process.

III. COORDINATED CONTROL METHODS

Employing the linear quadratic control theory and considering the decoupled case ($g_1 = 0, g_2 = 0$), we optimize each subsystem

$$\frac{d}{dt}z_1 = A_1 z_1 + B_1 v_1 \tag{10}$$

$$\frac{d}{dt}z_2 = A_2 z_2 + B_2 v_2 \tag{11}$$

with respect to the quadratic performance indices

$$J_{s} = \int_{0}^{\infty} e^{2\pi t} [z_{s}'Q_{s}z_{s} + v_{s}'R_{s}v_{s}]dt \quad s = 1,2 \quad (12)$$

where Q_s is an $r \times r$ symmetric, nonnegative definite matrix (r=6 for subsystem 1 and r=3 for subsystem 2), R_s is an $t \times t$ is an symmetric, positive definite matrix (t=2 for subsystems 1 and 2), and π is a nonnegative number. As known [8], under the assumption that the pair (A_s, B_s) is completely controllable, there exists a unique optimal control law

$$v_{s}^{*} = -K_{s}^{*}z_{s} = -R_{s}^{-1}B_{s}^{*}P_{s}z_{s}, \quad s = 1,2$$
 (13)

and P_s is an $r \times r$ symmetric, positive definite matrix which is the solution of the Riccati equation

$$P_{s}(A_{s} + \pi_{s}I) + (A_{s} + \pi_{s}I)^{t} P_{s} - P_{s}B_{s}R_{s}^{-1}B_{s}^{t} P_{s} + Q_{s} = 0,$$

$$s = 1, 2$$
(14)

such that v_{s}^{*} minimizes J_{s} in (12). The associated optimal cost is

$$I_{s} = z_{s}^{t}(0)P_{s}z_{s}(0), \qquad s = 1,2 \qquad (15)$$

Under the assumption that Q_s can be factored as $C_s C_s^t$, where C_s is an $p \times p$ constant matrix, so that the pair (A_s, C_s) is completely observable, each closed-loop subsystem

$$\frac{d}{dt}z_{s} = (A_{s} - B_{s}R_{s}^{-1}B_{s}^{t}P_{s})z_{s}, \quad s = 1,2 \quad (16)$$

is globally exponentially stable with the degree π [8]. That is, the solution \mathcal{Z}_s of (16) approaches the equilibrium at the origin at least as fast as $e^{-\pi t}$ for all initial conditions.

A. Method 1

Next, to compensate for the interactions $(g_1 \neq 0, g_2 \neq 0)$, and preserve the subsystems' autonomy, we apply the controls $v_s = v_s^* + v_s^c$, s = 1, 2 (17)

The controls v_s^c are selected in the form

 $v_s^c = -K_s^c z_s$, s=1,2 where the gains K_s^c are compensatory controls to adjust the dynamic behavior of the aerated basin and settling tank subsystems. It follows from [6] that

$$v_{s}^{c} = \begin{bmatrix} v_{1}^{c} \\ v_{2}^{c} \end{bmatrix} = -\begin{bmatrix} R_{1}^{-1}B_{1}^{t} & 0 \\ 0 & R_{2}^{-1}B_{2}^{t} \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix}$$
(18)

Where

$$\begin{split} F_{11} &= -[G_{11}^{-1}G_{12}G_{22}][K_{2}^{*}A_{21} - A_{12}^{t}K_{1}^{*}], \\ F_{21} &= -D_{22}[K_{12}^{*}A_{21} - A_{12}^{t}K_{1}^{*}] \\ F_{12} &= [G_{11}^{-1} + G_{11}^{-1}G_{12}D_{22}G_{21}G_{11}^{-1}][K_{12}^{*}A_{21} - A_{12}^{t}K_{1}^{*}] \\ G_{12} &= A_{21}^{t}, \quad G_{21} &= A_{12}^{t}, \\ F_{22} &= -[D_{22}G_{21}G_{11}^{-1}][K_{1}^{*}A_{12} - A_{21}^{t}K_{2}^{*}], \\ G_{11} &= [K_{1}^{*}B_{1}R_{1}^{-1}B_{1}^{t} - A_{1}^{t}], \\ G_{22} &= [K_{2}^{*}B_{2}R_{2}^{-1}B_{2}^{t} - A_{21}^{t}], \\ D_{22} &= [G_{22} - G_{21}G_{11}^{-1}G_{12}]^{-1}, \end{split}$$

From (13), (18) and (19), the coordinating controls take the form

$$v_{s} = \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix} = -R \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix}$$
(19)

and the coordinating gain matrix R is given by

$$R = -\begin{bmatrix} R_1^{-1}B_1'(K_1^* + F_{11}) & R_1^{-1}B_1'F_{12} \\ R_2^{-1}B_{12}'F_{21} & R_2^{-1}B_2'(K_2^* + F_{22}) \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

This scheme is depicted in Fig. 2 where the controller scheme consists of two-levels: local controls at the lower level and coordinating control at the higher level.



Fig. 1 Waste-water treatment process



Fig. 2 Coordinated control structure

B. Method 2

An alternative scheme is to constrain the gains K_s^c so as to neutralize the effect of coupling between the subsystems. It follows from [7] these gains can be computed by the formula

$$K_{s}^{c} = (BB^{t})^{-1}B^{t} \begin{bmatrix} 0 & A_{2} \\ A_{3} & 0 \end{bmatrix}$$
(20)

In the next section, we perform numerical simulation based on the coordinated schemes (18) and (19).

IV. SIMULATION RESULTS

Starting with the specification of the weighting matrices, we ran several computer simulations on the open loop nonlinear and linearized models. Satisfactory simulation results were attained using

$$\begin{split} & Q = diag \; [10, \, 10, \, 10, \, 1, \, 0.1, \, 0.5, \, 1, \, 1, \, 0.01] \\ & R = diag \; [1, 1, \, 1, \, 0.5]. \end{split}$$

Then we evaluate expressions (18) and (19). The difference between the two expressions was found to be small. The gain matrices are given by:

$$\begin{split} \mathbf{K}_{2}^{*} &= \begin{bmatrix} -28.7 & -0.31 & -1.01 & 1.65 & -0.85 & 0.32 \\ 4.45 & -0.12 & 0.57 & -1.75 & 0.67 & 0.07 \end{bmatrix}, \\ \mathbf{K}_{2}^{*} &= \begin{bmatrix} 0.04 & 0.01 & -0.02 \\ -1.12 & 0.11 & -0.38 \end{bmatrix} \\ \mathbf{K}^{*} &= \begin{bmatrix} -8.6 & -0.15 & -1.01 & 0.56 & -0.33 & 0.18 & 0.47 & 0.33 & -0.51 \\ 1.55 & -0.04 & 0.09 & -0.27 & 0.06 & 0.04 & 0.42 & -0.19 & -0.82 \\ -6.43 & -0.05 & -0.01 & 0.02 & -0.01 & 0.31 & 0.01 & 0.01 \\ -7.35 & 0.13 & 0.05 & -1.08 & 0.27 & -0.82 & 0.11 & 0.05 & 0.04 \end{bmatrix} \end{split}$$

The corresponding state (9) and control (4) trajectories are plotted in the figures below. From the ensuing results, it is quite clear that the developed coordinated control methods have been effective in regulating the dynamic behavior of the waste water treatment variables.





Fig.4 Trajectory of readily biodegradable substrate.



Fig.5 Trajectory of particulate inert organic matter



Fig.7 Trajectory of heterotrophic biomass



Fig.6 Trajectory of slowly biodegradable substrate.



Fig.8 Trajectory of oxygen dissolved in the diet



Fig.9 Trajectory of heterotrophic biomass in the diet



Fig.10 Trajectory of organic matter recycled inert particulate



Fig.11 Trajectory of slowly biodegradable substrate recycled



Fig.12 Trajectory of inlet flow



Fig.13 Trajectory of flow recycling between the clarifier and the reactor



Fig.14 Trajectory of Air flow in the aeration tank



Fig.15 Trajectory of Flow Purge

V. CONCLUSION

In this work, we have developed coordinated control methods for the regulation policies of high-strength waste water treatment process. A dynamic model for the activated sludge process with wastewaters has been presented and linearized for control studies. The coordinated control method consists of three components: two components have independent direct effects on the behavior of the aerated basin

and the settling tank and the third component coordinates the overall operation. Simulation results have shown the effectiveness of the developed methods.

APPENDIX

A. Variables and model parameters:

- S_i : concentration of soluble inert organic matter (mg / l)
- $S_{s}{:}\ concentration \ of \ readily \ biodegradable \ substrate \ (mg \ / \ l)$
- X_i : concentration of particulate inert organic matter (mg / l)
- $X_{s}\!\!:$ concentration of slowly biodegradable substrate (mg /l)
- X_h : concentration of heterotrophic biomass (mg / l)
- $X_{h,rec}$: concentration of recycled heterotrophic biomass (mg / l) $S_{i,nin}$: concentration of soluble inert organic matter in the diet (mg / l)

X_{1,rec}: concentration of organic matter recycled inert particulate (mg / 1)

 $X_{s,rec}$: concentration of slowly biodegradable substrate recycled (mg / l)

 $S_{i,in}$: concentration of soluble inert organic matter in the diet (mg / 1)

 S_{ssin} : concentration of readily biodegradable substrate in feed (mg / l)

 X_{isin} : concentration of particulate inert organic matter in the diet (mg / l)

 $X_{s,in}$: concentration of slowly biodegradable substrate in feed (mg / l)

 $X_{h,in}$: concentration of heterotrophic biomass in the diet (mg / l)

 $S_{\rm osin}$: concentration of oxygen dissolved in the diet (mg / l) Q_{in} : inlet flow (l / h)

- p_1 : Speed specific heterotrophic growth (1 / h)
- p₂: Speed specific mortality of heterotrophic (1 / h)

 p_3 : Speed specific hydrolysis of organic matter absorbed (1 / h)

 b_{H} : coefficient of mortality of heterotrophic organisms (1 / h) fx_1 : Fraction of inert COD generated by the death of the biomass

 Q_r : flow recycling between the clarifier and the reactor (1 / h) Q_w : Flow Purge (1 / h)

Yh: Coefficient of Performance of heterotrophic biomass

- Q_L : Air flow in the aeration tank (1 / h)
- C_s : constant saturation of dissolved oxygen (mg / l)
- V_r: Volume of aeration basin (s)
- V_{dec}: Volume of the settler (s)

 μ_{max} : maximum growth rate of heterotrophic microorganisms (1 / h)

- K_s: coefficient half-saturation of readily biodegradable
- substrate for heterotrophic biomass (mg / l)
- K_h: maximum specific rate for hydrolysis (1 / h)

K_x: Coefficient of half-saturation for hydrolysis of slowly biodegradable substrate

C. Constant values

$$\begin{split} S_{I,In} = &30, \ S_{S,In} = &50, \ X_{I,In} = &25, \ X_{S,In} = &125, \ V_r = &2000, \\ V_{dec} = &1500, \ Y_H = &0.67, \ K_S = &20, \ K_H = &3, \ K_X = &0.03, \\ \hline X_{S,In} = &125, \ \overline{X_{H,In}} = &30, \ \mu_{max} = &0.67, \ b_H = &0.62, \\ C_S = &10, \ f_{x_1} = &0.086 \end{split}$$

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