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The Derivation and Impact of an Optimal Cut-Off Grade Regime Upon Mine Valuations

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Abstract—

When mining companies extract from a mine they are faced with many decisions. Prior to extraction the mine is divided up into 'blocks', and the order of extraction of these blocks is decided. However, because commodity prices are uncertain, once each block is extracted from the mine the company must decide in real-time whether the ore grade is high enough to warrant processing the block further in readiness for sale, or to waste the block. The optimal cut-off ore grade, below which a block should be wasted, is not simply a function of the current commodity price and the ore grade of the block itself, but as we show, is a function also of the ore-grades of subsequent blocks, the costs of processing, and the bounds on the rates of processing and extraction. By using a price uncertainty contingent claims approach, this paper shows how to derive an efficient mathematical algorithm to calculate and operate an optimal cut-off grade criterion throughout the extraction process. The model is applied to a real mine composed of some 60,000 blocks, and shows that an extra 10% of value can be created by implementing such an optimal regime.

Keywords: Real-Options, Stochastic Control, Reserve Valuations.

1 Introduction

The planning and scheduling of extraction from a mine is a complicated process which is made in the presence of uncertainties such as the future commodity price and estimated ore-grade. Prior to extraction, the mine is graphically divided up into blocks, each containing its own estimated quantity of ore. From these blocks one can determine the optimal extraction pathway through the mine using software such as the Gemcom-Whittle package [13], which allows companies to construct feasible pit shapes that obey slope constraints on the angle of the pit, transportation needs and work-force limitations. Whilst this algorithm may be used several times throughout a mine's life, so as to make sure the mine plan is consistent with market conditions, on a day-to-day basis the mine must take more detailed scheduling decisions in real-time. The key real-time decision to take is whether or not to process the latest extracted block (e.g. by milling or electrolysis). The block's intrinsic value varies with its ore grade and with the underlying commodity price. However the mine owners must also strive to fill the operating capacity of the processing unit, since the cost of processing can be many times the cost of extraction. We define a 'cost-effective' block as one whose ore grade is high enough to pay the cash costs of processing, at the current price. However the cut-off ore grade, above which a block should be processed, need not be set as low as the grade above which the block will be cost-effective to process. Disparity between the rate of extraction and the maximum processing capacity mean that there can be an opportunity cost to processing all cost-effective material, since the small short-term gain of processing a low grade block could be surpassed by bringing forward the processing of more valuable blocks instead. The optimal wasting of potentially cost-effective material is the focus of this paper.

To highlight the above point, let us consider a trivial case where the mine has a stock of 3 blocks awaiting processing, extracted in order, A, B and C, whose current market values after processing costs are $V_A = \$1, V_B = \$50$, and $V_C = \$1000$. Whilst, classically, analysis has often been indifferent to the order of processing, with enough discounting applied one can see that by an optimal cut-off criterion, it would be best to simply waste A and get on with processing B and C. This is because the value gained in processing A is less than the time value of money lost in waiting to process B and C at a later date. This lack of consideration of the discount rate has been highlighted before as a drawback in current mine planning [12] but, as yet, little progress has been made with it. Another consequence of an optimal cut-off grade decision is that it can become efficient to increase the rate of extraction of poor quality ores to keep the processing plant loaded. This paper also shows how to determine the corresponding optimal local variation in the extraction rate, enabling mining engineers to respond promptly to joint variations in ore quality and underlying price.

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Other approaches to mine valuation have relied upon simulation methods to capture the uncertainty of price and ore-grade [8], [10], [7]. These methods can be extremely time consuming, with running times of several hours [3], and can often lead to sub-optimal and incomplete results. Using these simulation techniques, optimal cut-off grades were investigated, [9], although little insight into the core dynamics, performance or robustness was supplied. To make a step-change away from these methods, partial differential equations (PDEs) can be implemented to capture the full mine optimisation process. Using PDEs, optimal extraction rates were investigated by [6], which built on work by [2] and [4]. The inclusion of stochastic ore-grade uncertainty was then tested [5]. This enabled mine valuations to be produced in under 10 seconds and showed that the effect on mine value of stochastic ore-grade variation is much less than the effect of stochastic price. Whilst the mathematics and numerics of this PDE approach are relatively complex at the outset, once solved they produce highly accurate results in short times - complete with model input sensitivities. This paper extends the use of PDEs, adding a model for tactical processing decisions under foreseeable variations in ore grade and unforeseeable fluctuations in price. This shows that when processing capacity is constrained, the ability to maximise the value of processing by varying the cut-off ore grade can add significantly to mine value when optimally applied. By solving rapidly under a range of processing constraints, the scale of the processing plant can itself be optimised.

In Section 2 we derive the model using a price uncertainty contingent claims approach, and show how the optimal cut-off decision rule works in Section 3. We then apply the model to a mine composed of some 60,000 blocks in Section 4, to show how much extra value the running of an optimal cut-off grade regime can add to a valuation. We draw together our concluding remarks in Section 5.

2 Model Construction

To create the framework for a mine valuation, V, under an optimal cut-off grade, we first prescribe three state-space variables. These are the price S per unit of the underlying resource in the ore, the remaining amount of ore within the mine Q and time t.

Before specifying how to determine V, we first define the underlying price uncertainty process. Within this paper we assume the underlying price S to follow a geometric Brownian motion,

$$dS = \mu S dt + \sigma_s S dX_s, \tag{1}$$

where μ is the drift and σ_s the volatility of S. The random variable dX_s, is normally distributed as $N(0, \sqrt{dt})$. We use this price process without loss of generality, since other price processes (such as mean-reverting Brownian motion) can easily be implemented by the techniques described here.

Using this notation, we may apply Ito's lemma [14] to write an incremental diffusive change in V as,

$$\begin{aligned} dV =& \sigma_s \frac{\partial V}{\partial S} dX_s + \frac{\partial V}{\partial Q} dQ \\ &+ \left(\frac{\partial V}{\partial t} + \frac{1}{2}{\sigma_s}^2 \frac{\partial^2 V}{\partial S^2} + \mu \frac{\partial V}{\partial S} \right) dt, \end{aligned} \tag{2}$$

where we have taken powers of $(dt)^2$ and $(dQ)^2$ to be negligible. We are able to remove the dQ term via the relationship between Q and t by specifying the rate of extraction, q_e, namely,

$$dQ = -q_e dt. \tag{3}$$

This extraction rate is the function we wish to solve for in our optimal cut-off regime, as it determines both how we progress through the mine and, as a consequence, which blocks we choose to waste. The rate of extraction will obviously have limitations on its size, $q_e \in [0, q_{max}]$, which itself could be a function of time. The rate of extraction is closely linked to the rate of processing, which should be kept as close as possible to its physical maximum q_p. Hence q_e must be high enough for the processing unit to operate at its capacity, q_{max}, i.e. there must always be enough cost-effective ore-bearing material available to be processed. Optimal variation in the extraction rate has already been shown to produce improved valuations, [6], although this was achieved without considering processing limitations or grade variation.

With this relationship, (3), equation (2) can be transformed into,

$$\begin{aligned} dV =& \sigma_1 \frac{\partial V}{\partial S} dX_s \\ &+ \left(\frac{\partial V}{\partial t} - q_e \frac{\partial V}{\partial Q} + \frac{1}{2}{\sigma_s}^2 \frac{\partial^2 V}{\partial S^2} + \mu \frac{\partial V}{\partial S} \right) dt. \end{aligned} \tag{4}$$

To follow the conventional approach in creating and valuing risk-free portfolios we construct a portfolio, Π, in which we are instantaneously long in (owning) the mine and short in (owing) γ_s amounts of commodity futures contracts. This defines $\Pi = V - \gamma_s S$, such that,

$$d\Pi = dV - \gamma_s dS. \tag{5}$$

This portfolio is designed to contain enough freedom in γ_s to be able to continually hedge away the uncertainty of dX_s, which is the standard approach in creating risk-free portfolios [1], [11], [14]. It also means that within a small time increment, dt, the value of Π will increase by the risk-free rate of interest, minus any economic value generated and paid out by the mine during the increment. This economic value is typically composed of two parts, the first, negative, being the cost to extract, $q_e \epsilon_M$, and

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the second, positive, the cash generated by selling the resource content of the ore processed, $q_p(SG - \epsilon_P)$. Here ϵ_M is the cost of extraction per ore tonne, ϵ_P is the processing cost per ore tonne, and G is the ore-grade (weight of commodity per ore tonne). The reason why the economic functions contain the factors q_e or q_p is that we wish to maximise value by varying q_e in real time, so as to maintain q_p at its upper bound. To have turned the block model into one in which we use the continuous function, G, we have assumed that blocks are small enough that they can be approximated as infinitesimal increments of volume.

As discussed in Section 3, the decision whether to process or waste the next block must be optimised. Before or after optimisation the incremental change in portfolio value may be written as

$$d\Pi = r\Pi dt - \gamma_S \delta S dt - q_p(GS - \epsilon_P)dt - q_e\epsilon_M dt. \quad (6)$$

By setting the appropriate value of γ_s to be

$$\gamma_s = \frac{\partial V}{\partial S},$$

and substituting equations (1), (4) and (5) into (6), we may write our mine valuation equation as,

$$\frac{1}{2}{\sigma_s}^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - q_e \frac{\partial V}{\partial Q} + (r - \delta) S \frac{\partial V}{\partial S} - rV + q_p(GS - \epsilon_P) - q_e\epsilon_M = 0. \quad (7)$$

This is of the same form as that derived by Brennan and Schwartz (eq. 15, [2]), except that they added taxation terms, but did not model processing constraints or variations of ore grade.

We next need to prescribe boundary conditions for (7). The boundary condition that no more profit is possible occurs either when the reserve is exhausted $Q = 0$, or when a lease to operate the mine has reached its expiry date $t = T$, hence:

$$V = 0 \quad \text{on} \quad Q = 0, \quad \text{or} \quad t = T. \quad (8)$$

Since the extraction rate will have a physical upper bound, the extraction rate and cost will not vary with S when S is large. This permits a far field condition of the form,

$$\frac{\partial V}{\partial S} \to A(Q, t) \quad \text{as} \quad S \to \infty. \quad (9)$$

When the underlying resource price is zero we need only solve the reduced form of equation (7) with $S = 0$, which reduces to

$$V = e^{-rt} \int_0^T q_e \epsilon_M(z) e^{rz} dz. \quad (10)$$

This therefore completes the determination of our core equation, and its boundary conditions. We can now define the optimising problem which we wish to solve: we must determine the optimal extraction rate, q_e^*, at every point in the state space which maximises the value V, where V satisfies equation (7), with $q_e = q_e^*$, subject to the defined boundary conditions. Problems of this type may be solved numerically using standard finite-difference techniques [4]. All results in this paper have been thoroughly tested for numerical convergence and stability.

We must now show how the optimal q^* and its corresponding cut-off grade is to be incorporated into the maximisation procedure.

3 Cut-Off Grade Optimisation

The selection of the cut-off grade criteria boils down to whether a cost-effective block should be processed or not. This is because there is the possibility a more valuable block could be brought forward in time to be processed, which otherwise would loose more time-value of money than the value gained from processing the first block. To highlight this point let us consider the order of extracted of blocks from a mine, which we (hypothetically) place in a chronologically ordered row. As we operate the processing unit of the mine, we must pass along this row and decide which blocks to process and which blocks to waste. In reality, although we know the (estimated) ore-grades of the blocks in advance, until we know for certain the market price at the time of processing we cannot know what cashflow it will generate. Yet even if we assume a constant price, we can still show how this cut-off grade decision can be achieved, and what drives its determination.

Consider a highly simplified mine, as shown in Figure 1, which is composed of only two blocks. We allow the mine to have the capacity within the rate of extraction to immediately process either the first block, Block1, or its successor, Block2. As such, the comparison is between the value of processing both blocks in order, or the value of only processing Block2. With a constant price we can write down the net present value of these two (already extracted) blocks, where we shall process both,

$$V_1 = (SG_1 - \epsilon_P) + (SG_2 - \epsilon_P)e^{-r\delta t}. \quad (11)$$

Here δt is the time it takes to process one block, and G_1 and G_2 are amounts of ore within each Block1 and Block2, respectively. This value must be compared to the decision to waste the first block and process only the second block, which would have value,

$$V_2 = (SG_2 - \epsilon_P). \quad (12)$$

This comparison between V_1 and V_2 is one the algorithm must continually take throughout solution space. To demonstrate how the selection depends upon the underlying price, Figure 1 shows the choices available for two different commodity prices, one high ($S = \$10,000$

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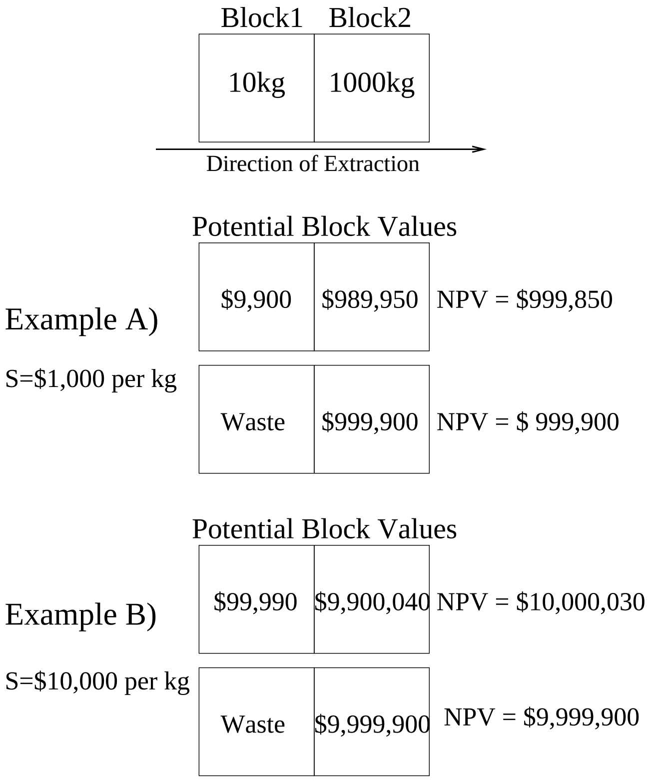

Figure 1: Two examples of how price may effect the order in which blocks are processed so as to maximise a mines NPV. Example A is made with a low commodity price, $S = \$1,000$ kg^{-1}, and Example B is made with a high commodity price, $S = \$10,000$ kg^{-1}.

kg^{-1}) and one low ($S = \$1,000$ kg^{-1}). These are made with prescribed parameter values,

$$r = 10\%, \quad \epsilon_P = \$100 \text{ block}^{-1}, \quad \delta t = 0.1 \text{ year}. \tag{13}$$

As can be seen, in the low-price case, Example A), it is best to process only the second block. But in the high commodity price case, Example B), it is best to process both blocks. This simple example demonstrates (albeit with rather exaggerated parameter values) how the selection need be actively taken, and how different values of the underlying price, and discount rate, will effect the optimal cut-off decision to take. Another consequence of this optimal decision taking, is that the mine will be exhausted earlier than might have been previously thought, since we wasted the first block and only processed the second, hence a mine owner could agree a shorter lease (expiry date T) on the mine.

4 Example Valuation

4.1 Mine Data

We now apply our optimal cut-off grade model to a real mine of some 60,000 blocks, whose block by block ore-grade and sequence of extraction were supplied by Gemcom Software International. This mine has an initial capital expenditure of some \$250m. They also supplied a fixed reference price S_{ref}, for us to compare valuations with. We ourselves assumed a maximum extraction rate of five times the processing rate. This seems broadly realistic, and it restricts the mine to wasting no more than 80% of any section of cost-effective ore. The other parameter values are,

$$\begin{aligned} &r = 10\% \text{ year}^{-1}, \quad \delta = 10\% \text{ year}^{-1}, \\ &\sigma_s = 30\% \text{ year}^{-\frac{1}{2}} \quad S_{ref} = \$11,800\text{kg}^{-1}, \\ &\epsilon_P = \$4 \text{ tonne}^{-1}, \quad \epsilon_e = \$1 \text{ tonne}^{-1}, \\ &Q_{max} = 305,000,000 \text{ tonnes}, \\ &q_p = 20,000,000 \text{ tonnes year}^{-1}. \end{aligned} \tag{14}$$

Whilst the ore-grade is quite volatile, it was shown in [5] that a suitable average of the estimated grade quality could be used without any sizeable alteration to the valuation. Using this average, Figure 1 shows the economic worth throughout extraction for each part of the mine, where we have assumed the price to remain at its prescribed reference price, $S_{ref}G - \epsilon_P$. This highlights how the grade varies through the extraction process, and it is with reference to this grade variation that we shall compare the regions where it is optimal to speed up extraction and consequently waste certain parts of the ore-body.

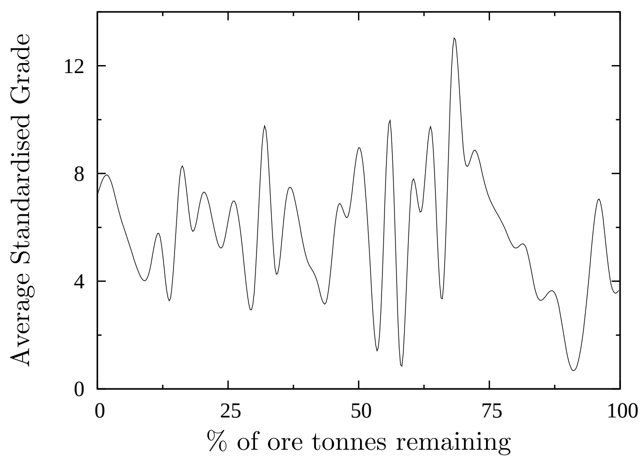

Figure 2: Given a block ordering in the mine, the average standardised grade value is the cash value of ore (against reference price) minus processing costs per tonne of ore.

4.2 Results

For the example mine, we first calculate and compare two different valuations made with, and without, the optimal cut-off criterion. Figure 3 shows two sets of valuations: the lower pair of lines shows the valuations made assuming a constant price ($\sigma_s = 0\%$), and the upper pair of lines shows the effect of including both price uncertainty ($\sigma_s = 30\%$) and the option to abandon the mine when

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the valuation becomes negative (which is a standard option to include in a reserve valuation [2]). In each pair of lines the lower, dotted lines show valuation without a cut-off regime, and the higher, solid lines show valuation with the optimal cut-off regime. The optimal cut-off regime increases the mine valuation by up to 10%, with increasing benefit at higher prices. This may seem surprising, but although the mine is always more profitable at higher prices, the opportunity cost of not allocating the finite processing capacity to the best available block does itself grow.

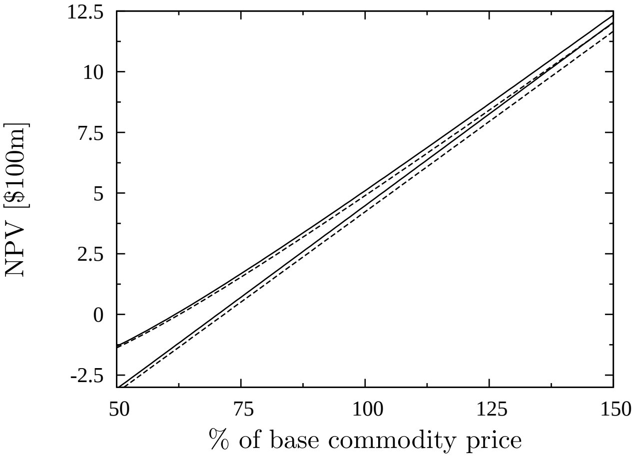

Figure 3: NPV of the mine against percentage of reference price for two different sets of valuations. The two lower lines are for a constant price while the two upper lines include price volatility and the abandonment option. NPV for the optimal cut-off regime is shown by solid lines, and no cut-off by dashed lines.

An obvious question which arises from this analysis is, how do we decide which ore-grades we should waste, and when? Since we know what the current underlying price is, we can look at the corresponding slice through the 3-D surface of the optimal cut-off grade and see for which regions we would waste the ore. With this we can refer back to the corresponding grade of Figure 2 and easily calculate what these grades actually are. For example, by looking at the closed regions of Figure 4 we can see the optimal cut-off grades for two different commodity prices, $S = 100\%$ and $S = 200\%$ of the reference price. In these two particular cases, they both appear to correspond to a standardised cut-off grade (Figure 2) of 2 units. In addition, one can also see the points in the state space at which the optimal trajectory (thick solid line in Figure 4) speeds up extraction, thereby wasting ore. The trajectory is calculated by integrating the ODE in (3) for a given extraction regime. The difference between the dotted line (trajectory for the no cut-off situation), and the thick straight line of the optimal cut-off regime therefore give an indication of the total amount of ore wasted.

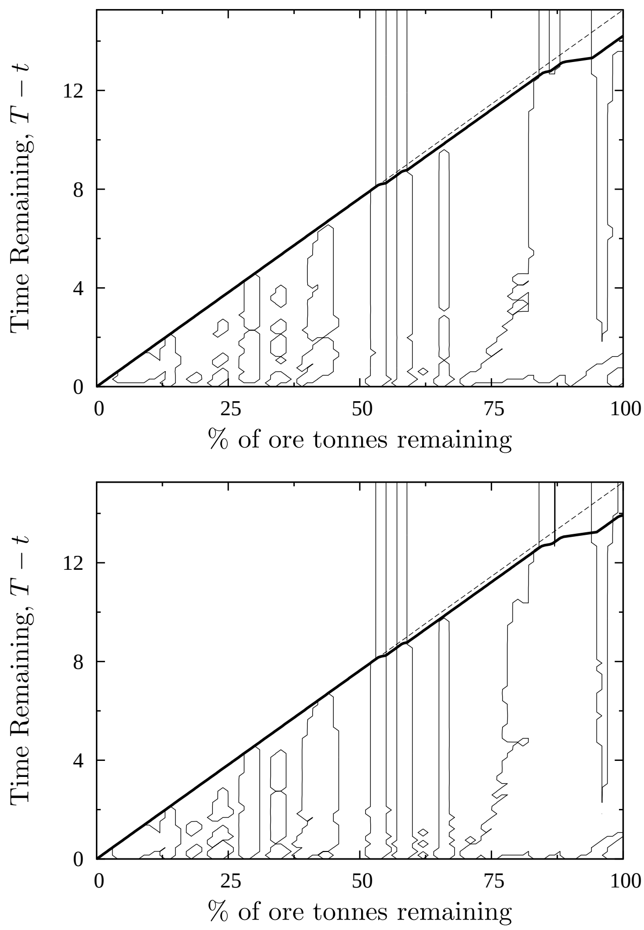

Figure 4: Graphs showing the optimal cut-off regions for an extraction project for two different price levels, medium (top), and high, (bottom). The closed regions contained within the thin solid lines show where ore is wasted and the extraction rate is increased. The dashed line represents the one realisation of a trajectory followed with no cut-off, while the thick solid line represents the realisation of the trajectory followed with optimal cut-off.

Finally, Figure 5 shows how the NPV depends upon the expected expiry time for extraction if one operates an optimal cut-off regime (solid line) or not (dotted line). If the mine chooses the optimal regime, the maximum NPV occurs just after 14 years, as opposed to the life of the mine being maximal at mine exhaustion at 15 years (as it is with no cut-off). This reflects the fact the under an optimal regime the mine will occasionally increase its extraction rate, thereby reaching the final pit shape in a shorter time.

5 Conclusions

This paper has shown how to derive the PDE framework for a mine to operate an optimal cut-off grade regime. We have valued the ‘option’ to process or not to process, and given the optimal decision rule for implementing it.

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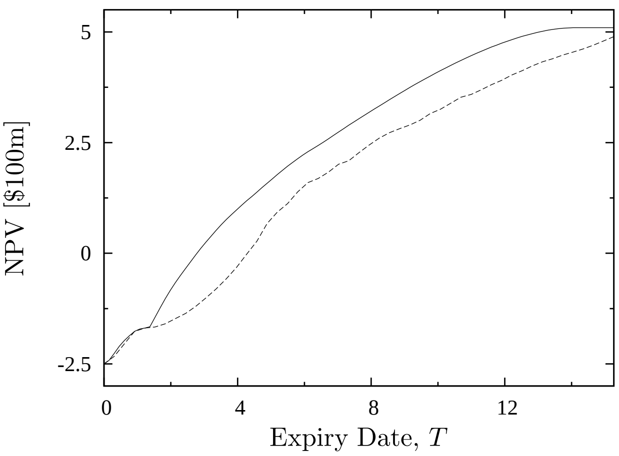

Figure 5: The NPV of the mine against time remaining on the option on the mine given that 100% of the mine is present. The solid line is with optimal cut-off, dashed without.

The model relies upon a contingent claims framework for price uncertainty, which allows the mine to react to all future market conditions. In our model the option adds around 10% to the expected NPV of an actual mine of 60,000 blocks. One natural extension will be to add - and to optimise - a dynamic stockpile. This will allow any block which it is not optimal to process immediately to be retained for potential processing at an optimal later stage, thus increasing the mine valuation further.

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