Theoretical Model of Stock Trading Behavior with Biases

Ke Liu, Kin Keung Lai, Jerome Yen

***Abstract*—Stock investors are not fully rational during their trading, and many behavioral biases affect their trading behavior, such as representative bias and disposition effect. However, most of the literature on behavioral finance cast efforts on explaining empirical phenomena observed in financial markets, but little on how individual investors' trading performance is affected by their behavioral biases. As against the common perception that behavioral biases are always detrimental to investment performance, we conjecture that these biases can sometimes yield better trading outcomes for investors. Focusing on representative bias and disposition effect, we construct a mathematical model in which the representative investor follows a Bayesian trading strategy based on an underlying Markov chain, switching between Trending regime and Mean-reversion regime. By this model, we are able to undertake scenario analysis to track investor behavior and performance along the time, under different patterns of market movements. Results validate our conjecture by showing that the effect of behavioral biases can sometimes be positive on investor performance.**

***Index Terms*—Representative Bias, Disposition Effect, Bayesian Investor, Trading Behavior**

I. Introduction

Although EMH (Efficient Market Hypothesis, by Fama [1]) sounds theoretically beautiful and serves as the foundation for most modern financial models, many empirical researches in finance have widely found evidence of anomalies against EMH [2][3]. This incompetence of modern finance with explaining many observed phenomena is due to the building blocks of it, i.e. expected utility theory and absolute arbitrage assumption. Thus, rather than treating agents as rational in modern finance, behavioral finance argues that people can make systematic errors when making decisions because of mistaken beliefs and psychological biases. Research in this field is mainly cast in two directions. The first direction is to discover those behavioral biases by psychological experiments, or applying cognitive psychology and theories on behavioral biases for the explanations of empirical phenomena observed in financial markets, especially those anomalies modern finance fails to expound [4][5]. However, in spite of the great explanatory power this line of research offers, contradictions among various biases together with the qualitative and descriptive nature of it cause large difficulties, when applying these findings to practical problems solving ex ante. Therefore, the further development of behavioral finance is growingly demanding the second direction of research: modeling of behaviors quantitatively and precisely. Though, owing to the uncertain and equivocal property of human behaviors, existed literature in this line often construct models with weight too simplified assumptions and largely differ from reality. For example, many simply use the price difference to determine the trading amount of shares, in order for modeling momentum traders. Obviously, momentum traders can not be trading in so simple a strategy.

In this light, this paper assumes the representative investor following a Bayesian trading strategy, whereas the investor believes that market state can switch between trending and mean-reversion, as modeled by an underlying Markov chain. By this model, we are able to undertake scenario analysis to track the investor's behavior and performance along the time, under each typical pattern of market movements. The main purpose of this paper is to see whether the effect of behavioral biases can sometimes be positive on investor's performance.

The rest of this paper is organized as follows: Section 2 elaborates in details the model construction and formulation; In Section 3 we simulate different typical market scenarios to investigate the investor's behavior and performance under each market pattern, and the impact of disposition effect; Finally, Section 4 concludes.

II. Modeling of Investor Trading Behavior

A. Model Rationale

Representative bias can cause a two-edged effect, i.e. on one hand it may yield investors' habit of extrapolative expectation (assuming the previous price trend pattern to continue in future price movements and then chasing the trend); while on the other hand it may foster investors' belief in mean-reversion. In addition, the interesting experiment designed by Andreassen and Kraus [6] provides striking evidence that, subjects can switch their trading behavior to trend chasing from tracking an average price level (i.e. sell in price rise and buy in price fall), as significant changes in price have occurred over some periods. In such sense, we direct our effort to a modeling framework in which investor's trend chasing behavior together with mean-reversion belief can be reconciled. Fortunately, the basic concept used to model investor's response to company's earnings, in the

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eminent BSV model [7], can also be employed for our aim to model investor's response to price evolution.

We assume that there is a category of investors, whose trading strategy is quite reasonable and sophisticated, and seems to represent the reality better. They are aware of the importance of recognizing the market regime first. Obviously, the market will not always walk in trends, nor will it always revert to mean swiftly once it drifted away. Thus they set up a Markov chain governing market regime switching in their mind. For the sake of formulation expedience, they are modeled as a single representative investor. This investor follows a Bayesian strategy to update his/her belief on market using new price evidences continually along the time. The investor's belief can switch between "Trend-chasing" and "Mean-reverting". If he/she believes the market moving currently in Regime 1 (Trend-chasing), he/she would probably buy after observing an uprise of price; whereas if in Regime 2 (Mean-reversion), he/she buys after a market fall because he/she believes the market would much likely to revert to normal.

Here, one may argue that, the investor represented in this model only makes use of market information (price evolution) to determine his/her trading behaviors, while in practice investors may rely on other fundamental or economic information for trading decisions. However, this paper aims not at modeling all kinds of investors together, which may become a tremendously arduous endeavor. Indeed, the Bayesian investor in our model can be viewed as representative of chartists, technical traders and all those who use historical market information to predict the market. This type of traders occupies a large share of all market participants, and their performance under psychological biases is of considerable interest. Additionally, in many stock markets especially of those emerging markets, most individual investors tend to trade just by watching patterns of market movements, irrespective of other types of information.

B. Model Deduction

Of the two-state Markov chain in the investor's belief, the transitional probability matrix ***TP*** is:

$$TP = \begin{bmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{22} & \lambda_{22} \end{bmatrix} \quad (1)$$

$$0.5 \le \lambda_{11} < 1,\ 0.5 \le \lambda_{22} < 1$$

Where, $\boldsymbol{\lambda}_{11}$ is the believed probability of market remaining in Regime 1, and $\boldsymbol{\lambda}_{22}$ is the believed probability of market remaining in Regime 2.

Price change at time $\boldsymbol{t}$ $\boldsymbol{Z_t}$ is defined as below, and $\boldsymbol{X_t}$ is the market price.

$$z_t = X_t - X_{t-1} \quad (2)$$

One main concern of the investor is the sign of $\boldsymbol{Z_t}$ which indicates the price movement direction.

$$z_t \begin{cases} > 0 & \text{price up} \\ = 0 & \text{price unchanged} \\ < 0 & \text{price down} \end{cases}$$

The posterior probability of Regime 1 given current price information at time $\boldsymbol{t}$ is defined as:

$$q_t = \Pr(S_t = 1 \mid z_t, q_{t-1}) \quad (3)$$

Then, following the probability transition along the two-regime Markov chain, the prior probability of Regime 1 estimated at previous time period is:

$$\Pr(S_{t+1} = 1 \mid z_t, q_t) = \lambda_{11} q_t + (1-\lambda_{22}) \cdot (1-q_t) \quad (4)$$

Obviously, the prior probability of Regime 2 estimated at previous time is one minus prior probability of Regime 2. According to the Bayes Theorem, we can update the investor's belief on the current state (at time $\boldsymbol{t+1}$) being Regime 1, using the new price evidence $\boldsymbol{Z_{t+1}}$, as formulated below:

$$\begin{aligned} q_{t+1} = {} & \Pr(z_{t+1} \mid S_{t+1} = 1, z_t) \cdot \Pr(S_{t+1} = 1 \mid z_t, q_t) \ / \\ & [\Pr(z_{t+1} \mid S_{t+1} = 1, z_t) \cdot \Pr(S_{t+1} = 1 \mid z_t, q_t) \\ & + \Pr(z_{t+1} \mid S_{t+1} = 2, z_t) \cdot \Pr(S_{t+1} = 2 \mid z_t, q_t)] \end{aligned} \quad (5)$$

The two market regimes can be seen as hidden states, whereas what the investor can observe is price movements along the time and this measurable evidence is used to update belief on the hidden states. If the market currently dwells in Regime 1, then the probability of price momentum (continuation) denoted as $\boldsymbol{\lambda}_T$ can be defined as below:

$$\begin{aligned} \Pr(z_{t+1} > 0 \mid S_{t+1} = 1, z_t > 0) = \lambda_T \\ \Pr(z_{t+1} \le 0 \mid S_{t+1} = 1, z_t \le 0) = \lambda_T \end{aligned} \quad (6)$$

Similarly, the probability of price mean-reversion given market dwelling in Regime 2 is defined as below:

$$\begin{aligned} \Pr(z_{t+1} \le 0 \mid S_{t+1} = 2, z_t > 0) = \lambda_M \\ \Pr(z_{t+1} > 0 \mid S_{t+1} = 2, z_t \le 0) = \lambda_M \end{aligned} \quad (7)$$

$$0.5 \le \lambda_T < 1,\ 0.5 \le \lambda_M < 1$$

Then, based on the estimated prior probability of Regime 1 in next period $\mathbf{Pr}(\boldsymbol{S_{t+1}}\mathbf{=1} \mid \boldsymbol{z_t, q_t})$, the Bayesian investor forms in his/her belief the probability for witnessing a rise or fall in next period, given current price information. The four situations are all considered and corresponded in (8)-(11).

$$\begin{aligned} & \Pr(z_{t+1} > 0 \mid z_t > 0, q_t) \\ & = \Pr(z_{t+1} > 0 \mid S_{t+1} = 1, z_t > 0, q_t) \Pr(S_{t+1} = 1 \mid z_t, q_t) \\ & + \Pr(z_{t+1} > 0 \mid S_{t+1} = 2, z_t > 0, q_t) \Pr(S_{t+1} = 2 \mid z_t, q_t) \\ & = \lambda_T \cdot (\lambda_{11} q_t + (1-\lambda_{22}) \cdot (1-q_t)) \\ & + (1-\lambda_M) \cdot (1 - \lambda_{11} q_t - (1-\lambda_{22}) \cdot (1-q_t)) \end{aligned} \quad (8)$$

$$\Pr(z_{t+1} \le 0 \mid z_t \le 0, q_t) = \Pr(z_{t+1} > 0 \mid z_t > 0, q_t) \quad (9)$$

$$\begin{aligned} & \Pr(z_{t+1} > 0 \mid z_t \le 0, q_t) \\ & = \Pr(z_{t+1} > 0 \mid S_{t+1} = 1, z_t \le 0, q_t) \Pr(S_{t+1} = 1 \mid z_t, q_t) \\ & + \Pr(z_{t+1} > 0 \mid S_{t+1} = 2, z_t \le 0, q_t) \Pr(S_{t+1} = 2 \mid z_t, q_t) \\ & = (1-\lambda_T) \cdot (\lambda_{11} q_t + (1-\lambda_{22}) \cdot (1-q_t)) \\ & + \lambda_M \cdot (1 - \lambda_{11} q_t - (1-\lambda_{22}) \cdot (1-q_t)) \end{aligned} \quad (10)$$

$$\Pr(z_{t+1} \le 0 \mid z_t > 0, q_t) = \Pr(z_{t+1} > 0 \mid z_t \le 0, q_t) \quad (11)$$

Under the condition that no short-sell is allowed, the Bayesian investor's trading behavior is assumed as follows: Based on expected possibility of experiencing a price rise at

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next period (i.e. p_{t+1}), currently the investor would make a decision of his/her position (denoted as H_t) depicted in (13).

$$p_{t+1} = \Pr(z_{t+1} > 0 \mid z_t, q_t) \tag{12}$$

$$\begin{aligned} H_t &= A * \max\{p_{t+1} - (1 - p_{t+1}),\ 0\} \\ &= A * \max\{2 p_{t+1} - 1,\ 0\} \end{aligned} \tag{13}$$

Where, A is a constant used to link position holding decision and expected probability of price rise.

Then, trading behavior in each period can be determined from the difference between positions held by the investor in two adjoining periods. Equation (14) & (15) depict the formulas calculating buy value and sell value in each period, respectively.

$$V_t^B = \max\{H_t - H_{t-1},\ 0\} \tag{14}$$

$$V_t^S = \max\{H_{t-1} - H_t,\ 0\} \tag{15}$$

The formulation derived above for the Bayesian investor's trading behavior doesn't account for the disposition effect (abbr. DE) yet. Therefore, in what follows, sell behavior influenced by the behavior bias is modeled. We assume that the sell value, should any, is impacted by a coefficient B, which is further determined by the DE coefficient denoted as D herein. Then, the investor's position at time t can be adjusted as (16) shows. From (17), the effect on sell value produced by the investor's loss aversion can be quantified by the DE coefficient D.

$$H_t^{DE} = \max\{H_{t-1}^{DE} + V_t^B - B * V_t^S,\ 0\} \tag{16}$$

$$B = \begin{cases} 1/(1-D) & \text{while earning} \\ 1 & \text{breakeven} \\ 1-D & \text{while losing} \end{cases} \tag{17}$$

III. Scenario Analysis

A. Scenario Setting

In this section, how the Bayesian investor would behave and perform under four different possible market scenarios, in terms of the process of evolution of price, is discussed. Fig. 1(a) illustrates three basic patterns of the price evolution process, i.e. linear uptrend (Scenario 1), downtrend (Scenario 2), and single-cycle oscillation (Scenario 3, where price first rises and then falls back). Subsequently, in case of price oscillation, one may wonder whether the frequency of oscillation matters in determining the investor's trading behavior and performance. For this, four sub-scenarios (Scenario 4) are designed to investigate possible investor behaviors under different rates of oscillation of the market, as shown in Fig. 1(b). The purpose is to analyze the Bayesian investor's beliefs and behaviors under each scenario from three angles, i.e. believed probability of a price rise, position held by the investor, and cumulative profit. Also, the disposition effect is intensively studied. Then, we are interested in how the investor would perform, given different sets of characteristic parameters of the investor.

B. Summary of Results

To summarize the results obtained in the scenario analysis, Table 1 lists three outcomes concerning the investor's performance under each price scenario. The investor realizes neither profit nor loss in the market of downtrend, because he/she never engages in any trading behavior during the course. However, he/she would profit from market situation of uptrend and rise-fall oscillation, since he/she can gradually recognize the upward trend or the downtrend in price oscillation. When trading in fluctuating price process with different oscillation frequency, either gain or loss can occur to the investor. With a sufficiently swift oscillation of stock price, the investor would assume the market as evolving in mean-reversion regime, thus will take advantage of short price movements by perfectly buying low and selling high. Nevertheless, as the price fluctuates repeatedly in much slower manner, the investor will be involved in a dilemma in which he/she has great difficulty in making choice between trend following strategy and mean-reversion arbitrage. By this way, he/she may be tricked by the market itself and entailed large loss. As the price oscillation frequency continues to decrease to a sufficiently low level, however, the investor can again realize gains with his/her Bayesian trading strategy. Actually, the single-cycle oscillation in Scenario 3 can be seen as a special case of multi-cycle oscillation in Scenario 4 when price fluctuation frequency is as low enough as 12 periods per cycle. Therefore, either sufficiently low frequency or high frequency of market fluctuation can render the investor better performance, while the investor would suffer from loss in midst of a range of market cycling speed.

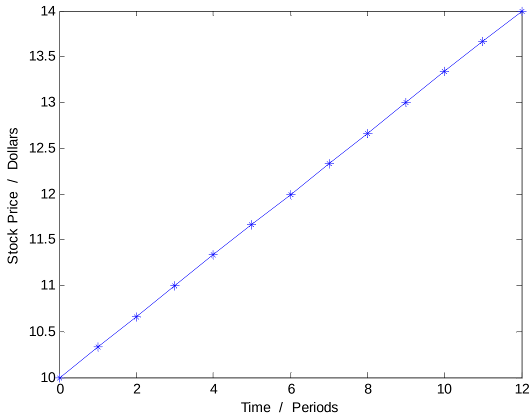

Figure 1(a). Basic Scenarios (1-3) of Stock Price
Scenario 1: Uptrend

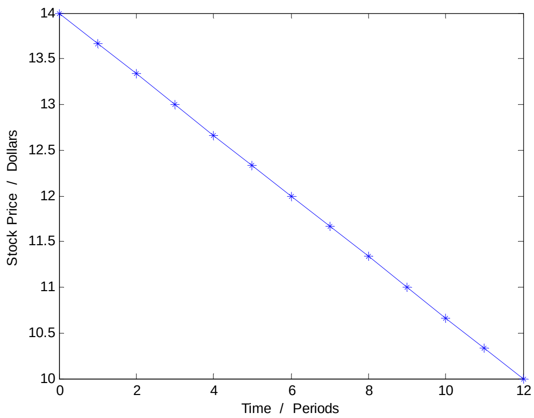

Figure 1(a). Basic Scenarios (1-3) of Stock Price
Scenario 2: Downtrend

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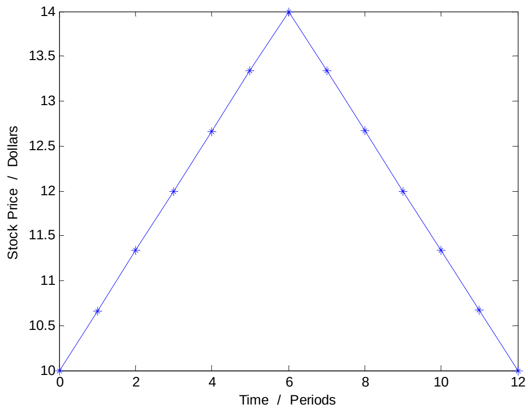

Figure 1(a). Basic Scenarios (1-3) of Stock Price
Scenario 3: Single-Cycle Oscillation

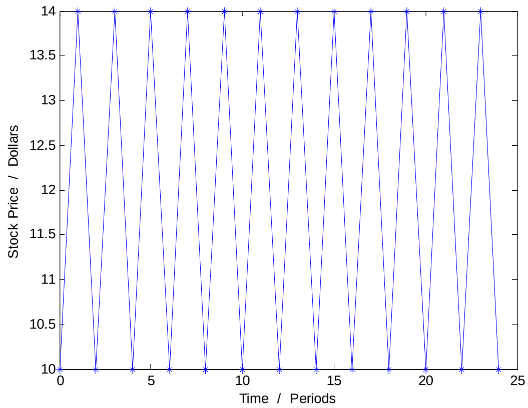

Figure 1(b). Scenario 4
Scenario 4.1: Two Periods per Cycle

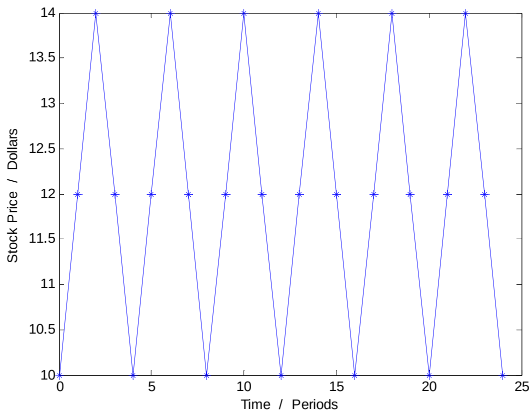

Figure 1(b). Scenario 4
Scenario 4.2: Four Periods per Cycle

Disposition effect exerts a direct influence upon the investor's selling behavior when he/she without DE shall sell. As a result, DE has no impact on the investor's trading when there is no selling at all, as is the case in Scenario 1 and Scenario 2. Whether DE turns out to be positive or negative upon trading is determined by two key factors: gain or loss the investor is entailed, and the market volatility which is represented by the fluctuation frequency. When market volatility is high enough (as in Scenario 4.1), the market can be seen as moving in mean-reversion regime and the investor fully realizes it while making profits. In this situation, DE should contribute to nice trading by making the investor more conservative, as he/she liquidates his/her position more decisively when he/she performs sell-high. However, the outcome for the investor is not improved by DE herein because with a zero DE the investor would also empty his/her position at higher price. In a market which oscillates in the way that tricks the Bayesian investor around, as in Scenario 4.2, the loss leads to the investor's reluctance to sell via the impact of DE, thus the investor still keeps a position that will partly capture short uptrend, although he/she fails to envision it. However, when the short downtrend lasts more periods, as in Scenario 4.3, the reluctance to liquidate position as caused by DE would incur more loss because the investor fails to timely escape from the downward market. As the periods for each uptrend and downtrend increase to sufficient level (as in Scenario 4.4), given a trading profit, the investor's conservatism caused by DE would impel his/her to more promptly withdraw his/her money from the downward market.

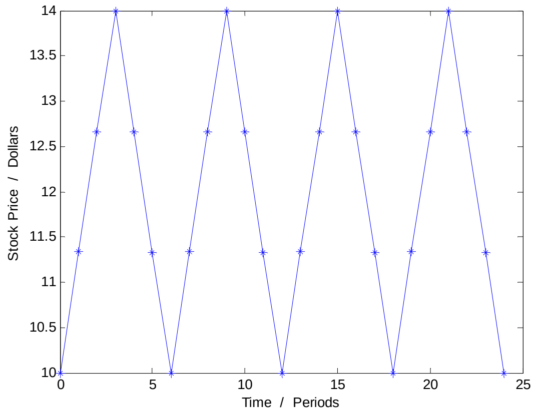

Figure 1(b). Scenario 4
Scenario 4.3: Six Periods per Cycle

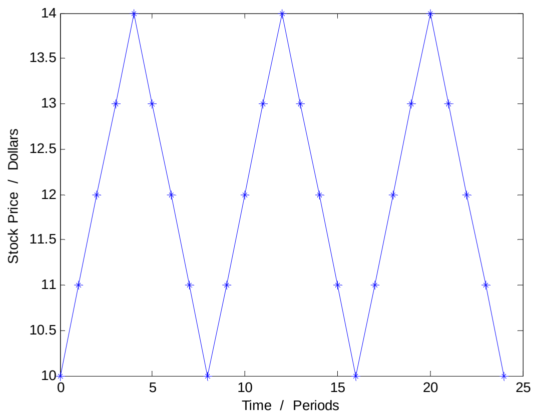

Figure 1(b). Scenario 4
Scenario 4.4: Eight Periods per Cycle

Further, the Bayesian investor's sensitivity (as represented by characteristic parameters) to market movements can largely affect his/her trading performance. Obviously, the agility of acting in the market direction is essential for market followers' performance. Especially in Scenario 4.3, if the sensitivity is sufficiently high, the investor would jump into a

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position of gain from loss. Generally speaking, more agile the investor is, more successful he/she would be in trading. However, this is in truth except a special case (as in Scenario 4.2), in which the market completely fools the investor around by engaging his/her into a disastrous buy-high-sell-low, while this underperformance is made even worse by higher sensitivity to market movements.

Finally, as depicted in Fig. 2, we vary the characteristic parameters over the full range to find the maximum as well as minimum of final cumulative profit, under each price scenario of oscillation. As the fluctuation frequency is mitigated, the span between maximum and minimum of profit is also reduced, which implicates that the investor's market sensitivity plays a less important role in slow market oscillation. Reasonably, highly volatile market demands higher agility to act, whereas low volatility market renders the investor more time to recognize the trends and then follow. We can also find that the best performance is attained in Scenario 4.1 with strong mean-reversion feature, while the worst one is in Scenario 4.2 with the tricky fluctuation fooling the investor to always buy high and sell low. Given a DE coefficient of 0.4, the picture of pattern turns out to be similar and the discussion above is applicable as well.

Table 1. Summary of Scenario Analysis

	Gain or Loss?	Impact of DE?	Impact of Investor's Higher Sensitivity?
Scenario 1: Uptrend	Gain	Nil	Positive
Scenario 2: Downtrend	Nil	Nil	Positive
Scenario 3: Single Cycle Oscillation (Rise-Fall)	Gain	Positive	Positive
Scenario 4: Multi-Cycle Oscillation			
2 periods per cycle	Gain	Positive	Positive
4 periods per cycle	Loss	Positive	Negative
6 periods per cycle	Loss	Negative	Positive
8 periods per cycle	Gain	Positive	Positive

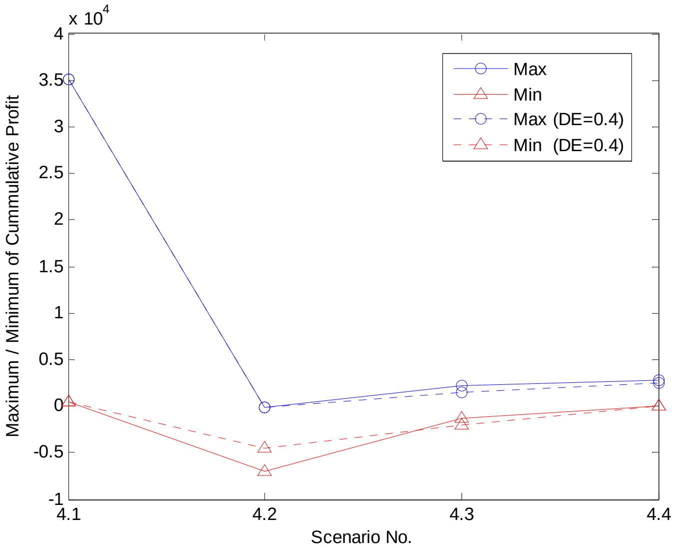

Figure 2. Max. / Min. of Cumulative Profit at the End Under Different Oscillation Scenarios

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IV. CONCLUSION

This paper attempts to answer a seldom asked question, i.e. are behavioral biases always harmful to investor trading performance? This is often taken for granted but still subject to proof. For this aim, a theoretical model is formulated in order to describe precisely a Bayesian investor's trading behavior and the process of how he/she forms and updates her belief on market movements. Further, scenario analysis under typical market patterns (uptrend, downtrend, and oscillation with different frequencies) presents some interesting findings, and behavioral bias can be favorable under certain circumstances. For future extension work, other psychological biases may also be incorporated into our model, e.g. confirmation bias, conservatism, overconfidence. In addition, this paper serves as a preparation essential for some potential topics, including: how different types of investors interact with each other? What strategies can arbitrageurs possibly adopt to take advantage of investor’s biases to make profit?

REFERENCES

[1] Fama E. (1965). “The behavior of stock market prices”. Journal of Business, 38 (1), 34–105.
[2] Barber, B., Odean, T. (2001). “Boys will be boys: gender, overconfidence, and common stock investment”. Quarterly Journal of Economics, 116, 261-292.
[3] Hirshleifer, D. (2001). “Investor psychology and asset pricing”. Journal of Finance, 56, 1533-1597.
[4] Garvey, R. and Murphy, A. (2004). “Are professional traders too slow to realize their losses?”. Financial Analysts Journal, 60(4), 35–43.
[5] Shu, P.-G., Yeh, Y.-H., Chiu, S.-B. and Chen, H.-C. (2005). “Are taiwanese individual investors reluctant to realize their losses?”. Pacific-Basin Finance Journal, 13, 201–223.
[6] Andreassen, P., Kraus, S. (1990). “Judgmental extrapolation and the salience of change”. Journal of Forecasting, 9, 347-372.
[7] Barberis, N., Shleifer, A. and Vishny, R. (1998). “A model of investor sentiment”. Journal of Financial Economics, 49, 307-343.

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