Ranking of the GSM Operators with Fuzzy ANP

Nihal ERGİNEL & Sevil ŞENTÜRK

***Abstract*— Customers can easily change their GSM operator after accepting the legal regulation on moving the number to the new operator in Turkey. In this study, fuzzy analytic network process (FANP) is used to rank for three Global Systems for Mobile Communications (GSM) operators. Five main criteria are described and also sub-criteria are determined. Because of the pairwise comparisons is linguistic expression, FANP method is preferred to rank the GSM operators in Turkey.**

***Index Terms*— Analytic network process, fuzzy analytic network process, ranking of the GSM operators**

I. Introduction

Many multi-criteria decision making problems (MCDM) can be structured hierarchically, in some cases they involve interaction of various factors in real life. When hierarchically structure can be modeled with Analytical Hierarchical Process (AHP), if we mentioned inner dependence between factors, Analytical Network Process (ANP) can be used in this situation. ANP technique allows for complex interrelationships among criteria.

In the real world applications, decision-makers use linguistic variables for pairwise comparisons between criteria, sub-criteria and alternatives in ANP method. Fuzzy set theory approach is an inevitable tool for these problems. Imprecision may arise from a variety of reasons: unquantifiable information, incomplete information, unobtainable information and practical ignorance. Conventional MCDM methods cannot effectively handle problems with such imprecise information. Combined with fuzzy approach and ANP can be appropriate technique in order to eliminate this ambiguity.

ANP technique is a multi-criteria decision making technique also developed by Saaty in 1996 [1]. Several MCDM problems for ranking the alternatives with ANP method are in literature:

ANP technique is used in the interdependent information system project selection process [2]. In supplier selection, ANP approach is handled to evaluate the relations between supplier selection criteria's in a feedback systematic [3]. An integrated multi-objective decision-making process by using ANP and mixed integer programming to optimize supplier selection process is presented [4].

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Also, there are many fuzzy ANP applications in literature: Mikhailov and Singh developed fuzzy extension of the ANP.

A prototype decision support system realizing the proposed method is developed, and its performance is illustrated by examples [5]. Combination of ANP and a fuzzy logic approach is proposed to incorporate the customer needs and the product technical requirements systematically into the product design phase in QFD. The coefficient of the objective function is obtained from a fuzzy ANP approach. FAHP is also used in the proposed framework. An application in a Turkish company producing PVC window and door systems is presented to illustrate the proposed model [6]. An evaluation model using fuzzy analytic network process is developed [7]. The proposed model can provide Taiwan's hospital accreditation policy a reference material, making it highly applicable for academic and government purposes.

FANP method is applied on to the large–sized real life problem related to the transportation project between Turkey and Germany [8]. FANP technique is applied to the company and is given the opportunity of comparing the results of the FANP method with the current preferences. Validation of the model achieved because the FANP results obtained are similar with preferences.

Faulty behavior risk is determined by using fuzzy analytical network process which is an extension of analytical hierarchy process and analyzed complex systems [9]. Also, in their study, the weight of factors and sub-factors necessary to calculate the fault behavior risk are determined by using fuzzy ANP and by this way it was possible to make better decision in this process. The best shipyard location is selected by FANP [10].

Dağdeviren and Yüksel provide a successful application with the help of expert people and the correct fuzzy analytical network process's success over the criteria which have close relationship with each other. Measuring the sectoral competition level of an organization within the framework of Porter's five forces analysis are made by using fuzzy analytic network process technique [11]. Balanced Scorecard approach is applied incorporate with fuzzy ANP technique to determine the performance indicators with different structures [12]. Felek et al is used fuzzy ANP for ranking the GMS operators in Turkey by evaluating the criteria with hierarchically and inner dependency. Studies on the ranking of GSM operators in literature took a few numbers. Classical AHP and ANP methods are compared to select the best GSM operators with several criteria [13]. ANP method is used for determining the market share of GSM operators in Turkey [14] .Also, The ranking of the GSM operators are determined with fuzzy TOPSIS because of the linguistic variables [15]. Tektas and Gozlu are investigated the international transfer process of general packet radio service (GPRS) technologies, which supports the wireless access to external Internet protocol-based networks, and to propose a multi-criteria decision making (MCDM) approach to evaluate the alternative transfer of companies [16]. In their study, a multi-criteria decision making (MCDM) approach is proposed to evaluate the mobile phone options in respect to the users' preferences order. Firstly, the most desirable features influencing the choice of a mobile phone are identified. This is realized through a survey conducted among the target group, the experiences of the telecommunication sector experts and the studies in the literature. Two MCDM methods (AHP and TOPSIS) are used in the evaluation procedure [17]

The rest of the paper is organized as follows: fuzzy ANP method is given shortly in Section II, ranking of the GSM operators is analyzed with fuzzy ANP in Section III, and conclusion is given in the Section IV.

II. FUZZY ANP METHOD

Fuzzy ANP technique uses both interdependence of criteria and inner dependence of criteria with pairwise comparison matrix. Chang's extent analysis method is used to evaluate fuzzy pairwise comparisons [18, 19]. Chang's (in 1992, and 1996) extent analysis approach is explained in details.

Let $X = \{x_1, x_2, ..., x_n\}$ be an object sets, and $G = \{g_1, g_2, ..., g_n\}$ be a goal set. According to the method of Chang's (1992) extent analysis, each object is taken and extent analysis for each goal, g_i, is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{gi}^1, M_{gi}^2, ..., M_{gi}^m \qquad i = 1,2,...,n \tag{1}$$

where $M_{gi}^j \quad j = 1,2,...,m$ are triangular fuzzy numbers (TFNs). The steps of Chang's extent analysis can be given as in the following:

Step1: The value of fuzzy synthetic extent with respect to the *i*th object is defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \tag{2}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{3}$$

and to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1}$, perform the fuzzy addition operation of M_{gi}^j ($j = 1,2,...,m$) values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \tag{4}$$

and then compute the inverse of the vector above, such that:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \tag{5}$$

Step2:

The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as:

$$V(M_2 \geq M_1) = \sup[\min(\mu_{M_1}(x), \mu_{M_2}(y))] \tag{6}$$

and can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2)$$

$$= \mu_{M_2}(d) = \begin{Bmatrix} 1, & if \quad m_2 \geq m_1, \\ 0, & if \quad l_1 \geq u_2, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{Bmatrix} \tag{7}$$

where d is the ordinate of the highest intersection point D between μ_{M1} and μ_{M2}. To compare M_1 and M_2, we need both values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 3:

The degree possibility for a convex fuzzy number to be greater than *k* convex fuzzy numbers

$M_i \ (i = 1,2,...,k)$ can be defined by

$$V(M \geq M_1, M_{21}, ..., M_k)$$
$$= [V(M \geq M_1 \ and \ (M \geq M_2) \ and.......and \ (M \geq M_k)]$$
$$= \min V(M \geq M_i) \tag{8}$$

Assume that:

$$d' = \min V(S_i \geq S_k) \tag{9}$$

for $k = 1, 2, ..., n$; $k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T \tag{10}$$

where $A_i \ (i = 1, 2, ..., n)$ are *n* elements.

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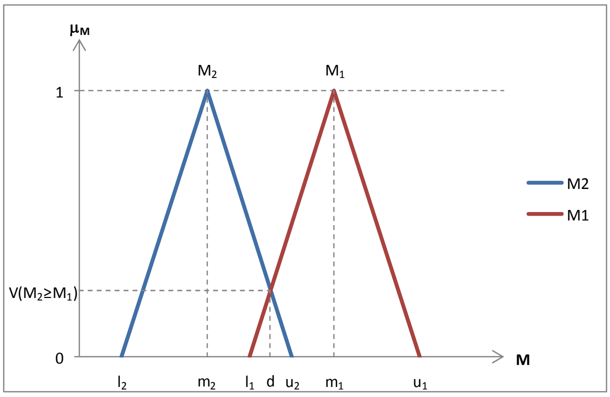

Fig.1. The intersection between *M1* and *M2*

Step 4:

Via normalization, the normalized weight vectors are

$$W' = (d(A_1), d(A_2), \ldots\ldots\ldots, d(A_n))^T \qquad (11)$$

where W is a nonfuzzy number.

III. RANKING GSM OPERATORS WITH FUZZY ANP

Three GSM operators are active in Turkey. The system of number moving from one operator to the second operator is obtained by legal regulation in 2009. After that the criteria of preferring GSM operators are be worthy study areas. Criteria and sub-criteria on selecting GSM operators are determined from literature and voices of customers. The importance level of linguistic expression is given as Table 1 and also the triangular membership functions are given as Figure 2. The hierarchical and inner dependence model of criteria is given as Figure 3.

TABLE 1
LINGUISTIC SCALE FOR IMPORTANCE

Linguistic scale for importance	**Triangular fuzzy scale**
Just Equal (JE)	(1,1,1)
Weakly more important(WMI)	(1,3,5)
Strongly more important(SMI)	(3,5,7)
Very strongly more important(VSMI)	(5,7,9)
Absolutely more important(AMI)	(7,9,9)

TABLE 2
THE LINGUISTIC EXPRESSION BETWEEN CRITERIA

Factors	Price	Services	Advertisement	Reliability	Campaign
Price	JE	SMI	WMI	SMI	
Services		JE		SMI	
Advertisement		WMI	JE		WMI
Reliability			WMI	JE	
Campaign	WMI	SMI		SMI	JE

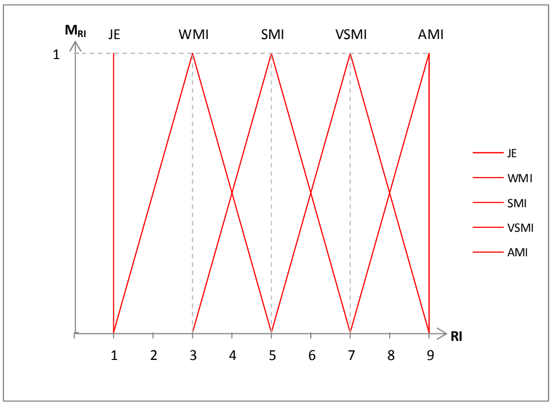

Fig.2. Linguistic scale for relative importance

The linguistic expression of between criteria is defined as Table 2. A symmetric cell presents inverse of relationship.

The linguistic expression between criteria (Table2) are rewrite by using fuzzy numbers (Table 1) and given in Table 3. Chang's extent analysis method is used for calculating the local weights of interdependence of criteria and given in the last column of Table 3. The calculation steps of Chang's extent analysis method are given as follows:

Interdependence of criteria

$$S_P = (8.2, 14.33, 21) \otimes (1/71.67, 1/47.66, 1/26.71) = (0.11, 0.30, 0.79) \qquad (12)$$

$$S_S = (4.48, 6.73, 9.66) \otimes (1/71.67, 1/47.66, 1/26.71) = (0.06, 0.14, 0.36) \qquad (13)$$

$$S_A = (3.4, 7.67, 13) \otimes (1/71.67, 1/47.66, 1/26.71) = (0.047, 0.16, 0.49) \qquad (14)$$

$$S_R = (2.43, 4.6, 7) \otimes (1/71.67, 1/47.66, 1/26.71) = (0.034, 0.096, 0.26) \qquad (15)$$

$$S_C = (8.2, 14.33, 21) \otimes (1/71.67, 1/47.66, 1/26.71) = (0.11, 0.30, 0.79) \qquad (16)$$

$V(S_P \geq S_H) = 0.61$, $V(S_P \geq S_A) = 0.73$,
$V(S_P \geq S_R) = 0.42$, $V(S_P \geq S_C) = 1$;
$V(S_S \geq S_P) = 1$, $V(S_S \geq S_A) = 1$, $V(S_S \geq S_R) = 0.82$,
$V(S_S \geq S_C) = 1$; $V(S_A \geq S_P) = 1$, $V(S_A \geq S_S) = 0.94$,
$V(S_A \geq S_R) = 0.77$, $V(S_A \geq S_C) = 1$; $V(S_R \geq S_P) = 1$,
$V(S_R \geq S_S) = 1$, $V(S_R \geq S_A) = 1$, $V(S_R \geq S_C) = 1$;
$V(S_C \geq S_P) = 1$, $V(S_C \geq S_S) = 0.61$, $V(S_C \geq S_A) = 0.73$,
$V(S_C \geq S_R) = 0.42$. (17)

$$d'(P) = \min(0.61; 0.73; 0.42; 1) = 0.42 \qquad (18)$$

$$d'(S) = \min(1; 1; 0.82; 1) = 0.82 \qquad (19)$$

$$d'(A) = \min(1; 0.94; 0.77; 1) = 0.77 \qquad (20)$$

$$d'(P) = \min(1; 1; 1; 1) = 1 \qquad (21)$$

$$d'(P) = \min(1; 0.61; 0.73; 0.42) = 0.42 \qquad (22)$$

TABLE 3
LOCAL WEIGHTS AND PAIRWISE COMPARISON MATRIX OF MAIN FACTORS

Factors	Price	Services	Advertisement	Reliability	Campaign	Local Weights
Price	(1,1,1)	(3,5,7)	(1,3,5)	(3,5,7)	(1/5,1/3,1)	0.12
Services	(1/7,1/5,1/3)	(1,1,1)	(1/5,1/3,1)	(3,5,7)	(1/7,1/5,1/3)	0.25
Advertisement	(1/5,1/3,1)	(1,3,5)	(1,1,1)	(1/5,1/3,1)	(1,3,5)	0.22
Reliability	(1/7,1/5,1/3)	(1/7,1/5,1/3)	(1,3,5)	(1,1,1)	(1/7,1/5,1/3)	0.29
Campaign	(1,3,5)	(3,5,7)	(1/5,1/3,1)	(3,5,7)	(1,1,1)	0.12

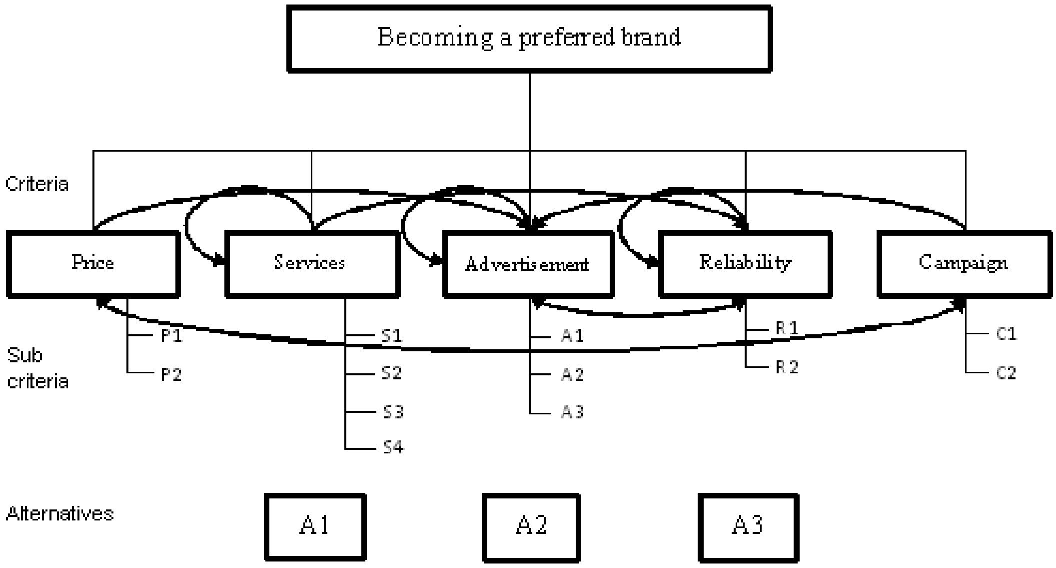

Fig.3.The hierarchical and inner dependence model of criteria

TABLE 4
THE INNER DEPENDENCE MATRIX OF FACTORS RESPECT TO 'ADVERTISEMENT'

Advertisement	Advertisement	Price	Campaign	Relative importance weights
Advertisement	(1,1,1)	(1/5,1/3,1)	(1,3,5)	0.30
Price	(1,3,5)	(1,1,1)	(3,5,7)	0.06
Campaign	(1/5,1/3,1)	(1/7,1/5,1/3)	(1,1,1)	0.64

TABLE 5
THE INNER DEPENDENCE MATRIX OF FACTORS RESPECT TO 'RELIABILITY'

Reliability	Services	Reliability	Relative importance weights
Services	(1,1,1)	(1/5,1/3,1)	0.70
Reliability	(1,3,5)	(1,1,1)	0.30

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Inner dependence of criteria

As Figure 3; price, services and campaign have only one input. So, the inner dependence of campaign for price is 1.00. Also, the inner dependence of services for services is 1.00, because of itself inner dependence. Then, the inner dependence of price for campaign is 1.00, also. The inner dependence relationship with fuzzy number is given as Table 4 and Table 5. By multiplying the relative importance weights and the local weights, global weights are obtained. Global weights include both interdependence and inner dependence of criteria.

$$\begin{bmatrix} 0.127 \\ 0.350 \\ 0.143 \\ 0.190 \\ 0.190 \end{bmatrix} \tag{23}$$

TABLE 6
THE LOCAL WEIGHTS OF SUB-CRITERIA AND THE WEIGHTS OF ALTERNATIVES

Factors and local weights	Sub Factors and local weigths	Global weights	A1	A2	A3
Price (0.127)	P1(0.60)	0.076	(0.50)	(0.20)	(0.30)
	P2(0.40)	0.051	(0.40)	(0.30)	(0.30)
Service (0.350)	S1(0.20)	0.070	(0.20)	(0.40)	(0.40)
	S2(0.25)	0.088	(0.15)	(0.55)	(0.30)
	S3(0.30)	0.105	(0.25)	(0.35)	(0.40)
	S4(0.25)	0.088	(0.15)	(0.45)	(0.40)
Advertisement (0.143)	A1(0.50)	0.072	(0.50)	(0.25)	(0.25)
	A2(0.15)	0.021	(0.20)	(0.45)	(0.35)
	A3 (0.35)	0.050	(0.25)	(0.40)	(0.35)
Relability (0.190)	R1(0.40)	0,076	(0.20)	(0.35)	(0.45)
	R2(0.60)	0,114	(0.15)	(0.45)	(0.40)
Campagin (0.190)	C1(0.50)	0.095	(0.50)	(0.25)	(0.25)
	C2(0.50)	0.095	(0.35)	(0.35)	(0.40)

After obtaining the local weights of criteria, the local weights of sub-criteria and the weights of alternatives for each criterion are defined with expert persons and given in Table 6. The global weights of sub-criteria are handled by multiplying the global weights of criteria and the local weights of sub-criteria. The ranking of the GSM operators in Turkey are calculating by multiplying the global weights of sub-criteria and the weights of alternatives for each criteria and also given in Table 7.

TABLE 7
THE RANKING OF THE GSM OPERATORS IN TURKEY

	A2	A3	A1
Total weight	0.36	0.35	0.29

IV. CONCLUSION

In many MCDM problems, linguistic expressions are used to evaluate the alternatives with multi criteria and sub-criteria. Also, the pairwise comparisons of criteria is used to analysis the linguistic expressions in AHP and ANP methods. If the linguistic expressions are there in the study, the fuzzy approach is suitable tool for modeling these linguistic expressions with fuzzy numbers.

This study presents the ranking of the GSM operators in Turkey with many criteria and sub-criteria. Therefore GSM operators' decision-makers can realize which criteria and sub-criteria are important for customers and can take improvement action to gain the customer. In the future study, also the relationship both between sub-criteria, and sub-criteria and alternatives are determined with fuzzy numbers.

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