Project Scheduling to Maximize Fuzzy Net Present Value

İrem UÇAL and Dorota KUCHTA

Abstract— **In this paper a fuzzy version of a procedure for project scheduling is proposed to maximize the fuzzy net present value of projects with fuzzy cash flows. Fuzzy equivalents of cash flow weight and discounted cash flow weight are defined which are used to find the importance of the activities with respect to the fuzzy net present value of the project. The procedure is applied to an example and results are discussed in conclusions.**

Index Terms— **fuzzy cash flow weight, fuzzy net present value, project scheduling**

I. Introduction

PROJECT management has become extremely important in organizations. The primary objective of project management is to achieve project goals while recognizing the project constraints such as: time, budget, and resources. The main stages of project management are project planning and project control.

Project planning involves among others:

--Identification of a complete list of basic activities. The list should be complete in the sense that it should include all the activities which are necessary to achieve the project goals. An activity is called basic if it is considered not possible or not useful from the management point of view to divide it into smaller units;

--Identification of precedence relations between the activities;

--Identification of resources required by each activity and resources available for the project;

--Identification (budgeting) of costs, revenues and cash flows;

--Identification of the cost of capital especially in long term projects (i.e. the discounting rate)

--Building of a project schedule. A project schedule consists of the project activity list with determined start and finish times for each activity.

--Risk analysis and risk prevention/management planning.

Project schedules can be constructed in different ways, according to the goals of the project. The project goals differ with company strategies but often one of the most important goals is to get a maximum cash flow from the project. That is why project scheduling with the objective function net present value (*NPV*) of the project (maximized) is becoming more and more popular.

There are several different algorithms proposed in the literature which maximize *NPV* of the project as summarized in Section 2. Most of them assume crisp values of cash flows. However, in many projects the risk analysis shows that the cash flows planned are very risky. If we want to take the risk into account and plan the project risk management we have to express the planned cash flows as random or fuzzy variables. As determination of random variables is often more difficult than the use of fuzzy numbers, we have chosen the latter to model uncertain cash flows. Fuzzy logic which takes into accounts the ambiguous and vague information enables us to deal with the uncertainty.

The aim of this paper is to develop a procedure to maximize fuzzy net present value of the project when the project has fuzzy cash flows. To achieve this objective, a crisp heuristic with known (good) ratio of result quality and time consumption has been used. Fuzzy equivalents of cash flow weight and the discounted cash flow weight, which are used in the crisp heuristics, are proposed for the case in which the cash flows of the activities are determined by fuzzy numbers. Then these fuzzy notion are built into the fuzzy version of the heuristic whose product is a project schedule maximizing the fuzzy net present value of the project.

In Section 2, a literature review on project scheduling to maximize net present value of the project in the crisp and probabilistic case is given. In Section 3, a cash flow weight heuristic determining the project schedule with a maximal net present value of the project is explained. In Section 4, the necessary information on fuzzy logic is given to make clear the operations of the fuzzy procedure. In Section 5, fuzzy cash flow weight heuristics is proposed for scheduling projects which have fuzzy cash. An application of the procedure is given in Section 6, and with the interpretation of the results paper is concluded.

II. Literature Review

There are many recent works on project scheduling to maximize net present value of the project. The approaches offered for the solution of the problem of maximizing the net present value of a project through the manipulation of the times of realization of its key events are reviewed in [1]. An integer programming algorithm for project scheduling subject to resource limitations during each period of the

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schedule duration is described in [2]. An activity scheduling problem for a project where cash inflows and outflows are given and availability restrictions are imposed on capital and renewable resources is presented in [3]. The problem of scheduling activities in a project to maximize the net present value of the project is solved for the case where the activity cash flows are independent of the time of activity realization in [6]. The unconstrained project scheduling problem with discounted cash flows where the net cash flows are assumed to be dependent on the completion times of the corresponding activities to maximize the net present value of the project subject to the precedence constraints and a fixed deadline are examined in [9]. The test results show that a simulated annealing scheduling procedure performs in the best way and the discounted cumulative cash flow weight heuristic (used in this paper) performs also well. Heuristic procedures for obtaining improved solutions to maximize net present value in a network problem are developed in [7]. The multi-mode resource-constrained project scheduling problem with discounted cash flows is considered in [10]. Discrete–continuous project scheduling problems with discounted cash flows are considered in [11]. The extensions of the basic resource-constrained project scheduling problem are classified according to the structure of the resource-constrained project scheduling problem in [13]. As the project scheduling problem may be a multicriteria problem and the NPV of the project may be just one of the most important criteria, in the literature the NPV objective combined with other ones. Thus, the performance of seventeen scheduling heuristics is evaluated separately on maximization of project *NPV* and minimization of project duration in [5]. Resource leveling and net present value problems are studied on resource-constrained project scheduling problems with nonregular objective functions where general temporal constraints given by minimum and maximum time lags between activities are prescribed in [8].

There are also some works in literature which use probability theory on project scheduling to maximize the net present value of a project. The use of stochastic scheduling rules for maximizing the net present value of a project with probabilistic cash flows is examined in [4]. Test problems are used to evaluate the performance of nine scheduling heuristics in [4]. The problem of adaptively optimizing the expected value of a project's cash flow is formulated in [12] including randomness in activity durations, costs, and revenues. A continuous-time Markov decision chain is used on project scheduling with net present value objective and exponential activity durations in [14]. Project scheduling when the activity durations and cash flows are described by a discrete set of alternative scenarios with associated occurrence probabilities is examined to maximize the project's expected net present value [15].

In the literature review, it has been found that none project scheduling procedure for the projects which have fuzzy cash flows has been developed, however there are lots of studies on project scheduling to maximize net present value of the project. It seems to be useful to adopt some of the to the fuzzy case, so that they can be used to build a project schedule with a maximal *NPV* taking into account the risk and uncertainty connected to the cash flow estimation in practice.

III. Cash Flow Weight Heuristics

A project is a network with activities $(A_i, i = 1,2,..., N)$ represented as nodes, relations between activities represented as arcs, the resources required by activities denoted by r_{ik} ($i = 1,2,..., N$ and $k = 1,2,..., m$) the total resources available for the project denoted by r_{tk} ($k = 1,2,..., m$), and durations of the activities denoted by d_i $(i = 1,2,..., N)$. Net cash flows of activities occur at the beginning or end of the related activity and the value of it is independent of the starting or ending moment of the activity. The sum of all the cash flows from different activities starting or finishing in moment j will be denoted as CF_j ($j = 1,2,..., T^H$ where T^H denotes time horizon).

Present value ($\tilde{P}V$) of a single future payment occurred in the end of n^{th} year from now is given in (1) where F stands for amount of the payment and r denotes the interest rate (cost of capital).

$$PV = \frac{F}{(1+r)^n} \tag{1}$$

The goal is to find a schedule with a maximal *NPV* which is sum of all discounted cash flows formulated on (2):

$$NPV = \sum_{j=0}^{n} \frac{CF_j}{(1+r)^j} \tag{2}$$

Cash flow weight (*CFW*) heuristic [7] is a heuristic which dynamically selects a high priority activity from available activities for the assignment of resources. In the considered heuristic procedure, the priority of an activity is linked to the cash flows linked to the very activity and all the activities which follow it. The priority is measured by means of cash flow weighting.

A. Cash Flow Weighting

Cash flow weighting is an assignment of a weight to each activity with respect to the cash flow creating potential of the activity which means the sum of the cash flows occurred from the activity and its successor activities. The cash flow weight heuristic is a forward pass heuristic which selects the activity with the largest *CFW* from the list of available activities and attempts to assign it to the earliest possible period with considering precedence and resource constraints. After assignment of an activity, the resource constraints are updated. When the last activity is assigned, the procedure stops [7].

B. Cash Flow Weight Algorithm

There are three steps on cash flow weight procedure. In the first step, the cash flow weights of each activity are determined and all activities are included to the list of available activities in an order of $i\,(i = 1,2,..., N)$ without taking into account the predecessors. In the second step, the activity with the highest *CFW* is selected from the top of the list of available activities. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon

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as possible, the predecessors of the selected activity are assigned respectively in the increasing order of their indices $i\ (i=1,2,...,N)$ and as soon as possible with respect to the resources available. After assignment of the selected activity the available resources are updated. In the third step if there is any unassigned activity second step is repeated, otherwise the project schedule is completed [7].

C. Discounted Cash Flow Weight Algorithm

Discounted cash flow algorithm has the same procedure with cash flow weight algorithm while it deals with discounted cash flow weights (*DCFW*s) instead of *CFW*s. *DCFW* for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities [7].

IV. Fuzzy Logic

Fuzzy logic enables us to use expert knowledge and experience to model uncertain or risky values to achieve more efficient solutions in uncertain environments. Zadeh [16] first founded the fuzzy set theory which has become an important tool for modeling the uncertainty. An important notion is that of a fuzzy number.

A. Fuzzy numbers

There is a general definition of fuzzy numbers but usually their simplest form, triangular fuzzy numbers (*TFN*) are preferred to simplify the calculations. A *TFN* has linear membership (possibility) functions both on the left and right sides. The membership function of *TFN* is given by (3) and its graphic is given in Fig. 1:

$$\mu(x)=\begin{cases}\dfrac{x-M_l}{M_m-M_l}, & M_l\le x\le M_m\\ \dfrac{M_r-x}{M_r-M_m}, \quad if & M_m\le x\le M_r\\ 0, & otherwise\end{cases} \qquad (3)$$

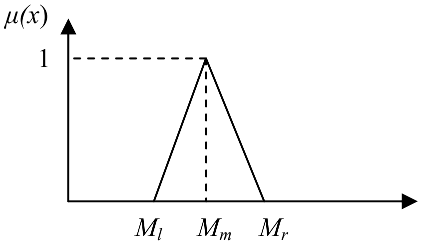

Fig. 1. Membership Function of a *TFN*

The number represented in Fig. 1 stands for an uncertain magnitude which with the highest degree of possibility will be (once it has occurred) equal to M_m, but it is also possible that it will be smaller or greater than M_m. It is however considered absolutely impossible that this values will be greater than M_r and smaller than M_l, and the more distant a value is from M_m, the less the possibility that it will the actual value of the modeled magnitude (on our case the magnitudes modeled will be cash flows). The difference interval ([M_l, M_r]) is called support of the fuzzy number $\tilde{M}$. M_m is called the mean or the mode of the fuzzy number $\tilde{M}$.

Algebraic operations for *TFN*s are given by (4)-(10) where all the fuzzy numbers are positive (here it is assumed to mean $M_l \ge 0, N_l \ge 0$) [17]:

$$(M_l,M_m,M_r)+(N_l,N_m,N_r)\cong(M_l+N_l,M_m+N_m,M_r+N_r) \qquad (4)$$

$$(M_l,M_m,M_r)-(N_l,N_m,N_r)\cong(M_l-N_r,M_m-N_m,M_r-N_l) \qquad (5)$$

$$(M_l,M_m,M_r)\otimes(N_l,N_m,N_r)\cong(M_lN_l,M_mN_m,M_rN_r) \qquad (6)$$

$$(M_l,M_m,M_r)\div(N_l,N_m,N_r)\cong\left(\frac{M_l}{N_r},\frac{M_m}{N_m},\frac{M_r}{N_l}\right) \qquad (7)$$

$$\lambda\otimes(M_l,M_m,M_r)\cong\begin{cases}(\lambda M_l,\lambda M_m,\lambda M_r), & \lambda\ge 0\\ (\lambda M_r,\lambda M_m,\lambda M_l), & \lambda\le 0\end{cases}\ if,\ \forall\lambda\in\Re \qquad (8)$$

$$\lambda\div(M_l,M_m,M_r)\cong\begin{cases}\left(\dfrac{\lambda}{M_r},\dfrac{\lambda}{M_m},\dfrac{\lambda}{M_l}\right), & \lambda\ge 0\\ \left(\dfrac{\lambda}{M_l},\dfrac{\lambda}{M_m},\dfrac{\lambda}{M_r}\right), & \lambda\le 0\end{cases}\ if,\ \forall\lambda\in\Re \qquad (9)$$

$$(M_l,M_m,M_r)^{\lambda}\cong\begin{cases}\left(M_l^{\lambda},M_m^{\lambda},M_r^{\lambda}\right), & \lambda\ge 0\\ \left(\dfrac{1}{M_r^{\lambda}},\dfrac{1}{M_m^{\lambda}},\dfrac{1}{M_l^{\lambda}}\right), & \lambda\le 0\end{cases}\ if,\ \forall\lambda\in\Re \qquad (10)$$

A negative fuzzy number is a positive fuzzy number multiplied by -1.

B. Ranking Fuzzy Numbers

Contrary to the crisp case, a comparison of two fuzzy numbers is not unequivocal: in case the supports of two fuzzy numbers overlap, it is not unequivocal to decide which fuzzy number is greater. In the literature many methods of ranking of fuzzy numbers are proposed [17]. Each method is different, because it is based on other features and preferences of the decision maker.

In this paper, we need a method of comparing fuzzy numbers, as we will compare fuzzy equivalents of the cash flow weights to decide which activity has a priority to be scheduled.

We will use the simplest ranking methods. In [20] there are discussed ranking methods based on the support of fuzzy numbers and on the attitude of the decision maker. If a high value of the magnitude in question was welcome, an optimist would rather expect values close to the upper bound of the support, a pessimist values closer to the lower bound of the support, someone with a neutral attitude would expect values "in the middle". So we can define three ranking methods for fuzzy numbers. Each corresponds to another decision maker attitude. Thus, in case high values are preferred, according to an optimist, fuzzy number $\tilde{M}$ is greater than fuzzy number $\tilde{N}$ if $M_r \ge N_r$, according to a pessimist fuzzy number $\tilde{M}$ is greater than $\tilde{N}$ if $M_l \ge N_l$. For a decision maker has neutral attitude the weighted method [18] which compares the fuzzy numbers by assigning relative weights to determine the preference of the fuzzy number is used for ranking. The preference of a *TFN* is given in (11) where w is the relative weight determined by nature and the magnitude of the most promising value (the mean of the fuzzy number):

$$N_{CP}=\frac{N_l+N_m+N_r}{3}+wN_m \qquad (11)$$

If the magnitude of the most promising value is important, a larger weight such as w=0.3 is recommended otherwise smaller weight such as w=0.1 is recommended.

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C. Fuzzy problem setting

We consider a project with fuzzy cash flows, linked to the beginning or ending of activities independent of their time setting, fuzzy interest rate. The goal is to find a schedule with a maximal fuzzy NPV, where in comparing the fuzzy NPV we choose one of the relations defined in Section 4.B.

Fuzzy present value ($\tilde{PV}$) of a single future payment occurred in the end of n^{th} year from now is given in (12) where $\tilde{F}$ stands for fuzzy amount of the payment and $\tilde{i}$ denotes the fuzzy interest rate.

$$\tilde{PV} = \frac{\tilde{F}}{(1+\tilde{i})^n} \quad (12)$$

The general formula of fuzzy net present value $\tilde{NPV}$ is given in (13), where $\tilde{CF}_j$ denotes net fuzzy cash flows occurred at time j, n denotes the useful life of the project and $\tilde{i}$ denotes the fuzzy interest rate [19].

$$\tilde{NPV} = \sum_{j=0}^{n} \frac{\tilde{CF}_j}{(1+\tilde{i})^j} \quad (13)$$

Fuzzy net present value formula for *TFN*s is generated on (12):

$$\tilde{NPV} = \sum_{\substack{j=0 \\ \tilde{CF}_j<0}}^{n} \frac{\tilde{CF}_{jl}}{(1+\tilde{i}_l)^j}, \frac{\tilde{CF}_{jm}}{(1+\tilde{i}_m)^j}, \frac{\tilde{CF}_{jr}}{(1+\tilde{i}_r)^j} + \sum_{\substack{j=0 \\ \tilde{CF}_j>0}}^{n} \frac{\tilde{CF}_{jl}}{(1+\tilde{i}_r)^j}, \frac{\tilde{CF}_{jm}}{(1+\tilde{i}_m)^j}, \frac{\tilde{CF}_{jr}}{(1+\tilde{i}_l)^j} \quad (14)$$

V. Fuzzy Cash Flow Weight Heuristics

A. Fuzzy Cash Flow Weighting

Fuzzy cash flow weighting is an assignment of a fuzzy weight to each activity with respect to the fuzzy cash flow creating potential of the activity which means the sum (in the sense of definitions on Section 4.B) of the cash flows occurred from the activity and its successor activities. In this procedure, the cash flows of the activities are assumed as either negative or positive fuzzy numbers.

B. Fuzzy Cash Flow Weight Algorithm

There are four steps on fuzzy cash flow weight algorithm. In the first step, the fuzzy cash flow weights of each activity which are denoted by $\tilde{CFW}_i$, are determined and all activities are added without predecessors to the available list. In the second step $\tilde{CFW}$ values are ordered with a method from Section 4.B. In the third step, the activity with the highest $\tilde{CFW}$ is selected from the list of precedence available. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively. After assignment of the selected activity the resource available list is updated. In the fourth step if there is any unassigned activity the third step is repeated, otherwise the project schedule is completed.

C. Fuzzy Discounted Cash Flow Weight Algorithm

Fuzzy discounted cash flow algorithm has the same procedure with fuzzy cash flow algorithm while it deals with fuzzy discounted cash flow weights $D\tilde{CFW}$ instead of $\tilde{CFW}$. $D\tilde{CFW}_i$ for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities.

VI. Application

The fuzzy cash flows occurred at the beginning of the activity, immediate predecessors, durations, and resource requirements for each task are given in Table 1. The number of available resources for this project is determined as 5.

A network diagram of a project is given in Fig. 2 with the cash flows, resource requirements, and durations of the tasks. The project has just one type of resource which is limited to 5 over the project realization time.

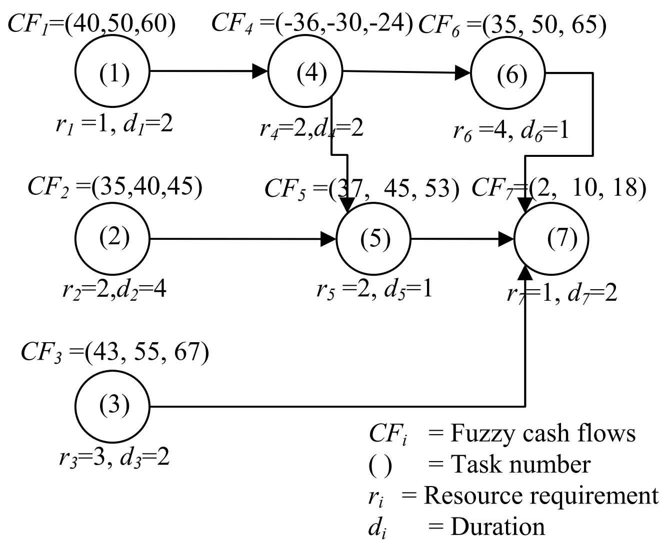

Fig. 2. Network Diagram of the Project

A. Fuzzy Cash Flow Weighting

The calculations of $\tilde{CFW}$ for the tasks 1 and 4 are given below as examples:

$$\begin{aligned}\tilde{CFW}_1 &= \tilde{CF}_1 + \tilde{CF}_4 + \tilde{CF}_5 + \tilde{CF}_6 + \tilde{CF}_7 \\ &= (-25,-20,-15) + (-13,-10,-7) + (35,50,65) + (36,45,54) + (2,10,18) \\ &= (35,70,115)\end{aligned}$$

$$\begin{aligned}\tilde{CFW}_4 &= \tilde{CF}_4 + \tilde{CF}_5 + \tilde{CF}_6 + \tilde{CF}_7 \\ &= (-13,-10,-7) + (35,50,65) + (36,45,54) + (2,10,18) = (60,95,130)\end{aligned}$$

$\tilde{CFW}$ and preference value with a weight of 0.2, pessimistic and optimistic values for each task are given on Table 2.

Ranking of $\tilde{CFW}$ values of activities are found as; $\tilde{CFW}_1 > \tilde{CFW}_2 > \tilde{CFW}_4 > \tilde{CFW}_3 > \tilde{CFW}_6 > \tilde{CFW}_5 > \tilde{CFW}_7$ for the optimistic and neutral ranking methods. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1-2. Activity 2 which has the next highest value is scheduled in periods 1-4 and available resources are updated as 2 for the periods 1-2, and as 3 for the periods 3-4. Activity 4 which has the third highest $\tilde{CFW}$ value is scheduled in periods 3-4 and available resources are updated as 1 for the 3-4. Activity 3 which has the next highest value is scheduled in periods 5-6 and available resources are updated as 2 for periods 5-6. Activity 6 which has the next highest value is scheduled in period 7 and available resources for period 7 are updated as 1. Activity 5 which has the next highest value is scheduled in period 5 and available resources for period 5 are updated as 0 and the last activity, Activity 7 is scheduled in periods 8-9 and available resources are updated for periods 8-9 as 4. After scheduling the last activity the algorithm is stopped.

The project schedules resulting from the neutral and optimistic ranking methods for $\widetilde{CFW}$ heuristic is given in Fig. 3.

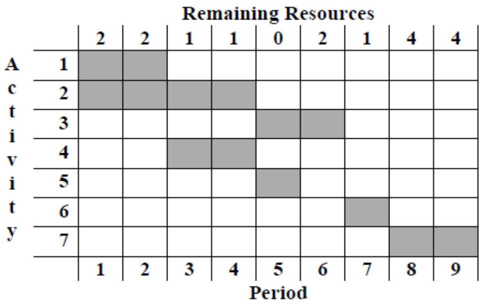

Fig. 3. Project Schedule Resulting From $\widetilde{CFW}$ Heuristic by Neutral and Optimistic Ranking Methods

Ranking of $\widetilde{CFW}$ values of activities are found as; $\widetilde{CFW}_1 > \widetilde{CFW}_2 > \widetilde{CFW}_3 > \widetilde{CFW}_5 > \widetilde{CFW}_6 > \widetilde{CFW}_4 > \widetilde{CFW}_7$ for the pessimistic ranking method. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1-2. Activity 2 which has the next highest value is scheduled in periods 1-4 and available resources are updated as 2 for the periods 1-2, and as 3 for the periods 3-4. Activity 3 which has the third highest $\widetilde{CFW}$ value is scheduled in periods 3-4 and available resources are updated as 0 for the 3-4. Activity 5 which has the next highest value but because of the predecessors, Activity 4 is scheduled in periods 5-6 and available resources are updated as 3 for periods 5-6. Activity 5 which has no predecessor constraint any more is scheduled in period 7 and available resources are updated as 3 for period 7. Activity 6 which has the next highest value is scheduled in period 8 and available resources for period 8 are updated as 1, and the last activity, Activity 7 is scheduled in periods 9-10 and available resources are updated for periods 9-10 as 4. After scheduling the last activity the algorithm is stopped. The project schedules resulting from the pessimistic ranking method for $\widetilde{CFW}$ heuristic is given in Fig. 4.

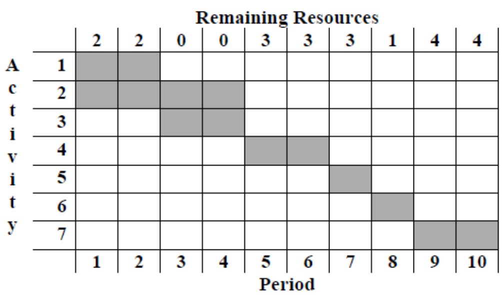

Fig. 4. Project Schedule Resulting From $\widetilde{CFW}$ Heuristic by Pessimistic Ranking Method

When the schedule shown in Fig. 3 is applied, fuzzy net present value of the project with a fuzzy interest rate $\tilde{i} = (0.08, 0.10, 0.12)$ is calculated as $\widetilde{NPV} \cong (113.61, 166.86, 225.53)$ \$

TABLE I
PROJECT DATA

TASK NUMBER	FUZZY CASH FLOW	IMMEDIATE PREDECESSORS	DURATION	RESOURCE REQUIREMENT
1	(40,50,60)	-	2	1
2	(35,40,45)	-	4	2
3	(43,55,67)	-	2	3
4	(-36,-30,-24)	1	2	2
5	(37,45,53)	2,4	1	2
6	(35,50,65)	4	1	4
7	(2,10,18)	3,5,6	2	1

TABLE II
FUZZY CASH FLOW WEIGHTS AND DISCOUNTED CASH FLOW WEIGHTS

TASK NO.	FUZZY CASH FLOW	$\widetilde{CFW}$	PREFERENCE VALUE	PESSIMIST VALUE	OPTIMIST VALUE
1	(40,50,60)	(78,125,172)	150	78	172
2	(35,40,45)	(74,95,116)	114	74	116
3	(43,55,67)	(45,65,85)	78	45	85
4	(-36,-30,-24)	(38,75,112)	90	38	112
5	(37,45,53)	(39,55,71)	66	39	71
6	(35,50,65)	(37,60,83)	72	37	83
7	(2,10,18)	(2,10,18)	12	2	18

TABLE III
FUZZY DISCOUNTED CASH FLOW WEIGHTS

TASK NO.	FUZZY CASH FLOW	$\widetilde{DCFW}$	PREFERENCE VALUE	PESSIMIST VALUE	OPTIMIST VALUE
1	(40,50,60)	(44.41,81.19,122.48)	98.93	44.41	122.48
2	(35,40,45)	(59.65,76.94,96.21)	92.99	59.65	96.21
3	(43,55,67)	(44.59,63.26,82.43)	76.08	44.59	82.43
4	(-36,-30,-24)	(8.25,37.74,71.19)	46.61	8.25	71.19
5	(37,45,53)	(38.79,54.09,69.67)	64.99	38.79	69.67
6	(35,50,65)	(36.79,59.39,81.67)	70.99	36.78	81.67
7	(2,10,18)	(2,10,18)	12	2	18

by (14). When the schedule shown in Fig. 4 is applied, fuzzy net present value of the project is calculated as $\widetilde{NPV} \cong (118.20, 170.69, 228.24)$ \$.

B. *Fuzzy Discounted Cash Flow Weighting*

The calculation of $\widetilde{DCFW}$ for the task 1 is given below as an example. The results, their preference values calculated with a weight of 0.2, their pessimistic and optimistic values are given on Table 3.

$$\widetilde{DCFW}_1 = \widetilde{DCF}_1 + \widetilde{DCF}_4 + \widetilde{DCF}_5 + \widetilde{DCF}_6 + \widetilde{DCF}_7$$
$$= \frac{(-25,-20,-15)}{(1.08,1.10,1.12)^0} + \frac{(-13,-10,-7)}{(1.08,1.10,1.12)^2} + \frac{(35,50,65)}{(1.08,1.10,1.12)^7} + \frac{(36,45,54)}{(1.08,1.10,1.12)^6} + \frac{(2,10,18)}{(1.08,1.10,1.12)^8}$$
$$= (-1.32, 27.72, 61.61)$$

Rankings of $\widetilde{DCFW}$ values of activities are found as; $\widetilde{DCFW}_1 > \widetilde{DCFW}_2 > \widetilde{DCFW}_3 > \widetilde{DCFW}_6 > \widetilde{DCFW}_5 > \widetilde{DCFW}_4 > \widetilde{DCFW}_7$ for the neutral ranking method, and $\widetilde{DCFW}_1 > \widetilde{DCFW}_2 > \widetilde{DCFW}_3 > \widetilde{DCFW}_6 > \widetilde{DCFW}_4 > \widetilde{DCFW}_5 > \widetilde{DCFW}_7$ for the optimistic ranking. The difference between neutral ranking method and optimistic ranking method is on Activity 4 and Activity 5. The Activity 4 should be scheduled first due to it is predecessor of Activity 5. So these two rankings result on the same schedule which is shown in Fig 5. The project schedules resulting from the neutral and optimistic ranking methods for $\widetilde{DCFW}$ heuristic is given in Fig. 5. Ranking of $\widetilde{DCFW}$ values of activities are

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found as; $DC\widetilde{F}W_2 > DC\widetilde{F}W_3 > DC\widetilde{F}W_1 > DC\widetilde{F}W_5 > DC\widetilde{F}W_6 > DC\widetilde{F}W_4 > DC\widetilde{F}W_7$ for the pessimistic ranking method. The project schedules resulting from the pessimistic ranking methods for $C\widetilde{F}W$ heuristic is given in Fig. 6.

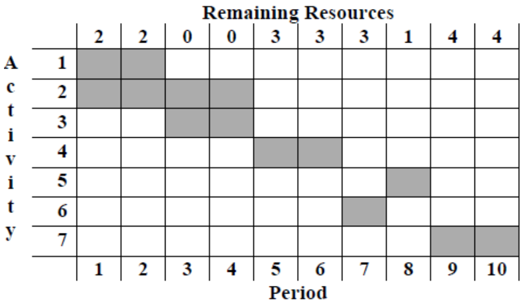

Fig.5. Project Schedule Resulting From $DC\widetilde{F}W$ Heuristic by Neutral and Optimistic Ranking Methods

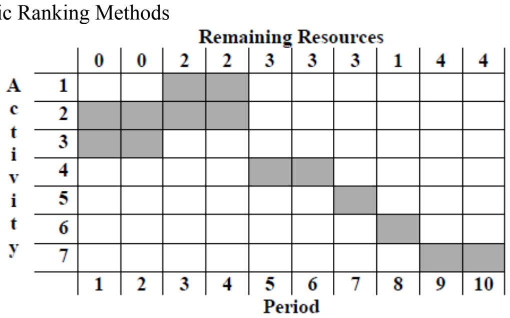

Fig.6. Project Schedule Resulting From $DC\widetilde{F}W$ Heuristic by Pessimistic Ranking Methods

When the schedule shown in Fig. 5 is applied, fuzzy net present value of the project with a fuzzy interest rate $\tilde{i} = (0.08, 0.10, 0.12)$ is calculated as $N\widetilde{P}V \cong (118.09, 170.94, 228.80)$ \$ by (14). When the schedule shown in Fig. 6 is applied, fuzzy net present value of the project is calculated as $N\widetilde{P}V \cong (121.22, 171.56, 225.63)$ \$.

VII. Discussion

In this paper, two different heuristic methods for project scheduling to maximize fuzzy net present value of a project are proposed. In the application section, the schedules resulting from $C\widetilde{F}W$ and $DC\widetilde{F}W$ heuristics are different which make differences on project's fuzzy net present value. Also the ranking method chosen for the ranking step of the algorithm could change the schedule, fuzzy net present value, and realization time of the project. The interpretation the decision maker gets from these algorithms is which activities are critical for fuzzy net present value of the project and cannot be moved (in our application Activities 2 and 7) and which activities are dependent on his/her attitude (in our application Activities 1,3,4,5, and 6). It is also worth mentioning that in our case the whole project duration is planned to be 9 or 10 time units and in one of the cases it is equal to the shortest possible project duration (which is 9) and in the other case that it is more advantageous to prolong the project realization by 1 time unit to achieve higher fuzzy net present value.

As a further research the proposed model could be expanded for different ranking methods to determine the best suitable ranking method for this approach.

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