

# A Modified Approach to Induction Motor Stator Voltage and Frequency Control

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**Abstract**—An inverter-fed three phase squirrel cage induction motor drive system with improved mechanical characteristics is presented. A detailed analytical review of the ideal constant flux control method and the basic  $v/f$  control strategy were undertaken before an improved  $v/f$  control method, which utilises a low frequency boost-voltage, was developed. This method, unlike the basic  $v/f$  control method, provides a boost-voltage at low frequencies thereby compensating for the stator impedance drop, offering constant flux operation with maximum motoring torque from zero to rated speed.

**Index Terms**—Induction Motor, Constant Volts/Hertz  $v/f$ , Constant Flux, Motor Torque and Speed)

## I. INTRODUCTION

Induction machine is the most used in industry because of its high robustness, reliability, low cost, high efficiency and good self-starting capability [1,2,3,4]. The induction motor, particularly with a squirrel cage rotor, is the most widely used source of mechanical power fed from an AC power system. Its low sensitivity to disturbances during operation make the squirrel cage motor the first choice when selecting a motor for a particular application [5]. In spite of this popularity, the induction motor has two inherent limitations: (1) the standard motor is not a true constant-speed machine, its full-load slip varies from less than 1% (in high-horse power motors) to more than 5% (in fractional-horsepower motors) and (2) It is not, inherently, capable of providing variable speed operations[6,7].

These limitations can be solved through the use of adjustable speed controllers [8,9]. The basic control action involved in adjustable speed control of induction motors is to apply a variable frequency variable magnitude AC voltage to

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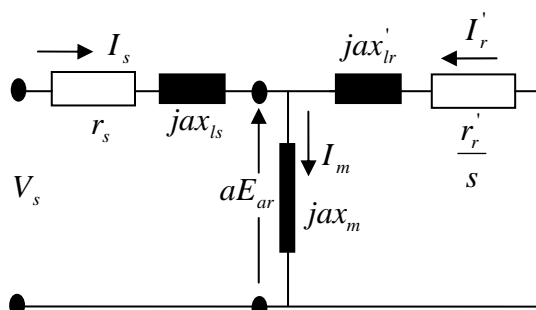
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the motor to achieve the aims of variable speed operation [10]. The most common AC drives today are based on sinusoidal pulse-width modulation SPWM. However, voltage source inverters with constant volts/hertz  $v/f$  are more popular, especially for applications without position control requirements, or where the need for high accuracy of speed control is not crucial [11]. However, since the introduction of field-oriented control theory, almost all research has been concentrated in this area and little has been published about constant  $v/f$  operation. Its application at low frequencies is still challenging due to the influence of the stator resistance and the necessary rotor slip to produce torque [12]. In addition, the nonlinear behaviour of pulse-width modulated voltage inverter in the low voltage range makes it difficult to use constant  $v/f$  drives at frequencies below 3Hz [13].

## II. CONSTANT FLUX CONTROL: PRINCIPLES AND MOTOR PERFORMANCE

The ideal of the variable frequency, variable voltage control methods is the constant flux control where the magnetizing current is kept constant [10, 14]. As the frequency varies, all the reactances vary accordingly. Taking the operating frequency as  $\omega_s$  and  $\omega_b$  or  $\omega_{sr}$  as the base (rated) frequency at which the reactances  $x_{ls}$ ,  $x'_{lr}$ , and  $x_m$  are determined, then the familiar equivalent circuit for the steady state analysis of squirrel cage induction motor modifies as shown in figure 1 below.



**Figure 1:** Steady State Equivalent Circuit of a Squirrel Cage Induction Motor for Constant Flux Control ( $a \leq 1$ )

The per unit frequency ‘‘a’’ is defined as,

$$a = \frac{f_s}{f_{rated}} = \frac{\omega_s}{\omega_b} = \frac{\omega_s}{\omega_{sr}} \quad (1)$$

Where  $E_{ar}$ , is the rated induced voltage or back emf. For constant  $I_m$ ,

$$\frac{E'}{ax_m} = \frac{E_{ar}}{x_m} \equiv E' = aE_{ar} \quad (2)$$

Next, the motor operation for this constant flux control is examined. Under this operating condition, the rotor referred current is

$$I'_r = \frac{aE_{ar}}{\sqrt{\left(\frac{r'_r}{s}\right)^2 + (ax'_{lr})^2}} = \frac{E_{ar}}{\sqrt{\left(\frac{r'_r}{as}\right)^2 + (x'_{lr})^2}} \quad (3)$$

The resulting motor torque is

$$T_{em} = \frac{3P}{2a\omega_{sr}} (I'_r)^2 \frac{r'_r}{s} = \frac{3P}{2\omega_{sr}} \frac{r'_r}{as} \frac{E_{ar}^2}{\left(\frac{r'_r}{as}\right)^2 + (x'_{lr})^2} \quad (4)$$

The maximum and minimum torques,  $T_{max/min}$ , are determined by setting  $\frac{dT_{em}}{ds} = 0$  to find  $s_{max/min}$ ; where

$$s_{max/min} \text{ are values of slip at which } T_{max/min} \text{ occur. Therefore,} \quad (5)$$

$$s_{max/min} = \pm \frac{r'_r}{ax'_{lr}}$$

Substituting  $s_{max/min}$  for  $s$  in equations 4,

$$T_{max/min} = \pm \frac{3P}{4\omega_{sr}} \frac{E_{ar}^2}{x'_{lr}} \quad (6)$$

The effect of the term ‘‘as’’ is obvious in equations 3 and 4. It is seen that if ‘‘as’’ is maintained constant, then  $I'_r$  and  $T_{em}$  are maintained constant analogous to armature voltage control of dc motor up to rated speed. From equation 6, it can be seen that  $T_{max/min}$  is independent of ‘‘as’’ and that  $T_{max}$  is equal to  $T_{min}$  in magnitude.

TABLE ONE: SAMPLE MACHINE DATA

Rated Voltage	400V
Winding Connection	Star
Rated Frequency	50Hz
Number of Poles	6
Rated Speed	960rpm
Stator Resistance	0.4Ω
Rotor Referred Resistance	0.2Ω
Stator Reactance	1.5Ω
Rotor Referred Reactance	1.5Ω
Magnetizing Reactance	30Ω

Figure 2 shows the Torque-Per Unit Frequency Curves of the Sample Motor of Table One under Constant Flux Control, at  $a \leq 1$ .

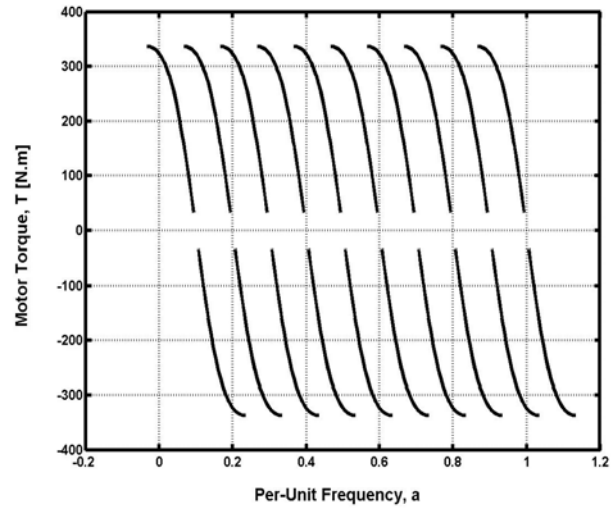


Figure 2: Torque-Per Unit Frequency Curves for Constant Flux Control ( $a \leq 1$ )

This control strategy applies up to the rated stator voltage  $V_s = V_{sr}$  at the rated frequency. Above the rated frequency,  $a > 1$ ,  $V_s$  must be kept constant at  $V_{sr}$  to avoid damage to insulation.

For control above base speed, where  $a > 1$ ,  $I_m$  is no longer constant but decreasing with increase in frequency. This implies increased magnetizing reactance and continuous decrease in magnetizing current. The approximate equivalent circuit is, therefore, used for analysis in this region of operation. The magnetizing reactance is transferred to the input terminal as shown in figure 3.

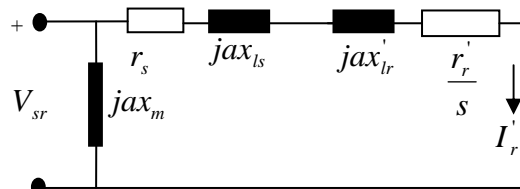


Figure 3: Approximate Equivalent Circuit for Control above Rated Speed ( $a \geq 1$ )

Under this operating condition, the rotor referred current and the motor torque are, respectively, as shown below.

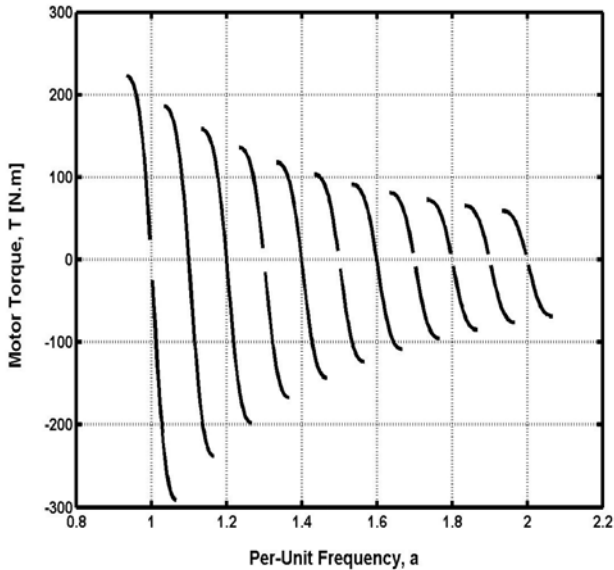
$$I'_r = \frac{V_{sr}}{\sqrt{\left(r_s + \frac{r'_r}{s}\right)^2 + a^2 x_L^2}} \quad (7)$$

$$T_{em} = \frac{3P}{2\omega_{sr}} \frac{r'_r}{as} \frac{V_{sr}^2}{\left(r_s + \frac{r'_r}{s}\right)^2 + a^2 x_L^2} \quad (8)$$

From analysis,

$$T_{max/min} = \frac{3P}{4a\omega_{sr}} \frac{V_{sr}^2}{\left(r_s \pm \sqrt{r_s^2 + a^2 x_L^2}\right)} \quad (9)$$

Figure 4 shows the Torque-Per Unit Frequency Curves of the Sample Motor of Table One for control above rated speed ( $a \geq 1$ ).

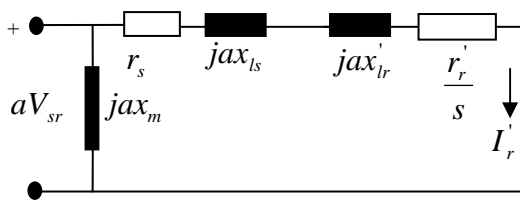


**Figure 4:** Torque-Per Unit Frequency Curves for Control above Rated Speed ( $a \geq 1$ )

The problem with direct implementation of constant flux drive is that flux (especially magnetizing flux or current) cannot be directly measured due to the lumped nature of winding parameters. Drive engineers have tried to solve this problem by using the constant  $\frac{v}{f}$  control strategy which permits the indirect control of flux.

### III. BASIC $\frac{v}{f}$ CONTROL: PRINCIPLES AND MOTOR PERFORMANCE

The approximate equivalent circuit is adopted for the basic  $\frac{v}{f}$  control analysis as shown in figure 5.



**Figure 5:** Approximate Equivalent Circuit for Basic  $\frac{v}{f}$  Control ( $a \leq 1$ )

In the equivalent circuit, the operating frequency to rated frequency ratio  $\frac{f_s}{f_{rated}} = a$  multiplies the rated terminal voltage  $V_{sr}$  and all inductive reactances. Where,  $f_{rated}$  is the rated frequency. It is deduced from figure 5, that:

$$I_r' = \frac{aV_{sr}}{\sqrt{(r_s + \frac{r_r'}{s})^2 + a^2(x_{ls} + x_{lr}')^2}} \quad (10)$$

From this, the operating torque  $T_{em}$  is derived as:

$$T_{em} = \frac{3P}{2a\omega_{sr}} \frac{r_r'}{s} \frac{a^2 V_{sr}^2}{(r_s + \frac{r_r'}{s})^2 + a^2(x_{ls} + x_{lr}')^2} \quad (11)$$

To obtain the slip  $s_{max/min}$  at which maximum/minimum torques occur, the first slip derivative of the operating torque,  $T_{em}$ , is equated to zero.

$$\frac{dT_{em}}{ds} = 0 \quad (12)$$

It is obtained, therefore, that

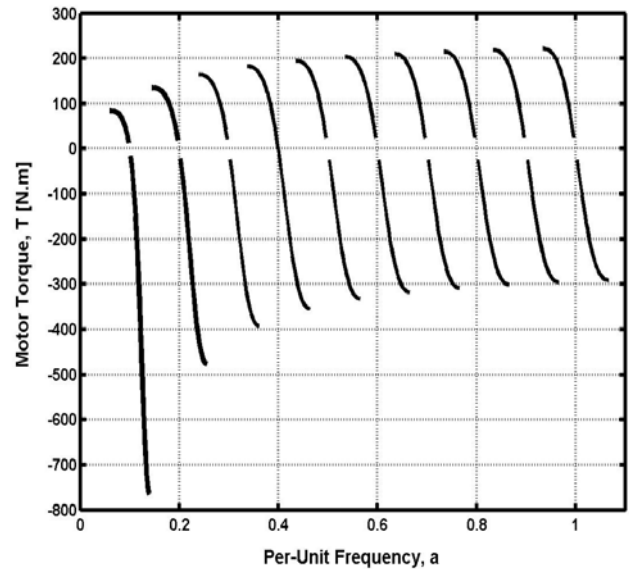
$$s_{max/min} = \pm \frac{r_r'/a}{\sqrt{(\frac{r_s}{a})^2 + (x_{ls} + x_{lr}')^2}} \quad (13)$$

Finally, the maximum and minimum torque for operation at  $a \leq 1$  is obtained as:

$$T_{max/min} = \frac{3P}{4\omega_{sr}} \frac{V_{sr}^2}{(\frac{r_s}{a} \pm \sqrt{(\frac{r_s}{a})^2 + (x_{ls} + x_{lr}')^2})^2} \quad (14)$$

The “+” sign is for maximum torque while the “-” sign is for minimum torque.

Figure 6 shows the Torque-Per Unit Frequency Curves of the Sample Motor of Table One under Basic  $\frac{v}{f}$  Control at  $a \leq 1$ .



**Figure 6:** Torque-Per Unit Frequency Curves for Basic  $\frac{v}{f}$  Control ( $a \leq 1$ )

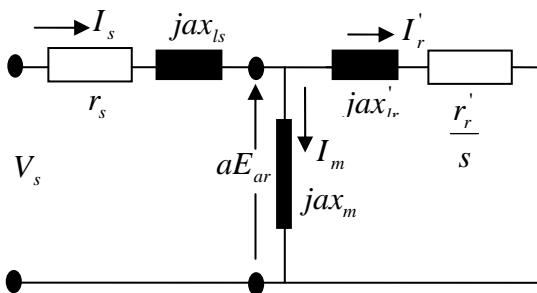
With this approach, for  $0 \leq a \leq 1$ , the magnetizing current  $I_m$  is far from being constant at low speeds because of large drops in the stator impedance when compared to  $aV_{sr}$ . Hence the need for modifications to compensate for this stator impedance drops at low frequencies [10].

#### IV. MODIFIED $\frac{v}{f}$ CONTROL STRATEGY AND EXAMPLE MOTOR PERFORMANCES

To make full use of the motor torque capability at the start and at low speed, the  $\frac{v}{f}$  ratio is increased to sustain magnetizing flux at its rated value thereby compensating for the stator resistance drop at low frequencies associated with the basic  $\frac{v}{f}$  control. The stator voltage is adjusted according to equation 15.

$$V_s = V_0 + Ka \quad (15)$$

This stator voltage is composed of two components. One is a constant term which is an off-set voltage and the other is a frequency dependent component. Where even at zero frequency,  $V_0$  compensates for the drop in stator series impedance to make the magnetizing current equal to its rated value. Where,  $a$ , is the per-unit frequency and  $V_0$  is the stator voltage chosen to give rated magnetizing current at zero speed. Now, to determine  $V_0$  and  $K$ , the voltage relationships in the exact equivalent circuit of figure 7 is analyzed.



**Figure 7:** Steady State Equivalent Circuit of a Squirrel Cage Induction Motor for Variable Voltage and Frequency Control

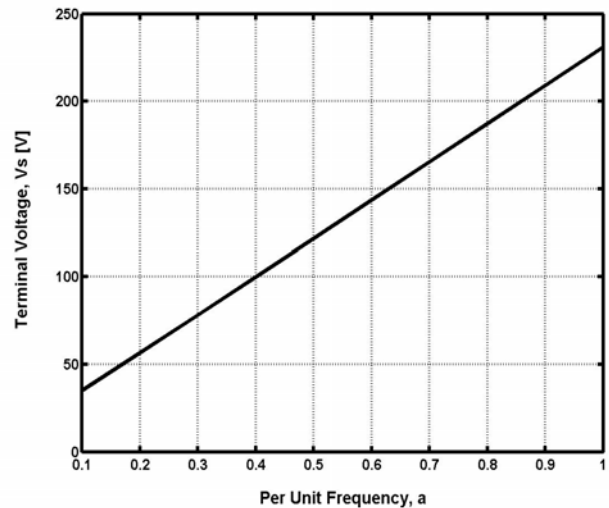
From figure 7,

$$V_s = aE_{ar} + (I_m + I'_r)(r_s + jax_{ls}) \quad (16)$$

$$\text{Where } I_m + I'_r = \left( \frac{jE_{ar}}{x_m} + \frac{aE_{ar}}{\frac{r'_r}{s} + jax'_{lr}} \right) \quad (17)$$

$$\text{Therefore, } V_s = aE_{ar} + (r_s + jax_{ls}) \left( \frac{jE_{ar}}{x_m} + \frac{E_{ar}}{\frac{r'_r}{s} + jx'_{lr}} \right) \quad (18)$$

Equation 18 is plotted as “ $a$ ” varies in the range  $0.1 \leq a \leq 1$ . The result is shown in figure 8 below.



**Figure 8:** Look-Up Plot for Modified  $\frac{v}{f}$  Control

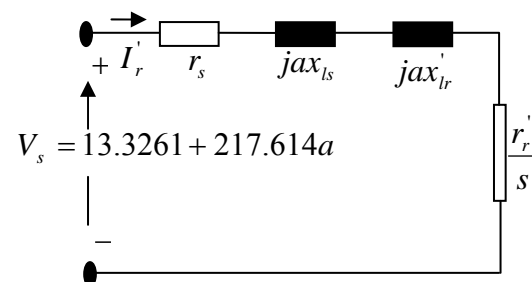
The constant,  $K$ , in equation 15 is the slope of figure 8 while  $V_0$  is obtained by extrapolating the straight line and determining the point of intersection at  $a=0$ .

The slope,  $K$ , is obtained as 217.614 and  $V_0$  obtained as 13.3261V (through extrapolation at  $a=0$ ) Then, equation 15 modifies to

$$V_s = 13.3261 + 217.614a \quad (19)$$

It is seen that even at zero frequency ( $a=0$ ), the low frequency boost-voltage  $V_s = 13.3261V$  compensates for the stator impedance drop.

Under this condition, the control characteristics are examined as  $a$  varies from 0.1 to 2 using the approximate equivalent circuit of figure 9.



**Figure 9:** Approximate Equivalent Circuit for Modified  $\frac{v}{f}$

Control Strategy

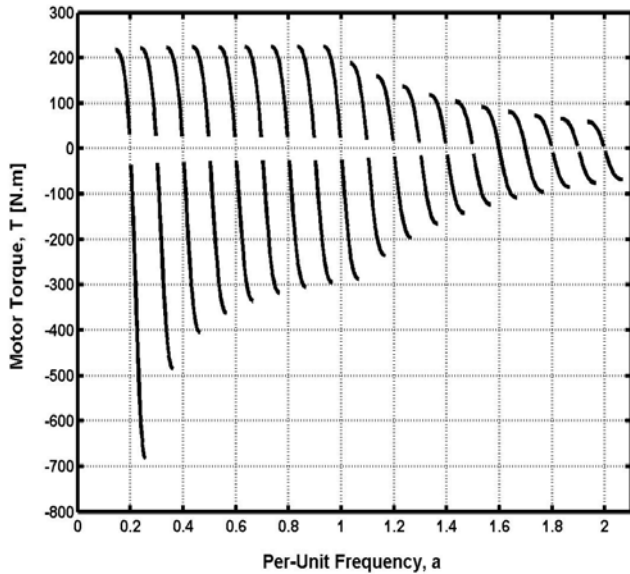
From figure 9,  $I'_r$  and  $T_{em}$  are obtained as follows:

$$I'_r = \frac{V_s}{\sqrt{\left(r_s + \frac{r'_r}{s}\right)^2 + a^2 x_L^2}} \quad (20)$$

$$T_{em} = \frac{3P}{2\omega_{sr}} \frac{r'_r}{as} \frac{V_s^2}{\left(r_s + \frac{r'_r}{s}\right)^2 + a^2 x_L^2} \quad (21)$$

Where,  $x_L = x_{ls} + x_{lr}$

Figure 10 shows the Torque-Per Unit Frequency Curves of the sample motor of Table One under the Modified  $\frac{v}{f}$  Control.

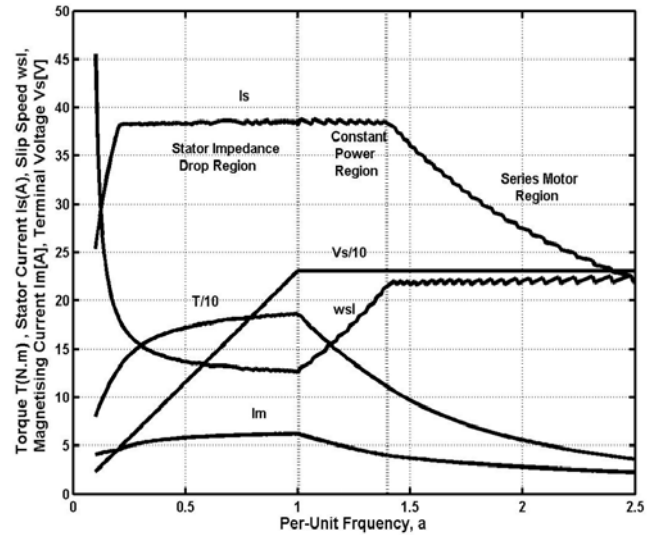


**Figure 10:** Complete Torque-Per Unit Frequency Curves for Modified  $\frac{v}{f}$  Control

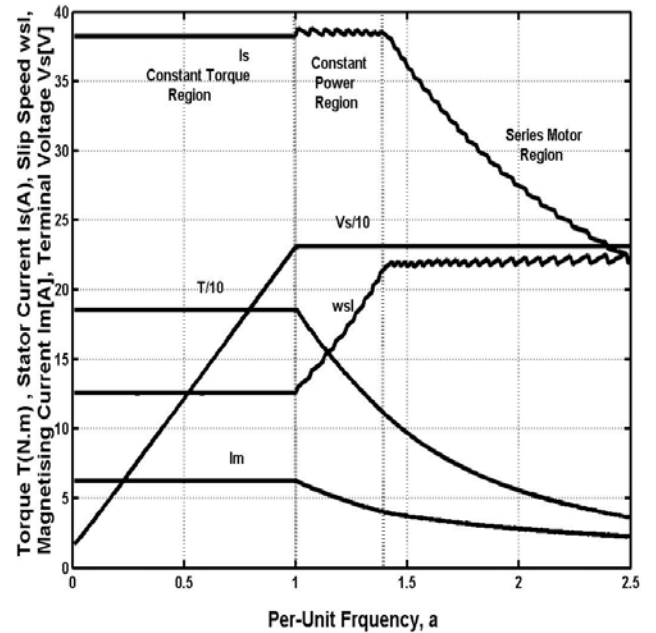
### V. PERFORMANCE COMPARISM BETWEEN THE BASIC AND THE MODIFIED $\frac{v}{f}$ CONTROL STRATEGIES

Due to the stator impedance drop at low frequencies associated with the basic  $\frac{v}{f}$  control, the magnetising flux and current gradually increases to rated values as frequency tends to rated. This leads to a corresponding gradual increase in the motor developed torque up to rated value at rated frequency. The complete drive strategy, as programmed in MATLAB, is shown in figure 11.

For the modified  $\frac{v}{f}$  control strategy where the low frequency boost voltage is injected to sustain the magnetizing flux thereby compensating for the stator resistance drop, at the maximum permissible current the drive operate essentially at a constant flux, providing constant torque operation from zero to rated speed [10]. The complete drive strategy, as programmed in MATLAB, is shown in figure 12.



**Figure 11:** The Complete Drive Strategy for Basic  $\frac{v}{f}$  Control



**Figure 12:** The Complete Drive Strategy for Modified  $\frac{v}{f}$  Control

For both the basic and the modified  $\frac{v}{f}$  drives strategies, the motor is driven beyond base speed by increasing the frequency further. However the voltage applied cannot be increased beyond rated value to avoid insulation breakdown. There are some applications, like traction, in which speed control in a wide range is required and the torque demand in the high-speed range is low. For such applications, control beyond the constant power range is required. To prevent the power from exceeding breakdown torque, the machine is operated at a constant slip speed and the machine current and power are allowed to decrease as shown in figures 11 and 12. Now, the motor current and torque decreases inversely with the speed. This characteristic is often called the series motor

characteristic. For speed below rated, the voltage and frequency are reduced with speed to maintain the desired  $\frac{v}{f}$  ratio or constant flux, and to keep the operation on the portion of the speed-torque curves with a negative slope.

## VI. CONCLUSION

The modified  $\frac{v}{f}$  control strategy, the central theme of which is the injection of low frequency boost-voltage, offers the opportunity to realize maximum torque from zero to rated speed, thereby compensating for the low frequency stator impedance drops associated with the basic  $\frac{v}{f}$  control. The control strategy generally allows the constant  $\frac{v}{f}$  control to operate up to the rated speed beyond which the motor terminal voltage is kept constant at its rated value to avoid damage to motor insulation. A comparison of the complete drives strategies of the basic and the modified  $\frac{v}{f}$  control methods, which describe the operation of the machine in a wide frequency range, were undertaken and the results, as shown in figure 11 and figure 12, confirms the modified  $\frac{v}{f}$  control as a very good approximation to the idealistic constant flux control.

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