Unsteady MHD Heat and Mass Transfer by Mixed Convection Flow in the Forward Stagnation Region of a Rotating Sphere in the Presence of Chemical Reaction and Heat Source

Ali J. Chamkha and Sameh E. Ahmed

***Abstract*— This paper is focused on the study of the problem of MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere in the presence of heat generation and chemical reaction effects. The surface of the sphere is maintained at constant fluid temperature and species concentration. The governing equations of the problem are converted into ordinary differential equations by using suitable similarity transformations. The self-similar equations are solved numerically using an efficient iterative implicit finite-difference method. The numerical results are compared with previously published results on special cases of the problem and found to be in excellent agreement. The obtained results are displayed graphically to illustrate the influence of the different physical parameters on the velocity components in x- and y-directions, temperature and concentration profiles as well as the local surface shear stresses and local heat and mass transfer coefficients.**

***Index Terms*— MHD, stagnation region, heat and mass transfer, mixed convection, heat generation, chemical reaction.**

I. Introduction

In view of their applications in industry and engineering, the study of uniform fluid flow on bodies of various geometries has been considered by many researchers using different analytical and numerical methods. For instance, the problem of mixed convection flow and heat transfer on a rotating sphere has been considered by many researchers [1-4]. Chen and Mcoglu [5] used the Keller box scheme to discuss steady mixed or free convection flow over stationary spheres. Kumari and Nath [6] investigated unsteady incompressible boundary layer flow over a rotating sphere. Takhar and Nath [7] presented a self-similar solution for unsteady flow in the stagnation-point region of a rotating sphere with a magnetic field. Anilkumar and Roy [8] reported a self-similar solution for unsteady mixed convection boundary layer flow in the forward stagnation-point region of a rotating sphere where the free stream velocity and the angular velocity of the rotating sphere vary continuously with time. A numerical study for free convection flow over a rotating sphere has been reported by Takhar et al. [9].

Manuscript received July 9, 2010.

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The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electronic chips and semi-conductor wafers. In addition, hydromagnetic incompressible viscous flows are of interest in many engineering and technological applications. In fact, the literature is replete with examples dealing with MHD and heat generation/absorption effects in laminar flow and heat transfer of viscous fluids. A similarity solution for natural convection boundary layers adjacent to vertical and horizontal surfaces in porous media with internal heat generation was reported by Pop and Postelnicu [10]. Chamkha [11] studied the problem of MHD flow of a uniformly-stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Unsteady heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects was studied by Chamkha and Al-Mudhaf [12]. Chamkha et al. [13] presented an analysis of the effects of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium. Bararnia et al. [14] investigated analytically the problem of MHD natural convection flow of a heat generation fluid in a porous medium. Andersson et al. [15] studied the flow and mass diffusion of a chemical species with first-order and higher order reactions over a linearly stretching surface.

Motivated by the investigations mentioned above, the purpose of the present work is to consider MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere in the presence of chemical reaction and heat generation effects.

II. Governing Equations

Consider unsteady laminar incompressible boundary layer flow of a viscous electrically-conducting fluid in the forward stagnation-point region of a sphere which is rotating with time-dependent angular velocity $\Omega(t)$ in the presence of magnetic field, chemical reaction and heat generation/absorption effects. The fluid properties are assumed to be constant and the chemical reaction is homogeneous and of first order. The velocity at the edge of

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

the boundary layer u_e is assumed to vary as follows (see [8]):

$u_e(x,t) = A^* x/t,\ A^* > 0,\ x > 0, t > 0.$
(1)

where A^* is the velocity gradients at the edge of the boundary layer in the x-direction. The viscous dissipation and Joule heating effects are assumed to be negligible. Under these assumptions as well as the Boussinesq approximation, the continuity, momentum, energy and concentration equations are given by

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \left[\frac{w^2}{r}\right]\frac{dr}{dx} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2}{\rho}(u - u_e) + \left[g\beta(T - T_\infty) + g\beta_c(C - C_\infty)\right] \quad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \left[\frac{uw}{r}\right]\frac{dr}{dx} = \upsilon\frac{\partial^2 w}{\partial y^2} \quad (4)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) \quad (5)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_c(C - C_\infty) \quad (6)$$

where t is time and x, y and z are the longitudinal, transverse and normal directions, respectively. r is the radial distance. u, v, and w are the dimensional velocity components in x, y and z directions, respectively. T and C are the dimensional temperature and concentration, respectively. The parameters υ, σ, β_0, ρ, β, β_c, α, g, k_c, C_p, D and Q_0 are the fluid kinematic viscosity, electrical conductivity, magnetic induction, fluid density, thermal expansion coefficient, compositional expansion coefficient, thermal diffusivity, acceleration due to gravity, chemical reaction parameter, specific heat of the fluid, mass diffusion coefficient and the heat generation/absorption coefficient, respectively. The subscript ∞ indicates ambient condition.

The initial and boundary conditions for this problem are given by

$$\begin{aligned} t = 0:\ & u(x,y,t) = u_i(x,y),\ v(x,y,t) = v_i(x,y), \\ & w(x,y,t) = w_i(x,y),\ T(x,y,t) = T_i(x,y), \\ & C(x,y,t) = C_i(x,y), \\ t > 0:\ & u(x,y,t) = 0,\ v(x,y,t) = V_w, w(x,y,t) = \Omega(t)r, \\ & T(x,y,t) = T_w, C(x,y,t) = C_w \quad at\ \ y = 0, \\ t > 0:\ & u(x,y,t) = u_e(x,t),\ w(x,y,t) = 0, \\ & T(x,y,t) = T_\infty,\ C(x,y,t) = C_\infty \quad as\ \ y \to \infty. \end{aligned} \quad (7)$$

where V_w, T_w and C_w are the normal velocity, temperature and concentration at the wall.

For this problem we can define the following transformations:

$$u = \left[\frac{A^* x}{t}\right] f'(\eta),\ v = -\sqrt{\frac{2\upsilon}{t}} A^* f(\eta),\ w = \left[\frac{Bx}{t}\right] s(\eta),\ r \approx x,$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ u_e = \frac{A^* x}{t},\ \Omega(t) = \frac{B}{t},$$
$$\lambda = \left(\frac{B}{A^*}\right)^2 = \left(\frac{w_w}{u_e}\right)^2,\ \Pr = \frac{\upsilon}{\alpha}, Sc = \frac{\upsilon}{D},\ \lambda_1 = \frac{Gr_x}{\mathrm{Re}_x^2}, \lambda_2 = \frac{Grc_x}{\mathrm{Re}_x^2}$$
$$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\upsilon^2},\ Grc_x = \frac{g\beta_c(C_w - C_\infty)x^3}{\upsilon^2},$$
$$\mathrm{Re}_x = \frac{u_e x}{\upsilon},\ M = \frac{\sigma\beta_0^2 A^* x^2}{\mu \mathrm{Re}_x}, \delta = \frac{Q_0 A^* x^2}{\mu c_p \mathrm{Re}_x}, \gamma = \frac{k_c A^* x^2}{\upsilon \mathrm{Re}_x}. \quad (8)$$

where B is the velocity gradients at the edge of the boundary layer in the y-direction and μ is the fluid dynamic viscosity.

Substituting (8) into (3)-(6) yields the following set of similarity equations:

$$f''' + Aff'' + \frac{A}{2}\left[1 - f'^2 + \lambda s^2 + \lambda_1\theta + \lambda_2\phi\right] - \frac{1}{2}\left[1 - f' - \eta f''/2\right] - \frac{M}{2}\left[f' - 1\right] = 0, \quad (9)$$

$$s'' + A\left[fs' - f's\right] + \frac{1}{2}\left[s + \eta s'/2\right] = 0, \quad (10)$$

$$\theta'' + \frac{\Pr}{2}\left[\frac{\eta\theta'}{2} + 2Af\theta' + \delta\theta\right] = 0, \quad (11)$$

$$\phi'' + \frac{Sc}{2}\left[\frac{\eta\phi'}{2} + 2Af\phi' - \gamma\phi\right] = 0, \quad (12)$$

where A is the acceleration parameter, $\Pr = \upsilon/\alpha$, and $Sc = \upsilon/D$ are the Prandtl number and the Schmitt numbers, respectively, $Gr_x = g\beta x^3(T_w - T_\infty)/\upsilon^2$ is the Grashof number, $Gr_{Cx} = g\beta_c x^3(C_w - C_\infty)/\upsilon^2$ is the modified Grashof number, $\lambda_1 = Gr_x/\mathrm{Re}_x^2$ and $\lambda_2 = Gr_{C_x}/\mathrm{Re}_x^2$ are the buoyancy parameters, $M = \sigma\beta_0^2 A^* x^2/\mu \mathrm{Re}_x$ is the magnetic field parameter, $\delta = Q_0 A^* x^2/\mu C_p \mathrm{Re}_x$ is the heat generation/absorption parameter and $\gamma = k_c A^* x^2/\upsilon \mathrm{Re}_x$ is the chemical reaction parameter.

The initial and boundary conditions (7) are transformed to

$$\begin{aligned} &\eta = 0:\ f = -f_w, f' = 0,\ s = 1, \theta = 1,\ \ \phi = 1, \\ &\eta \to \infty:\ f' = 1,\ s = 0, \theta = 0,\ \ \phi = 0\ \ . \end{aligned} \quad (13)$$

where $f_w = (V_w / A^* \sqrt{2\upsilon/t})$ is the suction/injection parameter such that $f_w > 0$ or $f_w < 0$ according to whether there is wall suction or injection, respectively.

The coefficients of the local skin friction, local Nusselt number and the local Sherwood number can be written as

$$C_{fx} = \frac{\left[2\mu\left(\frac{\partial u}{\partial y}\right)\right]_{y=0}}{\rho u_e^2} = \sqrt{\frac{8}{A^* \mathrm{Re}_x}}\, f''(0),$$
(14)

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$C_{fz} = \frac{\left[-2\mu\left(\frac{\partial w}{\partial y}\right)\right]_{y=0}}{\rho u_e^2} = -\sqrt{\frac{8\lambda}{A^* \operatorname{Re}_x}}\, s'(0), \tag{15}$$

$$Nu = \frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -\sqrt{\frac{2\operatorname{Re}_x}{A^*}}\,\theta'(0), \tag{16}$$

$$Sh = \frac{-x\left(\frac{\partial C}{\partial y}\right)_{y=0}}{C_w - C_\infty} = -\sqrt{\frac{2\operatorname{Re}_x}{A^*}}\,\phi'(0), \tag{17}$$

III. NUMERICAL METHOD AND VALIDATION

Equations (9)-(12) are nonlinear equations and it is difficult to get a closed-form solution for this system of equations. Therefore, these equations subject to the boundary conditions (13) are solved numerically by means of an efficient, iterative, tri-diagonal implicit finite-difference method discussed previously by Blottner [16].

These equations are discretized using three-point central difference formulae with f' replaced by another variable V. The η direction is divided into 196 nodal points and a variable step size is used to account for the sharp changes in the variables in the region close to the sphere surface where viscous effects dominate. The initial step size used is $\Delta\eta_1 = 0.001$ and the growth factor $G = 1.037$ such that $\Delta\eta_{n^*} = G\Delta\eta_{n^*-1}$ (where the subscript n^* is the number of nodes minus one). This gives $\eta_{\max} \approx 35$ which represents the edge of the boundary layer at infinity. The ordinary differential equations are then converted into linear algebraic equations that are solved by the Thomas algorithm discussed by Blottner [16]. Iteration is employed to deal with the non-linear nature of the governing equations. The convergence criterion employed in this work was based on the relative difference between the current and the previous iterations. When this difference or error reached 10^{-5}, then the solution was assumed converged and the iteration process was terminated.

The accuracy of the employed numerical method is tested by direct comparisons with the previously published work of Anilkumar and Roy [8] for special cases of the present problem and excellent agreement between the compared results was found. This lends confidence in the numerical results to be reported subsequently.

IV. RESULTS AND DISCUSSION

In order to get a clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The results of this parametric study are shown in Figs. 1-11. In all of the obtained results, we used $\Pr = 0.7$ to represent Hydrogen.

Figures 1 and 2 show the effects of the acceleration parameter A on the velocity components in the x- and y-directions (f', s) , temperature and concentration profiles (θ, ϕ) for injection condition $f_w = -1.0$. It is found that increasing the acceleration parameter A leads to increases in the velocity component in the x-direction. This can be explained as follows: from (1), the velocity at the edge of the boundary layer u_e increases with increasing values of the acceleration parameter A . Hence, the fluid inside the boundary layer gets accelerated in the x-direction which increases the velocity component f' . On the contrary, increases in the value of A tend to decay the fluid motion in the y-direction (rotational direction). On the other hand, both of the fluid temperature and the species concentration are reduced by increasing the acceleration parameter A . These behaviors are clearly shown in Figs. 1 and 2.

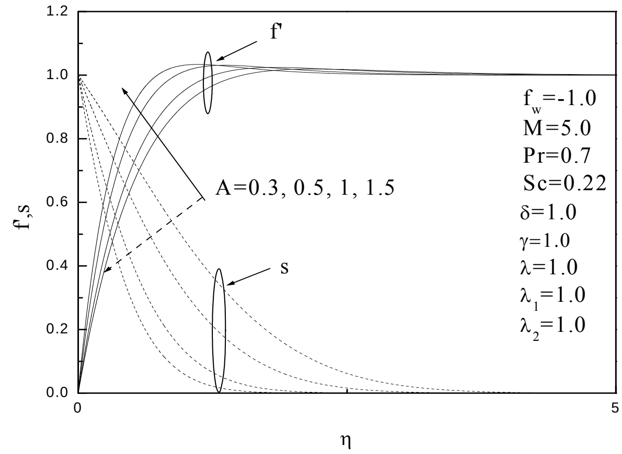

Fig. 1. Effects of A on f' and S

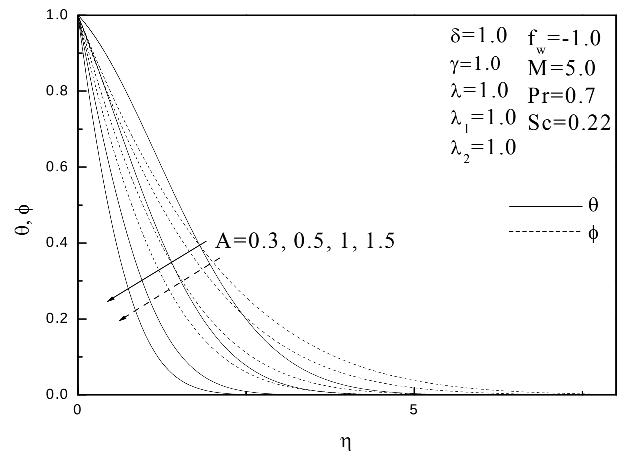

Fig. 2. Effects of A on θ and ϕ

The effects of the magnetic field parameter M and the suction/injection parameter f_w on the velocity components in the x- and y-directions (f', s) , temperature and concentration profiles (θ, ϕ) are depicted in Figs. 3 and 4. Application of a transverse magnetic field in the direction normal to the flow direction produces a drag-like force called the Lorentz force. This force tends to cause deceleration in the fluid motion and therefore, both of the velocity components decrease with increasing values of the magnetic field parameter. However, the corresponding fluid temperature and species concentration fields increase as the

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

magnetic field parameter M increases. It is also seen from Figs. 3 and 4 that increasing the suction/injection parameter f_w causes the x-component of velocity close to the sphere surface to decrease while it increases far downstream. However, the temperature and concentration increase close to the wall and decrease far downstream as f_w increases. The y-component of velocity shows a increasing trend with f_w everywhere.

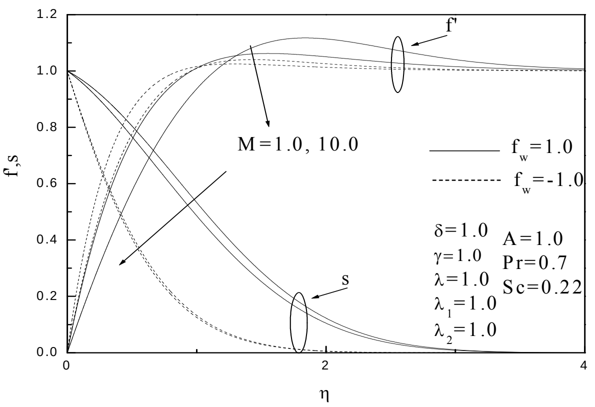

Fig. 3. Effects of M and f_w on f' and S

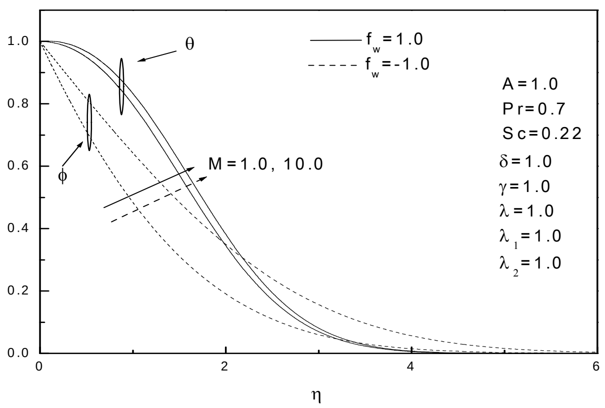

Fig. 4. Effect of M and f_w on θ and ϕ

Figures 5 and 6 display the effects of the heat generation parameter δ on the velocity component in x-direction f' and temperature profiles θ for the injection case $f_w = -1.0$, respectively. As the heat generation parameter δ increases, the fluid temperature increases. This increase in the fluid temperature has the tendency to increase the thermal buoyancy force. This produces higher buoyancy-induced flow along the sphere. This is represented in the increases in the x-component of fluid velocity as the heat generation parameter δ increases as seen from Fig. 5. It is also observed that the fluid temperature profile overshoots in the immediate vicinity of the sphere surface for higher values of heat generation parameter as depicted in Fig. 6.

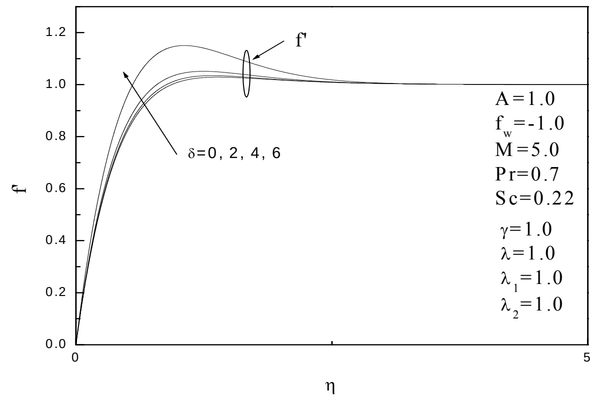

Fig. 5. Effect of δ on f'

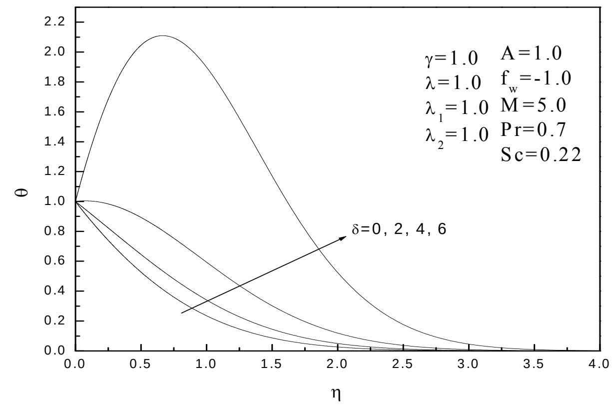

Fig. 6. Effect of δ on θ

The effects of the chemical reaction parameter γ on the velocity component in the x-direction f' and the concentration profiles ϕ for the injection case $f_w = -1.0$ are presented in Fig. 7. The effects of the chemical reaction parameter on the velocity component in the y-direction and temperature profiles are insignificant and therefore, not presented. It is observed that both of the velocity component f' and the concentration profiles ϕ decrease with increasing values of the chemical reaction parameter γ.

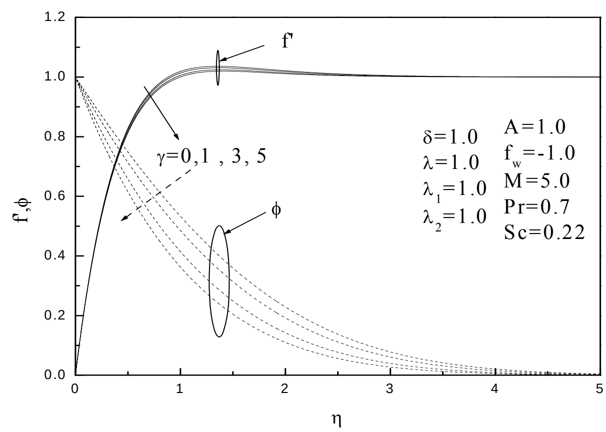

Fig. 7. Effects of γ on f' and ϕ

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

With the help of Figs. 8 and 9, the behaviors of the coefficients of the local surface shear stresses in the x- and y-directions ($f''(0), -s'(0)$) and the local heat and mass transfer coefficients (or reduced local Nusselt and Sherwood numbers) $(-\theta'(0), -\phi'(0))$ under the effects of the magnetic field parameter M and the heat generation parameter δ are observed. As expected, the local surface shear stresses coefficients ($f''(0), -s'(0)$) and the local reduced Sherwood number ($-\phi'(0)$) increase with increasing values of either the magnetic field parameter M or the heat generation parameter δ . However, the local reduced Nusselt number ($-\theta'(0)$) takes the opposite behavior as either M or δ increases.

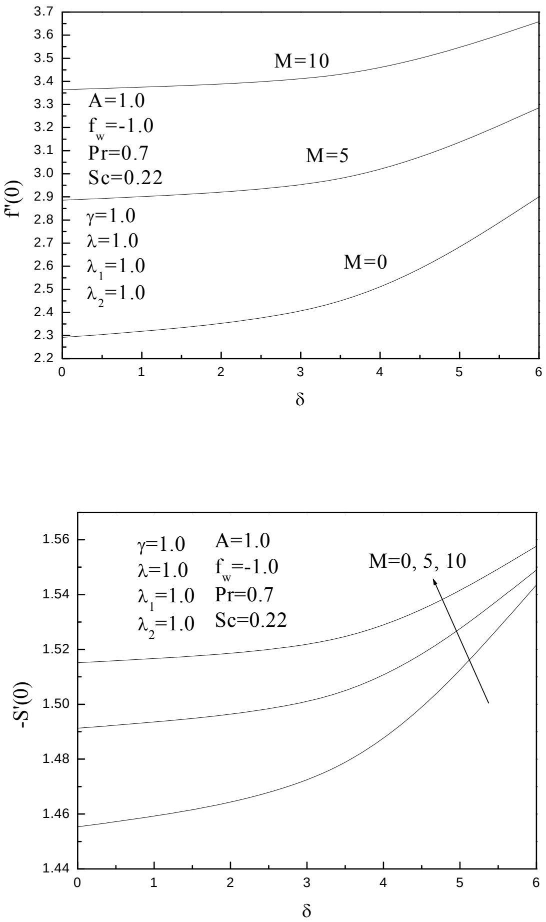

Fig. 8. Effects of M and δ on $f''(0)$ and $-s'(0)$

Figures 10 and 11 depict the effects of the acceleration parameter A and the chemical reaction parameter γ on the local surface shear stresses coefficients $(f''(0), -s'(0))$ and the local reduced Nusselt and Sherwood numbers $(-\theta'(0), -\phi'(0))$. It is observed that the coefficients of the local surface shear stresses $(f''(0), -s'(0))$ and the local reduced Sherwood number ($-\phi'(0)$) increase as the chemical reaction parameter increases while the local reduced Nusselt number ($-\theta'(0)$) decreases as the chemical reaction parameter γ increases for small values of the acceleration parameter A and increases for large values of the acceleration parameter A .

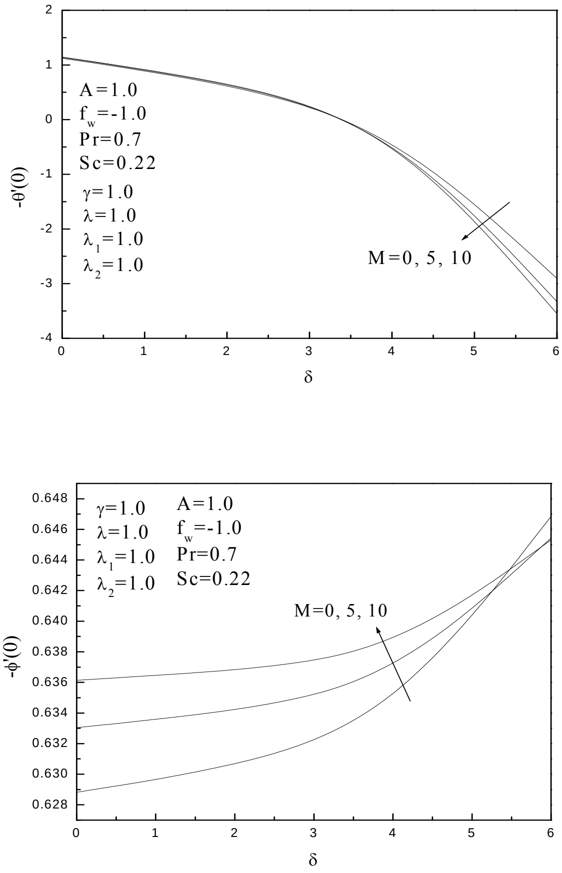

Fig. 9. Effects of M and δ on $-\theta'(0)$ and $-\phi'(0)$

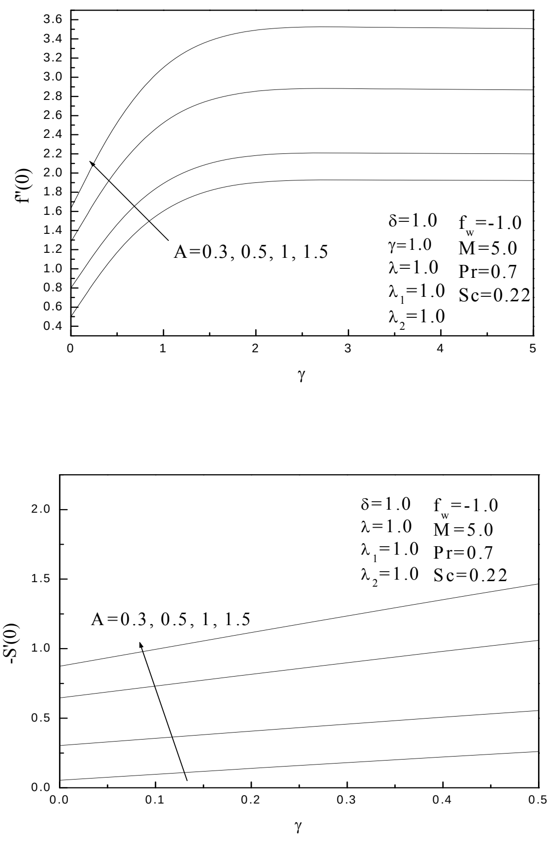

Fig. 10. Effects of A and γ on $f''(0)$ and $-s'(0)$

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

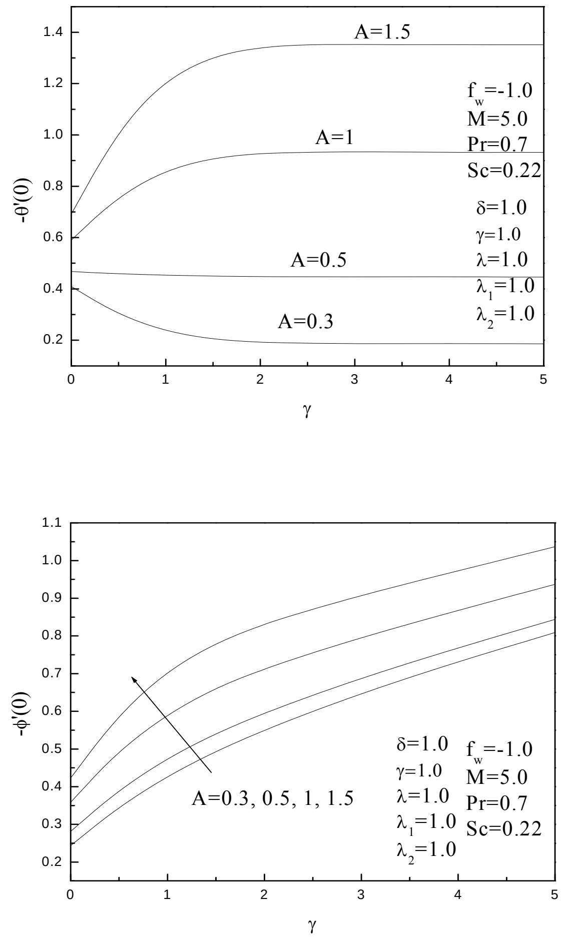

Fig. 11. Effects of A and γ on $-\theta'(0)$ and $-\phi'(0)$

V. CONCLUSION

The problem of unsteady MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere in the presence of chemical reaction and heat generation effects was studied. The governing equations were developed and transformed into a self-similar form. The similarity equations were solved numerically by an efficient, tri-diagonal, implicit finite-difference method. From the presented results of the problem, it was observed that

I. Increasing the value of the magnetic field parameter resulted in increases in both of the local coefficients of surface shear stresses, local reduced Sherwood number, temperature and solute concentration in the fluid whereas the local reduced Nusselt number and velocity components decreased with the increasing values of the magnetic parameter.

II. Imposition of fluid wall injection resulted in increases the x-component of velocity component close to the sphere surface and decreases in the temperature and concentration profiles close to the wall while the y-component of velocity decreased everywhere.

III. Increasing the value of the acceleration parameter led to increases in the velocity component in the x-direction, local coefficients of surface shear stresses and local reduced Nusselt and Sherwood numbers whereas the velocity component in the rotational direction, temperature and concentration in the fluid decreased.

IV. The local coefficients of surface shear stresses and local reduced Sherwood number increased as the chemical reaction parameter increased while the local reduced Nusselt number decreased as the chemical reaction parameter increased for small values of the acceleration parameter and increased for large values of the acceleration parameter.

ACKNOWLEDGMENT

The first author thanks and acknowledges the Kuwait Foundation for the Advancement of Sciences for its financial support of the scientific mission to present this work.

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ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)