Energy Function Analysis of a Single-Machine Infinite-Bus Power System

A. Cifci, Y. Uyaroglu, and A. T. Hocaoglu

***Abstract*—A power system at a given operating state and subjected to a given disturbance is voltage stable if the voltages near loads approach post-disturbance equilibrium values. In this paper, by using the energy function that maps the energy variation of the system, the effect of the slow chance of the system is analyzed and thus the system's energy level changes' effects on the system's stability is shown by using MATHCAD program. It is demonstrated that the stored energy measure is an indicator of the closeness of the operating point to the instability region of the system.**

***Index Terms*—Energy function, Lyapunov's second method, The variable gradient method, Voltage stability**

NOMENCLATURE

δ	Load angle in radians
V	Load voltage in p.u.
E_m	Generator voltage in p.u.
E_0	Infinite bus or slack bus voltage in p.u.
Y_m	Generator admittance in p.u.
Y_0	Infinite bus or slack bus admittance in p.u.
δ_m	Generator rotor angle in radians
C	Compensated load capacitor in p.u.
θ_m	Generator admittance angle in degrees
θ_0	Infinite bus admittance angle in degrees
M	Generator inertia in p.u.
D	Damping coefficient
P_m	Mechanical power in p.u.
K_{pw}, K_{pv}	Constant parameters
K_{qw}, K_{qv}, K_{qv2}	Constant parameters

I. INTRODUCTION

SINCE 1920s, electric power system stability has been considered as an important problem in terms of reliable system operation [1], [2]. The concept of voltage stability is expressed as the ability of keeping voltages' magnitudes of load buses, under both in steady state voltage stability and transient voltage stability conditions, within the specific operating limitations [3].

In the cases of not making voltage control and increase the load due to disabling, for any reason, the elements such as generator, line, transformer, bus etc if an uncontrolled voltage drop occurs, then there appears power system instability. The main reason of the voltage instability is that in the overloaded systems the system can not ensure the reactive energy needed by the system to keep voltage values in a certain amount [4]–[7]. Other reasons are generator reactive power limits, load characteristics, characteristics of load tap changer transformers, characteristics of reactive power compensation devices and behaviour of voltage control devices [8].Voltage stability and collapses began to play a significant role in power system analysis and control as a result of energy system collapses in various places of the world such as Egypt [9], Chile [10], The United States and Canada [11], [12].

This study is organized as follows respectively. Section II outlines the main idea of Lyapunov stability analysis. Section III examines a single-machine infinite-bus power system's energy function. Section IV presents simulation results of energy function analysis. Finally conclusions are given in Section V.

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II. LYAPUNOV STABILITY ANALYSIS

The constant exponents can be used in the study of nonlinear differential equations' stability was first shown by a Russian mathematician, Sonya Kovalevskaya, in 1889. Later in 1892 Kovalevskaya's study was developed by another Russian mathematician, Alexandr Mikhailovich Lyapunov.

Lyapunov's second method (also called Lyapunov's direct method) provides us with studying the stability of the system concerning on the dynamic system before finding the solution of differential equation. The second method is appropriate for the voltage stability of nonlinear systems which do not have accurate solutions. This method is the most common one in terms of the determination of stability conditions of time-dependent nonlinear systems and could be applied to all known systems.

Stability Analysis of Nonlinear Systems

Voltage stability of nonlinear systems is regional. Hence, Lyapunov function which obtains sufficient stability conditions in the largest region around the origin is searched for.

Some methods which arise from Lyapunov's second method are proper to examine the stability of nonlinear systems. One of them is the variable gradient method which is used for the generalization of Lyapunov functions.

The Variable Gradient Method

There are no generally applicable methods for finding Lyapunov functions. The variable gradient method is a formal approach to constructing Lyapunov functions. The variable gradient method assumes a certain form for the gradient of an unknown Lyapunov function, and then finding the Lyapunov function itself by integrating the assumed gradient [13].

Consider a nonlinear dynamical system described by

$$\dot{x} = f(x,t) \tag{1}$$

f: n×1 nonlinear vector function
x: n×1 state vector
n: numbers of states, order of the systems

Accept an equilibrium point at the origin of the space. Denote a test Lyapunov function by using V. Assume that in (1), V is x's open function but not t's. Then,

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \ldots + \frac{\partial V}{\partial x_n}\dot{x}_n \tag{2}$$

can be written. Hence,

$$\dot{V} = (\nabla V)^* \dot{x} \tag{3}$$

In (3), $(\nabla V)^*$ is ∇V's transpose. The gradient of V, denoted by ∇V as follows:

$$\nabla V = \begin{bmatrix} \frac{\partial V}{\partial x_1} \\ \cdot \\ \cdot \\ \frac{\partial V}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ \cdot \\ \cdot \\ \cdot \\ \nabla V_n \end{bmatrix} \tag{4}$$

∇V's line integral can be expressed by

$$V = \int_0^x (\nabla V)^* dx \tag{5}$$

In (5), integral's upper limit does not point that V is a vector magnitude, but integral is prefer to line integral of a random point $(x_1, x_2, \ldots, x_n)$ at the space. This integral can be done separately from integration method.

Investigation of Lyapunov Function Using Gradient System

A special class of dynamical system is particularly well suited to the Lyapunov method. This system arises from the gradient of a function [14]. A gradient dynamical system is given as

$$\dot{x} = -A\nabla v(x, x_0) \tag{6}$$

In (6), v: $\Re^n x \Re^n \rightarrow \Re$ can be a continuously differentiable. $A \in \Re^{nxn}$ is defined as det(A)≠0 and $v(x,x_0)=0$ for $x=x_0$. If $v(x,x_0)$'s Hessian is completely positive definite at x_0, equilibrium point is asymptotically stable at x_0.

Lyapunov function is given as

$$V(x) = \int_{x_0}^{x} [f(\xi)]^T d\xi \tag{7}$$

Lyapunov function which is given above will be used in order to find the single-machine infinite-bus power system's energy function.

III. Energy Function of a Single-Machine Infinite-Bus Power System

We consider the power system model shown in Fig. 1, which is taken from [15].

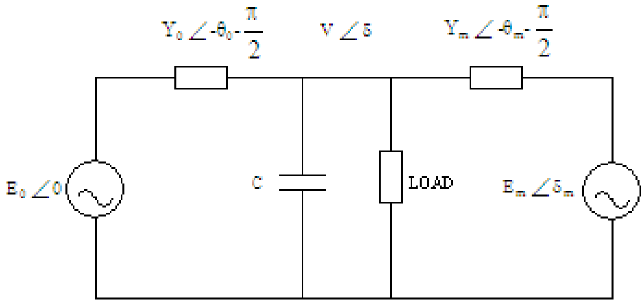

Fig. 1. Power system model.

This system consists of a load bus and two generator buses. One of the generator busses is treated as a slack bus. The load is modelled by a simplified induction motor in parallel with a constant P-Q load and constant impedance. The load also includes a fixed capacitor C to raise the voltage up to near 1.0 per unit [15]. The network, load and generator parameters have been presented in the Appendix.

First-order differential equations are expressed which show power system model's equations of state as follow [16].

$$\dot{\delta}_m = w \tag{8}$$

$$M\dot{w} = -Dw + P_m + E_m V Y_m \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m \tag{9}$$

$$K_{qw}\dot{\delta} = -K_{qv}V - K_{qv2}V^2 + Q - Q_0 - Q_1 \tag{10}$$

$$\begin{aligned} TK_{qw}K_{pv}\dot{V} &= K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V \ldots \\ &+ K_{pw}(Q_0 + Q_1 - Q) - K_{qw}(P_0 + P_1 - P) \end{aligned} \tag{11}$$

The system differential equations above can be written again under the condition that generator mechanical power is equivalent to active load requirement (P_m=P_l).

$$\dot{w} = -\frac{D}{M^2}Mw - \frac{1}{M}f(\delta, \delta_m, V) \tag{12}$$

$$\dot{\delta}_m = \frac{1}{M}Mw \tag{13}$$

$$\dot{\delta} = -g(\delta, \delta_m, V) \tag{14}$$

$$\dot{V} = -h(\delta, \delta_m, V) \tag{15}$$

Here,

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$$f(\delta,\delta_m,V) = -(P_m + E_m V Y_m \sin(\delta - \delta_m - \theta_m) + E_m^{\,2} Y_m \sin(\theta_m)) \quad (16)$$

$$g(\delta,\delta_m,V) = -\frac{1}{K_{qw}}(-K_{qv2}V^2 - K_{qv}V + Q - Q_0 - Q_1) \quad (17)$$

$$\begin{aligned} h(\delta,\delta_m,V) = -\frac{1}{TK_{qw}K_{pv}}(K_{pw}K_{qv2}V^2 + (K_{pw}Kqv... \\ -K_{qw}K_{pv})V + K_{pw}(Q_0 + Q_1 - Q) - K_{qw}(P_0 + P_1 - P)) \end{aligned} \quad (18)$$

The system differential equations expressed by the equations (8), (9), (10), and (11) are the definition of the simple system that involves highly complicated load modelling at around the high voltage operating point.

Defining Gradient System to the Form of Lyapunov Function for a Simple Power System

The derivation of Lyapunov Function for the system in Fig. 1, equations (12), (13), (14) and (15) could be determined as

$$\begin{bmatrix} \dot{\delta}_m \\ \dot{w} \\ \dot{\delta} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{1}{M} & 0 & 0 \\ \dfrac{1}{M} & \dfrac{D}{M^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f(\delta_m,\delta,V) \\ Mw \\ g(\delta_m,\delta,V) \\ h(\delta_m,\delta,V) \end{bmatrix} \quad (19)$$

The equation (19) for the system defined in (8), (9), (10) and (11) equations, is an alternative definition for this system's dynamics.

For the $(w_0,\delta_{m0},\delta_0,V_0)$'s equilibrium point, a candidate energy function which is seen on the right of the (19) equation ((4x1) gradient matrix seen on the right of the (19) equation) is obtained and therefore it can be used in (7) equation. The candidate energy function can be written in (7) equation as

$$v(w,\delta_m,\delta,V) = \int_{(w_0,\delta_{m0},\delta_0,V_0)}^{(w,\delta_m,\delta,V)} \begin{bmatrix} Mw \\ f(\delta_m,\delta,V) \\ g(\delta_m,\delta,V) \\ h(\delta_m,\delta,V) \end{bmatrix}^T \begin{bmatrix} dw \\ d\delta_m \\ d\delta \\ dV \end{bmatrix} \quad (20)$$

If $f(w,\delta_m,\delta,V)$, $g(w,\delta_m,\delta,V)$ ve $h(w,\delta_m,\delta,V)$ are replaced on (20) equation, the system's energy function is obtained [17].

The equilibrium point is $(w^*,\delta_m^{\,*},\delta^*,V^*)=(0.0,0.3,0.2,0.97)$.

IV. Simulation Results of a Single-Machine Infinite-Bus Power System

There are four important state variables in these analyses. These are the system frequency (w_1), generator rotor angle (δ_m), load angle (δ) and load voltage (V). The aim of these analyses is to show what kind of effects the load would have over the whole energy of the power system. The generator rotor angle will be changed, beginning with zero and will be increased to 1.6 by 0.4 rise each turn in order to observe the system's stability.

The sample of the energy function for the single-machine infinite-bus power system is given as follows:

$$v(\delta,V) = 2.008V^3 + a_2V^2 + a_1V + a \quad (21)$$

When the sample of the energy function given above is equalized the system's energy function which is obtained from (20) equation, a_2, a_1 and a are respectively obtained for each case.

The following cases are considered:

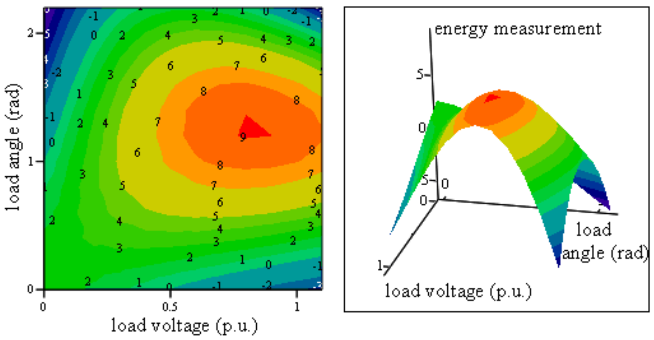
Case-1: Generator rotor angle δ_m=0 rad., system frequency w=1 p.u

a_2, a_1 and a are respectively obtained as

$$\begin{aligned} a_2 = 2.426 - 14.907\delta + 0.075\sin(\delta - 0.087)... \\ +0.3\sin(\delta - 0.209) - \cos(\delta - 0.087) - 4\cos(\delta - 0.209) \end{aligned} \quad (22)$$

$$\begin{aligned} a_1 = -0.405 + 2.8\delta + 5\cos(\delta - 0.213) - 5\cos(\delta + 0.087)... \\ +5\sin(\delta - 0.087) + 20\sin(\delta - 0.209) \end{aligned} \quad (23)$$

$$\begin{aligned} a = -1.336 - 1.3\delta - 0.063\sin(\delta - 0.087) - 0.254\sin(\delta - 0.209)... \\ +0.846\cos(\delta - 0.087) + 3.386\cos(\delta - 0.209) \end{aligned} \quad (24)$$



(a) (b)

Fig. 2. System's stored energy for δ_m=0 (a) Two-dimensional representation (b) Three- dimensional representation.

The system's energy density is in the range of $0.6 \leq V \leq 1$ and $1 \leq \delta \leq 1.6$, which is seen as in Fig. 2 and Table I. The system's energy density varies between 8 and 9 energy units around these points.

TABLE I
ENERGY MEASUREMENT FOR Δ_M=0

δ	Energy Measurement										
0	2,9	2,3	1,8	1,2	0,6	-0,1	-0,7	-1,3	-1,9	-2,4	-2,9
0,2	2,6	2,6	2,6	2,4	2,2	1,8	1,5	1,1	0,6	0,2	-0,3
0,4	2,2	2,8	3,2	3,5	3,6	3,6	3,5	3,3	3,1	2,7	2,3
0,6	1,6	2,7	3,6	4,3	4,8	5,2	5,4	5,4	5,3	5,0	4,7
0,8	0,9	2,5	3,8	4,9	5,8	6,4	6,8	7,1	7,1	7,0	6,7
1	0,0	2,1	3,8	5,2	6,4	7,2	7,9	8,3	8,4	8,3	8,0
1,2	-0,9	1,4	3,5	5,2	6,5	7,6	8,4	8,9	9,1	9,0	8,7
1,4	-2,0	0,7	2,9	4,8	6,3	7,5	8,3	8,8	9,0	8,9	8,4
1,6	-3,1	-0,3	2,1	4,1	5,6	6,8	7,6	8,0	8,1	7,8	7,2
1,8	-4,2	-1,3	1,1	3,0	4,6	5,6	6,3	6,5	6,4	5,8	4,9
V	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

For δ_m=0 rad. and w=1 p.u., Table I shows numerical values of the energy function for different load angles and different load voltages.

Case-2: Generator rotor angle δ_m=0.4 rad., system frequency w=1 p.u.

a_2, a_1 and a are respectively obtained as

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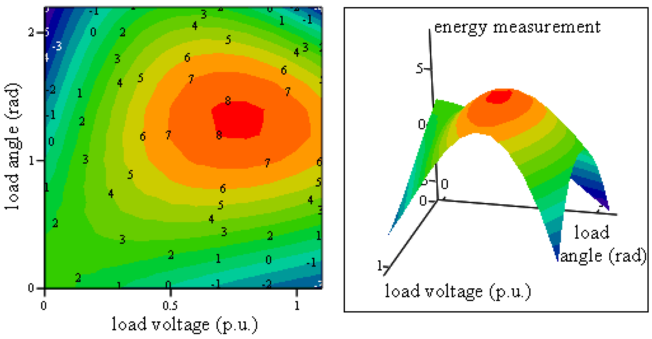
$$a_2 = 2.426 - 14.907\delta + 0.075\sin(\delta - 0.487)...$$
$$+0.3\sin(\delta - 0.209) - \cos(\delta - 0.487) - 4\cos(\delta - 0.209) \quad (25)$$
$$a_1 = 1.573 + 2.8\delta + 5\cos(\delta - 0.213) - 5\cos(\delta + 0.313)...$$
$$+5\sin(\delta - 0.487) + 20\sin(\delta - 0.209) \quad (26)$$
$$a = -1.562 - 1.3\delta - 0.063\sin(\delta - 0.487) - 0.254\sin(\delta - 0.209)...$$
$$+0.846\cos(\delta - 0.487) + 3.386\cos(\delta - 0.209) \quad (27)$$



(a) (b)

Fig. 3. System's stored energy for δ_m=0.4 (a) Two-dimensional representation (b) Three- dimensional representation.

The system's energy density is in the range of 0.5≤V≤1 and 1≤δ≤1.6, which is seen as in Fig. 3 and Table II. The system's energy density varies between 7 and 8 energy units around these points.

TABLE II
ENERGY MEASUREMENT FOR Δ_M=0.4

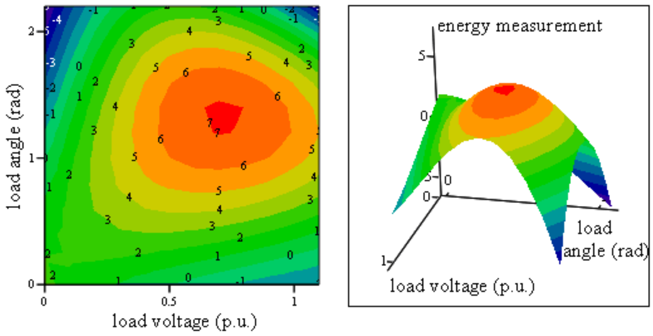
δ	Energy Measurement										
0	2,6	2,1	1,5	1,0	0,4	-0,2	-0,8	-1,3	-1,9	-2,4	-2,8
0,2	2,4	2,4	2,3	2,1	1,9	1,5	1,1	0,7	0,3	-0,2	-0,7
0,4	2,0	2,6	2,9	3,1	3,2	3,2	3,0	2,8	2,4	2,0	1,5
0,6	1,5	2,5	3,4	4,0	4,4	4,6	4,7	4,6	4,4	4,1	3,6
0,8	0,9	2,3	3,6	4,6	5,3	5,8	6,1	6,2	6,1	5,8	5,4
1	0,0	2,0	3,6	4,9	5,9	6,7	7,1	7,4	7,4	7,2	6,7
1,2	-0,9	1,4	3,3	4,9	6,2	7,1	7,7	8,1	8,1	7,9	7,4
1,4	-1,9	0,7	2,8	4,6	6,0	7,1	7,8	8,1	8,2	7,9	7,3
1,6	-3,0	-0,2	2,1	4,0	5,5	6,6	7,3	7,6	7,6	7,2	6,4
1,8	-4,1	-1,2	1,2	3,1	4,6	5,6	6,2	6,4	6,2	5,5	4,5
V	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

For δ_m=0.4 rad. and w=1 p.u., Table II shows numerical values of the energy function for different load angles and different load voltages.

Case-3: Generator rotor angle δ_m=0.8 rad., system frequency w=1 p.u.

a_2, a_1 and a are respectively obtained as

$$a_2 = 2.426 - 14.907\delta + 0.075\sin(\delta - 0.887)...$$
$$+0.3\sin(\delta - 0.209) - \cos(\delta - 0.887) - 4\cos(\delta - 0.209) \quad (28)$$
$$a_1 = 3.329 + 2.8\delta + 5\cos(\delta - 0.213) - 5\cos(\delta + 0.713)...$$
$$+5\sin(\delta - 0.887) + 20\sin(\delta - 0.209) \quad (29)$$
$$a = -1.788 - 1.3\delta - 0.063\sin(\delta - 0.887) - 0.254\sin(\delta - 0.209)...$$
$$+0.846\cos(\delta - 0.887) + 3.386\cos(\delta - 0.209) \quad (30)$$



(a) (b)

Fig. 4. System's stored energy for δ_m=0.8 (a) Two-dimensional representation (b) Three- dimensional representation.

The system's energy density is in the range of 0.5≤V≤0.9 and 1≤δ≤1.6, which is seen as in Fig. 4 and Table III. The system's energy density varies between 6 and 7 energy units around these points.

TABLE III
ENERGY MEASUREMENT FOR Δ_M=0.8

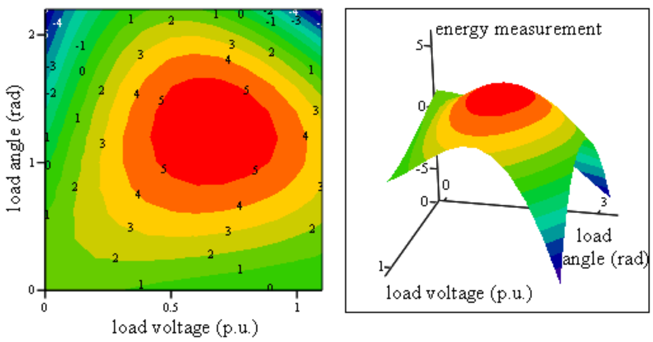
δ	Energy Measurement										
0	2,2	1,8	1,4	0,9	0,5	0,0	-0,4	-0,8	-1,2	-1,5	-1,8
0,2	2,0	2,1	2,1	2,0	1,8	1,5	1,2	0,9	0,5	0,1	-0,3
0,4	1,7	2,3	2,6	2,9	3,0	2,9	2,8	2,6	2,2	1,8	1,4
0,6	1,3	2,3	3,0	3,6	4,0	4,2	4,2	4,1	3,9	3,5	3,0
0,8	0,7	2,1	3,3	4,1	4,8	5,2	5,5	5,5	5,3	4,9	4,4
1	-0,1	1,8	3,2	4,4	5,4	6,0	6,4	6,5	6,3	6,0	5,4
1,2	-0,9	1,2	3,0	4,5	5,6	6,4	6,9	7,0	6,9	6,6	5,9
1,4	-1,9	0,5	2,6	4,2	5,5	6,4	6,9	7,1	7,0	6,5	5,8
1,6	-2,9	-0,3	1,9	3,7	5,0	5,9	6,5	6,6	6,4	5,8	4,9
1,8	-4,0	-1,2	1,0	2,8	4,2	5,1	5,5	5,6	5,2	4,4	3,2
V	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

For δ_m=0.8 rad. and w=1 p.u., Table III shows numerical values of the energy function for different load angles and different load voltages.

Case-4: Generator rotor angle δ_m=1.2 rad., system frequency w=1 p.u.

a_2, a_1 and a are respectively obtained as

$$a_2 = 2.426 - 14.907\delta + 0.075\sin(\delta - 1.287)...$$
$$+0.3\sin(\delta - 0.209) - \cos(\delta - 1.287) - 4\cos(\delta - 0.209) \quad (31)$$
$$a_1 = 4.584 + 2.8\delta + 5\cos(\delta - 0.213) - 5\cos(\delta + 1.113)...$$
$$+5\sin(\delta - 1.287) + 20\sin(\delta - 0.209) \quad (32)$$
$$a = -2.014 - 1.3\delta - 0.063\sin(\delta - 1.287) - 0.254\sin(\delta - 0.209)...$$
$$+0.846\cos(\delta - 1.287) + 3.386\cos(\delta - 0.209) \quad (33)$$



(a) (b)

Fig. 5. System's stored energy for δ_m=1.2 (a) Two-dimensional representation (b) Three- dimensional representation.

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The system's energy density is in the range of $0.4 \le V \le 1$ and $0.6 \le \delta \le 1.8$, which is seen as in Fig. 5 and Table IV. The system's energy density varies between 4 and 5 energy units around these points.

TABLE IV
ENERGY MEASUREMENT FOR Δ_M=1.2

δ	Energy Measurement										
0	1,6	1,5	1,2	1,0	0,8	0,6	0,4	0,2	0,0	0,0	-0,1
0,2	1,6	1,8	1,9	1,9	1,9	1,8	1,6	1,4	1,2	1,0	0,8
0,4	1,3	1,9	2,4	2,7	2,9	2,9	2,9	2,7	2,5	2,2	1,9
0,6	0,9	1,9	2,7	3,3	3,7	3,9	4,0	3,9	3,7	3,3	2,9
0,8	0,4	1,8	2,9	3,7	4,4	4,8	4,9	4,9	4,7	4,3	3,7
1	-0,3	1,4	2,8	3,9	4,8	5,3	5,6	5,6	5,4	4,9	4,3
1,2	-1,1	0,9	2,6	3,9	4,9	5,6	5,9	6,0	5,7	5,2	4,4
1,4	-2,0	0,3	2,2	3,6	4,7	5,5	5,8	5,9	5,5	4,9	4,0
1,6	-3,0	-0,5	1,5	3,1	4,2	5,0	5,3	5,3	4,9	4,1	2,9
1,8	-4,0	-1,4	0,7	2,3	3,4	4,1	4,4	4,2	3,6	2,6	1,2
V	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

For δ_m=1.2 rad. and w=1 p.u., Table IV shows numerical values of the energy function for different load angles and different load voltages.

Case-5: Generator rotor angle δ_m=1.6 rad., system frequency w=1 p.u.

a_2, a_1 and a are respectively obtained as

$$a_2 = 2.426 - 14.907\delta + 0.075\sin(\delta - 1.687)\ldots \\ + 0.3\sin(\delta - 0.209) - \cos(\delta - 1.687) - 4\cos(\delta - 0.209) \tag{34}$$

$$a_1 = 5.140 + 2.8\delta + 5\cos(\delta - 0.213) - 5\cos(\delta + 1.513)\ldots \\ + 5\sin(\delta - 1.687) + 20\sin(\delta - 0.209) \tag{35}$$

$$a = -2.241 - 1.3\delta - 0.063\sin(\delta - 1.687) - 0.254\sin(\delta - 0.209)\ldots \\ + 0.846\cos(\delta - 1.687) + 3.386\cos(\delta - 0.209) \tag{36}$$

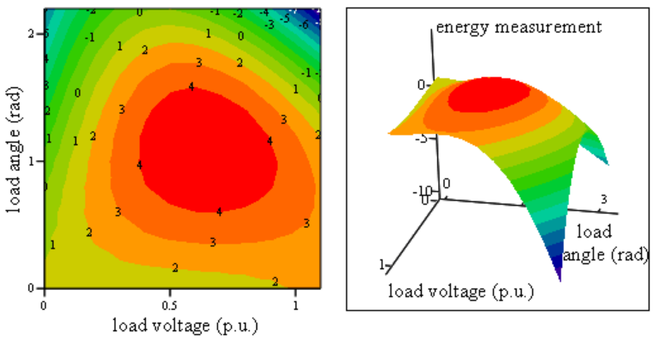

Fig. 6. System's stored energy for δ_m=1.6 (a) Two-dimensional representation (b) Three- dimensional representation.

The system's energy density is in the range of $0.3 \le V \le 1$ and $0.4 \le \delta \le 1.6$, which is seen as in Fig. 6 and Table V. The system's energy density varies between 3 and 4 energy units around these points.

TABLE V
ENERGY MEASUREMENT FOR Δ_M=1.6

δ	Energy Measurement										
0	1,1	1,1	1,2	1,2	1,2	1,3	1,3	1,4	1,6	1,8	2,1
0,2	1,0	1,4	1,7	1,9	2,1	2,2	2,3	2,3	2,4	2,4	2,4
0,4	0,8	1,5	2,1	2,6	2,9	3,1	3,2	3,2	3,2	3,1	2,9
0,6	0,5	1,5	2,4	3,0	3,5	3,8	4,0	4,0	3,9	3,6	3,3
0,8	0,0	1,3	2,5	3,3	4,0	4,4	4,6	4,6	4,4	4,1	3,5
1	-0,6	1,0	2,4	3,4	4,2	4,7	4,9	4,9	4,7	4,2	3,5
1,2	-1,4	0,5	2,1	3,3	4,2	4,7	5,0	4,9	4,6	3,9	3,1
1,4	-2,2	-0,1	1,6	2,9	3,9	4,4	4,6	4,5	4,0	3,3	2,2
1,6	-3,1	-0,9	0,9	2,3	3,3	3,8	3,9	3,7	3,1	2,1	0,7
1,8	-4,1	-1,7	0,1	1,5	2,4	2,8	2,8	2,4	1,6	0,3	-1,3
V	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

For δ_m=1.6 rad. and w=1 p.u., Table V shows numerical values of the energy function for different load angles and different load voltages.

After all of these cases, we observed that the value of the whole energy density decreased. This decrease in energy measurement is an indicator of operating point's movement towards instability region. In case-1, while δ=1.2 rad. and V=0.8 p.u., maximum energy level was 9.1 energy unit. But in case-5, energy level was 4.6 energy unit even the same value of δ and V. It is observed clearly that, any changes in load is going to continue decrease level of energy density to low values, even possible to see negative values.

V. CONCLUSION

Energy function has long been recognized as a useful way of analysing voltage stability. Our study shows that a more realistic energy function -which can clearly demonstrate the critical load angles gained on the energy measurement levels, corresponding to the representations of system works in the different levels and load voltages, and a single-machine infinite-bus power system's stability attitude- can be obtained. Thus, this shows the effect of energy fluctuations in the system on system stability, nearly definitely. Eventually, for the system dependency to load angle and load voltage, optimal range of load angle and load voltage can be defined with the energy fluctuation which is plotted the range of stability shown.

APPENDIX

The load parameter values used in the simulation are [15]:
K_{pw} = 0.4, K_{pv} = 0.3, K_{qw} = -0.03, K_{qv} = -2.8, K_{qv2} = 2.1
T = 8.5, P_0 = 0.6, Q_0 = 1.3, P_1 = 0.0, Q_1 = 0.0

The network and generator parameter values used in the simulation are [15]:
Y_0 = 20, θ_0 = -5, E_0 = 1, C = 12, Y_0' =8, θ_0' = -12
E_0' = 2.5, Y_m = 5, θ_m = -5, E_m = 1, P_m = 1, D = 0.05
M =0.3

All values are in per unit except for angles, which are in degrees.

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