The Truck Mounted Concrete Boom Pump: A Dynamic Numerical Model

G. Cazzulani, H. Giberti, F. Resta, F. Ripamonti

***Abstract*— Concrete pump booms are subjected to vibrations that increase mechanical stress and shorten their lifespan. This paper aims to study the problem by considering the two subsystems, the boom and the concrete pump, that have the greatest effect on the phenomenon. The authors supply numerical and experimental tools that can analyze the problem in depth in all its complexity. First, the systems were investigated independently, to identify their individual aspects. Then a numerical model was created to reproduce the behavior of the whole system and the interaction between boom and pump. The result was a new powerful tool for investigating passive and active solutions for suppressing vibration.**

***Index Terms*— flexible system, concrete pumping, co-simulation, nonlinear dynamics**

I. INTRODUCTION

In recent years construction machinery has undergone considerable development, in terms of increasing dimensions and reduced weight. This trend can be clearly seen in, for example, concrete pump booms. The need for weight reduction with regard for boom longer than 60m, to improve the performance of the systems, makes for structures characterized by high flexibility and low damping. These characteristics, coupled with the dynamic loads due to the large motion of the structure and the flow of the concrete, produce considerable mechanical stress on the material. These structures suffer from fatigue and instability issues, with several negative implications for safety. To reduce the spreading of cracks and increase the boom's working life, it is necessary to reduce the stress by suppressing vibrations of the entire system. Traditional external passive control methods are generally more invasive, since they involve introducing mass on to the structure, and are less effective in a large range of frequencies. On the other hand, active control is an attractive solution, especially considering the rapid development of computer hardware and the consequent reduction of costs. In both cases, a numerical tool that can reproduce the dynamics of the entire system is very useful for identifying the best solution for each specific model and specific application.

Some contributions can be found in the literature. For example, Khulief [1] used a FEM model of a flexible boom to define a control logic for suppressing vibration. Other authors created boom models for tip trajectory synthesis (Wang [2]) or the automation of the pouring process (Zhou [3]) and pump models for enhanced component design (Worthington [4]). Finally the papers on innovative long span machines by Kronenberg [5][6], specifically relating to the Putzmeister AG company, should be mentioned, though they contain no analysis of possible control laws. The aim of present paper is to investigate and reproduce the dynamics of the entire system, which have been little investigated in the scientific literature, creating a numerical and experimental development environment for investigating solutions aimed at increasing the performance of the system in terms of pumping capacity, vibration and safety, highlighting the most critical aspects of the system as a whole. The complexity of the system led first to consider the boom and the pump independently. In order to reproduce the characteristics of the systems and permit an intensive series of experiments, two test rigs were built. The results of these experiments were the starting point for defining the numerical models. For the boom, a non-linear flexible multibody (MB) model was created, while the pumping group numerical model was used to solve the oil and concrete continuity equations and the equations of motion of the hydraulic pistons. Then, in the second stage, the forces coming from the pump numerical model were used as the input for the boom model, that, on the other side, returns the system configuration.

II. THE TRUCK MOUNTED PUMP

Concrete boom pumps are complex dynamic systems with a variable number of segments linked together by kinematic joints and moved by hydraulic actuators. In this way the end-activator, i.e. the boom tip housing the device for placing the concrete, may be very far from the pump or in very high place (Figure 1). In general these systems have high flexibility and, as a result, very low natural frequencies. Therefore, because of the very low damping associated with these frequencies, there will be high amplitudes of vibration and severe mechanical stresses on the structure. Moreover, it is important to underline that the possibility of changing the configuration of the system

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spreads the natural frequencies over a quite wide range. This range can be only estimated in the design stage and doesn't exclude resonance forcing during the boom's lifespan. In fact, in addition to the forcing associated with the boom's motion that excites the system's vibration modes, during normal operation the structure is affected by the pumping of the concrete.

Fig. 1. The system: typical pumping configurations

The pumping unit is bolted to the chassis of the truck (Figure 2) and, through its connection to the truck's engine, works like an alternating pump, consisting of two actuators and four pressure chambers.

Fig. 2. The pumping group unit bolted on the truck

The movement of the pistons causes the pressure in the chambers to vary over time and generates forces on the boom. In particular, the pumping forces are due to the concrete flowing through the pipe that is rigidly connected to the boom links. The material mainly transfers the forces due to the friction and the pressure difference between the inlet and outlet surfaces. The resultant, balanced by the reaction of each single support, is defined by a longitudinal component on the generic i-th link. Figure 3 shows the acceleration of the tip of the final link. Figure 3-a shows the decay of the acceleration after movement ends; the harmonic component at the first natural frequency (0.47 Hz for the extended configuration, defined as “long span” in the following) is clearly visible. Figure 3-b shows the steady state response (in terms of acceleration) due to the pumping of the concrete; we can see that the fundamental forcing component (0.45 Hz) is very close to the system's first natural frequency in the “long span” configuration. For all these reasons, and in particular because of the complexity involved in studying the system whole, this paper will consider, initially, the boom and the pump independently.

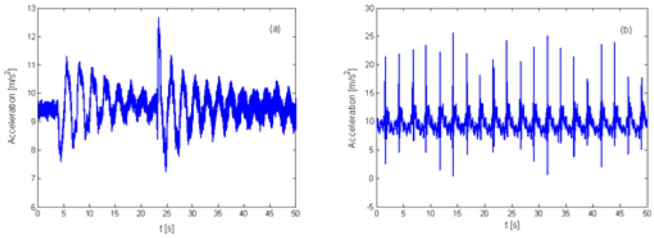

Fig. 3. The system forcing: boom tip acceleration due to "movement" (left) and to "pumping" in long span configuration (right)

This will allow the most important parameters of both subsystems to be highlighted and the best solutions for improving performance and safety identified. In particular, by considering the pump and the boom respectively as cause and effect, it is possible to reduce oscillations both by improving the pump design and/or by applying a control logic for suppressing vibration. Below two experimental test rigs are described: a full scale test rig for the boom and for the pump system.

III. THE BOOM

The full-scale boom test rig, reproducing a commercial one, was created to reproduce the dynamic behaviour of truck-mounted booms. It is a 40 m boom composed by 5 links and directly connected to the ground (Figure 4).

Fig. 4. The boom test rig

The movement is performed by hydraulic actuators, driven by a human operator. The test rig was equipped with several sensors. Each link has been instrumented with an inclinometer and a pressure sensor for each actuator chamber. The inclinometer measurements can be used for calculating link angles and hence the boom configuration. The pressure sensors measure the total force applied by the actuators to the boom. Finally two capacitive accelerometers, positioned on the tip of the 3rd and 5th link, were used to measure the vibrations in the boom. Several tests were carried out on the rig to identify its dynamic behavior. As the system is non-linear (both natural frequencies and damping depend on boom configuration) these tests were repeated for different configurations. The experimental parameters identifications was performed using different tests:

- actuator static force measurement, via pressure measurements, which was used to verify the model of the inertial contributions of the boom;
- natural frequencies identification, performed using

sweep sine tests;

- estimation of the damping factor associated to each mode, carried out by exciting the structure on its natural frequencies and considering the decay time histories.

The first two (static actuator forces and natural frequencies) were used to verify the accuracy of the boom numerical model. Modal damping was used to set the damping parameters of the model.

A. The boom numerical model

The numerical model of the boom was developed to simulate both its large motion and the vibrations due to its flexible nature. As the boom is perfectly symmetrical and its motion is planar, the system can be considered two-dimensional. This allows the model to be considerably simplified (lateral and torsion vibrations are neglected) without compromising its accuracy. Other assumptions are:

- small deformations: the relationship between strain and deformation can be considered linear; in addition axial and bending deformations can be considered as decoupled;
- small link rotations: this assumption means the contribution of centrifugal and Coriolis terms in the motion equation can be neglected.

Each link movement is described, using the so-called “floating frame of reference formulation" [7][8][9], by two sets of coordinates, describing respectively the large motion and the vibrations. The first set describes the rigid motion of the j^{th} link, in terms of its absolute displacement and rotation, considering the motion of the reference system of this link $\left(y^j O^j x^j\right)$ with respect to the global reference system $\left(yOx\right)$. The translation contribution can be written as a function of the degrees of freedom of the previous links, while the rotation contribution is described by the independent variable θ^j.

The second set of coordinates models the link vibration with the finite element method (FEM), using beam elements. The deformations of each element are described considering the displacement and rotation of the nodal reference system $\left(y^{jk} O^{jk} x^{jk}\right)$ with respect to the link reference system $\left(y^j O^j x^j\right)$. Thanks to the assumption of planar motion, each FEM node is described by a vector, $\underline{\mathbf{d}}^{jk}$, with three independent coordinates $\left(x, y, \vartheta\right)$. Defining the vector containing all the link rotations as $\underline{\boldsymbol{\theta}}$ and as $\underline{\mathbf{d}}$ the vector containing all the nodal coordinates, the total independent variable vector ($\underline{\mathbf{z}}$) can be written as:

$$\underline{\mathbf{z}} = \begin{Bmatrix} \underline{\boldsymbol{\theta}} \\ \underline{\mathbf{d}} \end{Bmatrix} \tag{1}$$

Using the Lagrange formulation, we obtain the non-linear equation of the boom

$$\left[\mathbf{M}\left(\underline{\mathbf{z}}\right)\right]\underline{\ddot{\mathbf{z}}} = \underline{\mathbf{f}}\left(\underline{\mathbf{z}}, \underline{\dot{\mathbf{z}}}\right) + \left[\boldsymbol{\Lambda}_{\text{act}}\left(\underline{\mathbf{z}}\right)\right]^T \underline{\mathbf{F}}_{\text{act}}\left(t\right) + \underline{\mathbf{F}}\left(t\right) \tag{2}$$

where $\left[\mathbf{M}\left(\underline{\mathbf{z}}\right)\right]$ represents the inertial contribution of the structure (depending on boom position), $\underline{\mathbf{f}}\left(\underline{\mathbf{z}}, \underline{\dot{\mathbf{z}}}\right)$ contains the elastic and damping terms, $\left[\boldsymbol{\Lambda}_{act}\left(\underline{\mathbf{z}}\right)\right]$ represents the kinematic relationship between the movement of the actuators and the independent coordinates and multiplies the actuators forces $\underline{\mathbf{F}}_{act}\left(t\right)$, and $\underline{\mathbf{F}}\left(t\right)$ contains all the external forces acting on the boom.

To study boom vibration, (2) can be linearized around a configuration "$\underline{\mathbf{z}} = \underline{\mathbf{z}}_j$". Considering small deformations, the linearization configuration is defined as

$$\underline{\mathbf{z}}_j = \begin{Bmatrix} \underline{\boldsymbol{\theta}}_j \\ \underline{\mathbf{0}} \end{Bmatrix} \tag{3}$$

and the linearized equation becomes

$$\begin{aligned} &\left[\mathbf{M}_j\right]\cdot\delta\underline{\ddot{\mathbf{z}}}_j + \left[\mathbf{R}_j\right]\cdot\delta\underline{\dot{\mathbf{z}}}_j + \left[\mathbf{K}_j\right]\cdot\delta\underline{\mathbf{z}}_j = \\ &= \underline{\mathbf{f}}_g\left(\underline{\mathbf{z}}_j\right) + \left[\boldsymbol{\Lambda}_j\right]^T \underline{\mathbf{F}}_{\text{act}} + \underline{\mathbf{F}}\left(t\right) \end{aligned} \tag{4}$$

where $\delta\underline{\mathbf{z}}_j$ is defined as $\delta\underline{\mathbf{z}}_j = \underline{\mathbf{z}} - \underline{\mathbf{z}}_j$ and describes the system’s vibration.

The damping matrix $\left[\mathbf{R}_j\right]$ was assumed to be the sum of two contributions:

- structural distributed damping $\left[\mathbf{R}_{j,\text{str}}\right]$, expressed by

$$\left[\mathbf{R}_{j,\text{str}}\right] = \alpha\left[\mathbf{M}_j\right] + \beta\left[\mathbf{K}_j\right] \tag{5}$$

as a linear combination of the inertial and elastic terms; the two coefficients α and β have been estimated experimentally;

- cconcentrated damping $\left[\mathbf{R}_{j,\text{conc}}\right]$ described by three concentrated dampers on the joints connecting the links.

The calculation of the damping parameters (α, β and the concentrated terms) was done minimizing the difference between estimated damping and model damping in different configurations. This numerical model can simulate the typical operating condition of concrete pump booms.

IV. THE CONCRETE PUMPING UNIT

This section describes the pumping unit layout. The full scale test rig was built to reproduce the real working pump and it was used to identify its parameters, especially the exchanged forces between pumping group and boom. These forces come from the internal pumping group pressures, from the interaction between concrete and pipeline and from the geometrical position of the boom links. The internal pressures depend on the oil flow, the chambers volume and fluid compressibility. The interaction between concrete and pipeline determines the friction force depending on the velocity of the concrete inside the pipeline. Finally the links position determines the concrete static pressure contribution. To better understand the functioning of the machine a

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detailed description is presented in Figure 5. The pumping unit is an alternative volumetric pump with two pistons (E). The pistons are driven by two hydraulic actuators (B and C). It is possible to divide the system into two subsystems: the "oil side" and "concrete side".

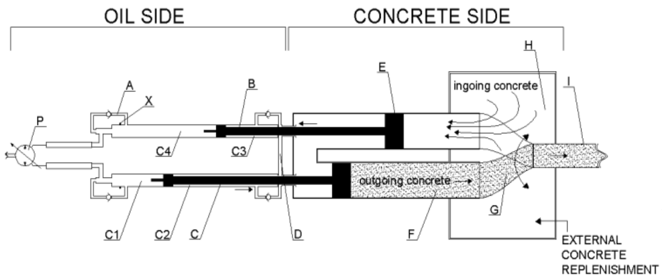

Fig. 5. A detailed diagram of the pumping group

The oil side represents the power unit while the concrete side is the driven unit and it is in direct contact with the concrete. Every pumping cycle is composed of two phases: "ejection" and "replenishment". In the first phase one of the two actuators (C for example) pushes out the concrete (see "outgoing concrete" in Figure 5) while the second actuator (B), driven by the first one (through an hydraulic circuit called slave (D)), refills its cylinder with the concrete (see "ingoing concrete" in Figure 5) moving in the opposite direction almost simultaneously. Figure 5 also shows a series of pipes called ducts (A), with retaining valves, which form a closed circuit on the oil side cylinder. They function as brakes, as oil conduits (because of oil leakage) and as a piston interlock during the operating cycles. These components are very important for the trade-off of pressure and flow between the adjacent chambers C1 and C2 (or C4 and C3). About the concrete side there is the S valve (G) and the concrete tank. The first one directs the flow of concrete connecting the end of the cylinder with the beginning of the pipeline. Its position is triggered by a signal from the proximity sensors (X) and changes every cycle. The second one is useful as a container of concrete. Beyond the physical components, the test rig is equipped with sensors for measuring pressure in each chamber and displacement of the pistons. To measure the input flow of oil and the output flow of concrete, flow sensors were used. All these instruments were necessary and used both for direct and derived measurements. In fact some measurements, for example the velocity and acceleration of the pistons, were extracted from the measurements of displacement and numerically calculated. In addition, to simulate the static load of the concrete due to the configuration of the boom, a series of ball valves are installed.

A. The pump numerical model

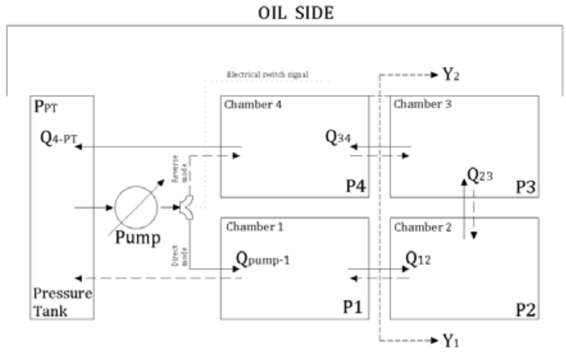

Fig. 6. Diagram of the pumping unit oil side

The pumping unit subsystem [10] was modelled with differential equations describing both the dynamics of oil chamber pressures and the motion of the pistons. The first part of the system (oil side) was modelled as a series of chambers in pressure coupled by the oil flows Q_{ij}. The model in Figure 6 is characterized by the input variables Q_{pump-i} and P_T, by the chamber state variables P_i and y_i and by the measurements Q_{ij}. All these variables define the input vector $\underline{\mathbf{u}}_{\text{oil}}$, the state vector $\underline{\mathbf{x}}$ and the measurement vector $\underline{\mathbf{y}}_{\text{oil}}$ for the oil side. About the pressures in the oil chambers, the following general equation is used

$$\dot{P}_i = \frac{\beta}{V_i} \cdot \left[\left(Q_{ij} - Q_{jk} \right) - \frac{dV_i}{dt} \right] \tag{6}$$

where β is the oil compressible coefficient and the is the chamber volume, a function of y_i. The oil flows can be evaluated using the general algebraic efflux equation

$$Q_i = C_v \cdot \sqrt{\frac{2\left|P_i - P_j\right|}{\rho}} \cdot \operatorname{sign}\left(P_i - P_j\right) \tag{7}$$

where C_v is the efflux coefficient experimentally identified. In addition to these equations, the state of the system is completed by the equations of motion of the two pistons.

These describe the motion of the system and couple the oil and concrete sides.

They are summarized as:

$$\begin{cases} \ddot{y}_1 = \dfrac{P_1 A_1 - P_2 A_2 - P_{cls6} A_{cls6} - f\left(\dot{y}_1\right) + mg \cdot \sin(\alpha)}{m} \\ \dot{y}_1 = \dot{y}_1 \\ \ddot{y}_2 = \dfrac{P_4 A_4 - P_3 A_3 - P_{cls7} A_{cls7} - f\left(\dot{y}_2\right) + mg \cdot \sin(\alpha)}{m} \\ \dot{y}_2 = \dot{y}_2 \end{cases} \tag{8}$$

The balance of forces is shown in Figure 7. It takes into account the pressure of the oil (P_i) and concrete chambers (P_{clsi}), the forces of gravity, the inertial and friction forces.

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The friction force $f(\dot{y}_1)$ is calculated with the LuGre friction model [12] for Translational Friction.

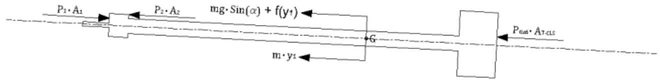

Fig. 7. Balance of the piston forces in the pumping unit

The concrete side (called the CLS side) is instead governed by algebraic equations based on an energetic approach. The input vector $\underline{\mathbf{u}}_{\mathrm{cls}}$ contains the variables P_{clsi} (pressure of concrete chambers) and h (maximum height of the boom). The pressures in the CLS chambers and the output flow of concrete $Q_{clsi} = \dot{y}_i \cdot A_i \cdot \eta$ (η is the coefficient of replenishment) define the CLS measurement vector $\underline{\mathbf{y}}_{\mathrm{cls}}$. The pressures are alternately equal to P_{atm} and P_{clsi}. The complete system is described by

$$\begin{cases} \dot{\underline{\mathbf{x}}} = & [\mathbf{A}]\cdot\underline{\mathbf{x}} \quad +[\mathbf{B}]\cdot\underline{\mathbf{u}} \\ \underline{\mathbf{y}} = & [\mathbf{C}]\cdot\underline{\mathbf{x}} \end{cases} \tag{9}$$

where $\underline{\mathbf{x}}$, $\underline{\mathbf{u}}$ and $\underline{\mathbf{y}}$ are obtained by combining the oil side (8) and CLS side variables.

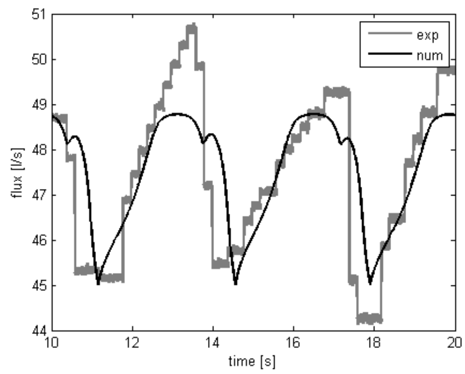

Fig. 8. Comparison between the numerical and experimental outputs; the concrete flow

On the pumping unit system, a sensitivity analysis, involving the values of oil flow and globe valve area (simulating different h values) was carried out. The tests carried out yielded important information on the forces transferred from the pumping group to the boom; these were mainly due to CLS pressure in the pipeline during the work cycle. As validation of the numerical model, Figure 8 and Figure 9 show a comparison with the experimental data, in terms of CLS flow and pressure of CLS. In both figures the numerical result shows a greater regularity in comparison with the experimental one. The comparison is very good; small percentage errors, lower than 5%, can be noticed in the CLS flow time history. Instead, in Figure 9, the irregularity of the pressure in experimental data is mainly due to a wrong functioning of the real machine. Whatever, this is a good starting point for the co-simulation between the pumping unit and boom environments.

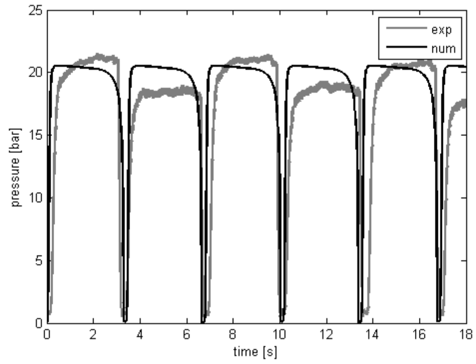

Fig. 9. Comparison between the numerical and experimental outputs; the back pressure

V. THE BOOM/PUMP INTERACTION MODEL

The final step of the work was a co-simulation using the two models. The aim was to simulate the behavior of a full-scale commercial boom in working conditions. Figure 10 shows the interaction model: the external inputs are the strokes of the boom actuators, which allow the boom configuration to be calculated and, as a consequence, the needed pump head.

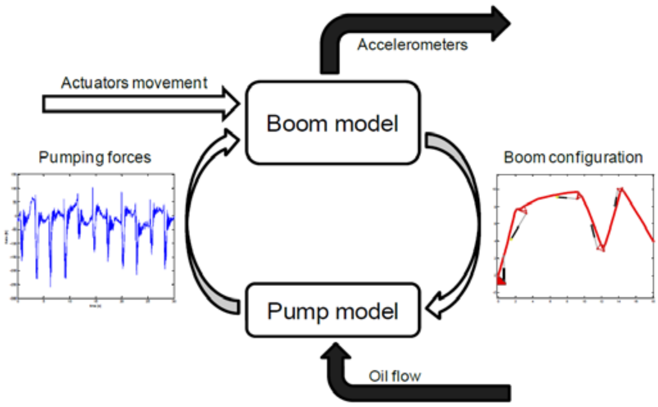

Fig. 10. Scheme of the data flow between the two numerical models

On the other side, the pump model will give the flow rate of the concrete and the pressure along the pipe, which can be used to calculate the pumping forces on the boom.

In fact, as a first approximation, the equilibrium of a generic concrete volume is described by:

$$\int_V \rho \frac{\partial v}{\partial t} dV = \int_{A_{in}} P_{in} dA_{in} - \int_{A_{out}} P_{out} dA_{out} - F_{cls} \tag{10}$$

where F_{cls} is the friction force that the concrete transfers to the boom. The concrete pumping force is transmitted to the boom through several links along the pipe length.

Assuming constant pipe section and uncompressible fluid, the friction force in a finite volume can be calculated by integrating (10) as:

$$F_{cls} = \rho V \frac{\partial v}{\partial t} + (P_{in} - P_{out}) A \tag{11}$$

This force acts on the boom model during motion integration as an external disturbance force. Figure 11 shows the result of a co-simulation using the boom/pump

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interaction model. For a comparison with experimental data (Figure 1), the simulation has been performed in the same conditions ("long span" configuration with a pumping frequency of 0.40 Hz). The comparison of boom tip acceleration shows a good agreement between numerical and experimental data. For this reason the complete boom/pump model can be considered as a tool able to reproduce the behavior of the entire system.

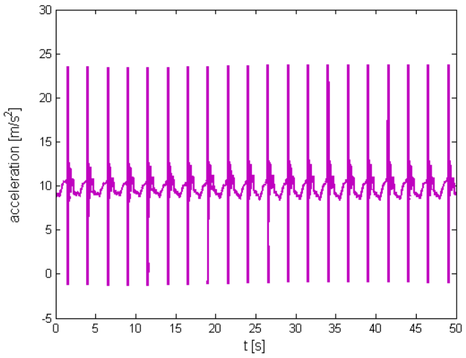

Fig. 11. Co-simulation results: acceleration of the boom tip in long span config. with pumping frequency of 0.40 Hz

VI. CONCLUSIONS

The article describes the design of a new experimental test rig created to study vibration control for a concrete pump truck boom. The experimental setup is split into two independent test rigs: the first a model of the boom, the second a concrete pump fitted with instrumentation and connected to purpose-built hydraulic test circuit. This article also presents mathematical models for describing the behavior of the physical systems. The mathematical models are validated and implemented in a software simulating system behavior under varying operating conditions. This experimental setup and mathematical model are unique. It is a development environment that makes it possible to improve the performance of the concrete pump truck boom in terms of pumping capacity, maximum pumping distance attainable and safety. Summarizing, the main goals achieved by this development environment are:

1. The possibility of developing and designing new boom position control strategies and new controls for pump flow. This result can be obtained simulating the boom and pump behavior separately, or together using the co-simulation tool.
2. The possibility of checking the control strategies developed on physical models with full instrumentation (the test rig). The test rig accurately reproduces the behavior of a concrete pump truck boom but in a completely safe and protected environment.
3. The possibility of verifying the robustness and dependability of the control strategies developed. In fact with this test rig it is easy to generate operating conditions difficult to reproduce in real systems and in real environments.
4. The ease of transferring the control strategies developed to real equipment.

In conclusion, the test rig created plays an important role both in the design of a control strategy and in the verification and analysis phase of the concrete pump boom's behavior. In addition, the test rig allows us to observe all the physical phenomena involved in the working of the concrete pump boom.

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