# Power Flows and Torque Analyses of an Independently Controllable Transmission with a Parallel Type

Guan-Shyong Hwang*, Chung-Chi Lin, Der-Min Tsay, Jao-Hwa Kuang, and Tzuen-Lih Chern

*Abstract*—**An independently controllable transmission (ICT) with a parallel type is a novel, infinitely, and continuously variable transmission mechanism. The parallel type ICT can produce a required output angular velocity, which is independently controlled by a controller and not affected by the angular velocity of the input power shaft. This study derives the equations of power flows and torques introduced in the parallel type ICT. A prototype of the parallel type ICT is implemented for demonstration, and correctness of the derived equations is investigated by the experiment of static torque for the prototype.**

*Index Terms*—**independently controllable transmission (ICT) with parallel type, planetary gear train, transmission-connecting member, free-transmission end**

## I. INTRODUCTION

In the automation engineering, many applications have widely employed infinitely and continuously variable transmission (IVT, CVT), because CVTs are capable of providing any ratio between output and input speeds. By CVTs' continuous ranges of transmission ratios and independent transmission of selected torques, the optimal conditions of power transmissions can be achieved. Therefore, to design an efficient CVT is frequently an important and interesting topic for many researches.

Various researches have focused on the design of CVT. For example, Gott classified continuously variable transmissions to be four different categories: traction, belt, ratcheting, and hydraulic systems [1]. Mucino et al., presented the system of continuously variable power split transmission (CVPST), which consists of a variable pulley set coupling a planetary gear train [2]. Parrish proposed CVT mechanisms comprising first and second planetary gear sets [3]. Benitez et al., described an IVT including one-way clutches and two planetary gear trains [4]. Hsu and Huang presented a systematic methodology to effectively simplify the design of automatic transmissions with parallel-connected planetary gear trains [5]. Bottiglione and Mantriota proposed a MG-IVT, which is composed of the coupling of a CVT, a planetary gear train and two ordinary transmissions with a constant transmission ratio [6]. Lahr and Hong presented the cam-based IVT of ratcheting drive type [7].

A novel mechanism with an independently controllable power transmission, referred to as an independently controllable transmission (ICT) with a parallel type, had been proposed in the previous studies [8-9]. And the kinematic characteristics of the parallel type ICT also had been demonstrated. The parallel type ICT is an infinitely and continuously variable transmission mechanism, and can produce a required output angular velocity, which is independently controlled by a controller and not affected by the angular velocity of the input power shaft. The parallel type ICT, for example, could be utilized in the automatic transmission systems and the variable speed wind turbines. This study derives the equations of power flows and torques introduced in the parallel type ICT. A prototype of the parallel type ICT is implemented for demonstration and the correctness of the derived equations is investigated by the experiment of static torque for the prototype.

## II. PARALLEL TYPE ICT

A parallel type ICT is a transmission mechanism possessing four rotational shafts used to connect to the input power source, the output power end, the controller, and the free-transmission end, respectively. As depicted in Fig. 1, a parallel type ICT is composed of two planetary gear trains, respectively indicated by *A* and *B*, and two transmission-connecting members, respectively indicated by *D* and *E*. Each planetary gear train has three rotational shafts indicated by *OP*, *AD*, *AE* and *CR*, *BD*, *BE*, respectively. In planetary gear train *A*, shaft *OP* connects to the output power end and shafts *AD*, *AE* connect to transmission-connecting members *D* and *E*, respectively. In planetary gear train *B*, shaft *CR* connects to the controller and shafts *BD*, *BE* also connect to the transmission-connecting members *D* and *E*, respectively. In transmission-connecting member *D*, shaft

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*SD* could connect to the source of input power, whereas transmission-connecting member *E* could connect to the free-transmission end by shaft *SE*.

### A. Basic Requirements of Kinematics

To achieve the function and performance of the parallel type ICT, the former study had established the basic requirements of kinematics as follows [8]:

$$n_{BD} = \alpha\, n_{AD} \tag{1}$$

$$n_{CR} = \beta\, n_{OP} \tag{2}$$

$$n_{BE} = n_{AE} \tag{3}$$

where $n$ denotes the angular velocity of the rotational shaft indicated by its subscript, $\alpha$ and $\beta$ are the constant multiples. The kinematic parameters $\alpha$ and $\beta$ can be used to determine the speed ratios between the rotational shafts, and then the configuration of the ICT mechanism.

### B. Positive-Ratio Planetary Gear Train

A positive-ratio planetary gear train, used in this study and shown in Fig. 2, includes a first sun gear *ps*1 mounted on the rotational shaft *pss*1, a second sun gear *ps*2 mounted on the rotational shaft *pss*2, at least one compound planet gear set including gears *pp*1, *pp*2, as well as meshing with the first and second sun gears, and a planet gear carrier *pa*. A positive-ratio planetary gear train means that the shafts of the first and second sun gears, when the carrier is fixed, have the same direction of rotation. Therefore, its basic speed-ratio, which is defined as the ratio of the relative velocities of the two sun gears' shafts respectively with respect to the carrier, is consequently positive and cannot be equal to 1 [10]. The basic speed-ratio of a positive-ratio planetary gear train, denoted by $i_0$, can be also mathematically expressed as

$$i_0 = \frac{n_{pss1} - n_{pa}}{n_{pss2} - n_{pa}} = \frac{N_{pp1} \times N_{ps2}}{N_{ps1} \times N_{pp2}} \tag{4}$$

where $N$ is the teeth number of the gear indicated by its subscript.

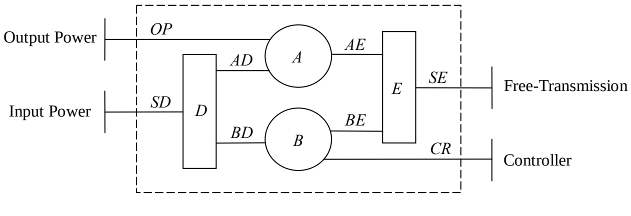

Fig. 1. Conceptual structure of the ICT with a parallel type

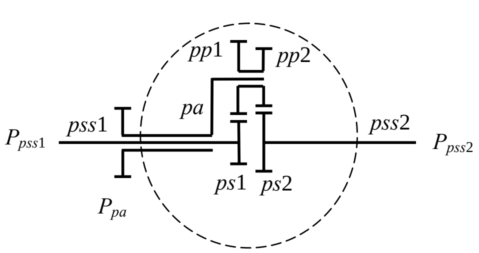

Fig. 2. Positive-ratio planetary gear train

### C. Arrangement of the Parallel Type ICT

A practical arrangement of the parallel type ICT is schematically shown in Fig. 3. In the positive-ratio planetary gear trains *A* and *B*, the shafts of the first sun gears *pss*1*A* and *pss*1*B*, similar to the shafts *OP* and *CR* shown in Fig. 1, are connected to the output power end and the controller, respectively. The function of the rotational shafts *paA*, *pss*2*A*, *paB* and *pss*2*B* are also similar to those of the shafts *AD*, *AE*, *BD* and *BE*, respectively. Therefore, by referring to Eqs. (1) and (3), the following equations can be also obtained

$$\frac{n_{paB}}{n_{paA}} = \frac{N_{cmg2D}}{N_{cmg3D}} = \alpha \tag{5}$$

$$n_{pss2B} = n_{pss2A} \text{ or } N_{cmg3E} = N_{cmg2E} \tag{6}$$

In the transmission-connecting members *D* and *E*, the rotational shafts *cmsD* and *cmsE*, similar to the shafts *SD* and *SE* shown in Fig. 1, are connected to the input power source and the free-transmission end, respectively. According to Eq. (4), the basic speed-ratios of the planetary gear trains *A* and *B*, denoted by $i_{0A}$ and $i_{0B}$, can be rewritten as follows:

$$i_{0A} = \frac{n_{pss1A} - n_{paA}}{n_{pss2A} - n_{paA}} = \frac{N_{pp1A} \times N_{ps2A}}{N_{ps1A} \times N_{pp2A}} \tag{7}$$

$$i_{0B} = \frac{n_{pss1B} - n_{paB}}{n_{pss2B} - n_{paB}} = \frac{N_{pp1B} \times N_{ps2B}}{N_{ps1B} \times N_{pp2B}} \tag{8}$$

### D. Design Formulas of Parallel Type ICTs

According to the former study [8], the design formulas of the parallel type ICT had been derived as follows:

$$\begin{cases} i_{0A} = \dfrac{\alpha-\beta}{\beta(\alpha-1)},\ i_{0B} = \dfrac{\alpha-\beta}{\alpha-1} & \text{if } \alpha \neq \beta, \alpha \neq 1 \text{ and } \beta \neq 1 \\ i_{0A} = i_{0B} & \text{if } \alpha = \beta = 1 \end{cases} \tag{9}$$

## III. POWER FLOWS AND TORQUE ANALYSES

In this study, an input shaft is defined as introducing a positive power into the ICT and consequently the torque and the speed have the same sense of rotation and carry the same sign. Conversely, an output shaft introduces a negative power while the torque and the speed carry opposite signs. The power introduced by the shaft of the ICT can be

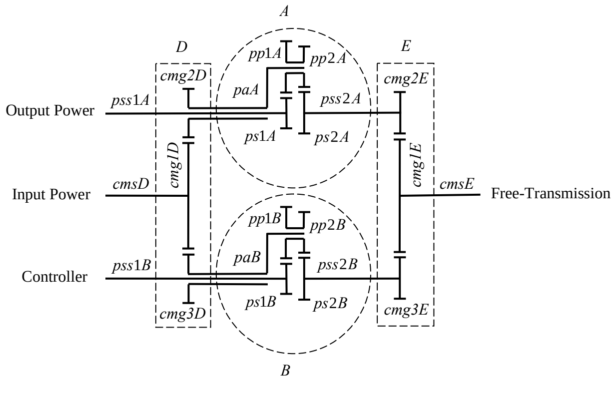

Fig. 3. Arrangement of ICT with a parallel type

mathematically expressed as

$$P_X = T_X n_X \tag{10}$$

where $P$ and $T$ denote the power and torque introduced by the shaft indicated by its subscript, respectively.

### A. Power Flows and Torque Analyses of a Planetary Gear Train

While neglecting friction losses, the sum of power introduced by a planetary gear train shown in Fig. 2 will be zero according to the conservation of energy, i.e.

$$\sum P = P_{pss1} + P_{pss2} + P_{pa} = 0 \tag{11}$$

In a stationary operating condition, a planetary gear train will also yield the equilibrium condition that the sum of all external torques acting on the shafts is equal to zero, i.e. [10-11]

$$\sum T = T_{pss1} + T_{pss2} + T_{pa} = 0 \tag{12}$$

By referring to Eqs. (4), (10)-(12), the following results can be also obtained

$$T_{pss2} = -i_0 T_{pss1} \tag{13}$$

$$T_{pa} = (i_0 - 1) T_{pss1} \tag{14}$$

### B. Power Flows and Torque Analyses of a Parallel Type ICT

When considering the parallel type ICT shown in Fig. 3, the power introduced by the controller shaft can be mathematically expressed as

$$\begin{aligned} P_{controller} &= T_{controller} n_{controller} = T_{pss1B} n_{pss1B} \\ &= \beta T_{pss1B} n_{pss1A} \end{aligned} \tag{15}$$

where $n_{pss1B} = \beta n_{pss1A}$, according to Eq. (2) and the similar function between the shafts $pss1A$, $pss1B$ and the shaft $OP$, $CR$, respectively.

The input power is transmitted into the planetary gear trains $A$ and $B$ by the shafts $paA$ and $paB$, therefore it can be expressed as

$$\begin{aligned} P_{input} &= T_{input} n_{input} = T_{cmsD} n_{cmsD} \\ &= T_{paA} n_{paA} + T_{paB} n_{paB} \end{aligned} \tag{16}$$

Referring to Eqs. (9) and (14) yields

$$T_{paA} = (i_{0A} - 1) T_{pss1A} = \frac{\alpha(1-\beta)}{\beta(\alpha-1)} T_{pss1A} \tag{17}$$

$$T_{paB} = (i_{0B} - 1) T_{pss1B} = \frac{1-\beta}{\alpha-1} T_{pss1B} \tag{18}$$

By substituting the results of Eqs. (5), (17), and (18) into Eq. (16), the input power can be rewritten as

$$P_{input} = \frac{\alpha(1-\beta)}{\beta(\alpha-1)} T_{pss1A} n_{paA} + \frac{\alpha(1-\beta)}{\alpha-1} T_{pss1B} n_{paA} \tag{19}$$

The torque introduced by the shaft $pss1A$ can be derived by rearranging Eq. (19), i.e.

$$T_{pss1A} = \frac{\beta(\alpha-1)}{\alpha(1-\beta)} \cdot \frac{P_{input}}{n_{paA}} - \beta T_{pss1B} \tag{20}$$

By substituting and referring to the results shown in Eqs. (15) and (20), the output power introduced by the shaft $pss1A$ can be obtained as

$$\begin{aligned} P_{output} &= T_{output} n_{output} = T_{pss1A} n_{pss1A} \\ &= \frac{\beta(\alpha-1)}{\alpha(1-\beta)} \cdot \frac{n_{pss1A}}{n_{paA}} P_{input} - P_{controller} \end{aligned} \tag{21}$$

From Fig. 3, the power introduced by the free-transmission end is

$$\begin{aligned} P_{free-transmission} &= T_{free-transmission} n_{free-transmission} = T_{cmsE} n_{cmsE} \\ &= T_{pss2A} n_{pss2A} + T_{pss2B} n_{pss2B} \end{aligned} \tag{22}$$

Referring to Eqs. (7), (9), and (13) also yields

$$\begin{aligned} n_{pss2A} &= \frac{1}{i_{0A}} \cdot n_{pss1A} + \frac{i_{0A}-1}{i_{0A}} \cdot n_{paA} \\ &= \frac{\beta(\alpha-1)}{\alpha-\beta} \cdot n_{pss1A} + \frac{\alpha(1-\beta)}{\alpha-\beta} \cdot n_{psA} \end{aligned} \tag{23}$$

$$T_{pss2A} = -i_{0A} T_{pss1A} = \frac{\beta-\alpha}{\beta(\alpha-1)} T_{pss1A} \tag{24}$$

$$T_{pss2B} = -i_{0B} T_{pss1B} = \frac{\beta-\alpha}{\alpha-1} T_{pss1B} \tag{25}$$

By substituting the results shown in Eqs. (6), (20), and (23)-(25), the power introduced by the free-transmission end can be rewritten as

$$P_{free-transmission} = \left( \frac{\beta(1-\alpha)}{\alpha(1-\beta)} \cdot \frac{n_{pss1A}}{n_{paA}} - 1 \right) P_{input} \tag{26}$$

The output torque shown in Eq. (20) can be further rewritten as

$$T_{output} = T_{pss1A} = \frac{1}{1-i_{0A}} \cdot \frac{N_{cmg2D}}{N_{cmg1D}} T_{input} - \beta T_{controller} \tag{27}$$

where $\frac{N_{cmg2D}}{N_{cmg1D}} = -\frac{n_{cmsD}}{n_{paA}}$.

From Eqs. (22) and (26), the torque introduced by the free-transmission shaft can be also rewritten as

$$\begin{aligned} T_{free-transmission} &= T_{cmsE} \\ &= \frac{i_{0A}}{1-i_{0A}} \cdot \frac{N_{cmg2D}}{N_{cmg1D}} \cdot \frac{N_{cmg1E}}{N_{cmg2E}} T_{input} \end{aligned} \tag{28}$$

where $\frac{N_{cmg1E}}{N_{cmg2E}} = -\frac{n_{pss2A}}{n_{cmsE}}$.

## IV. DEMONSTRATION OF ICT PROTOTYPE

To investigate and demonstrate the correctness of the torque analyses shown in the previous section, this study proposes a prototype of the parallel type ICT and implements a test-bed of the prototype for experiment.

### A. Dimensional Parameters of ICT Prototype

Figure 4 schematically shows the design of the prototype of the parallel type ICT. In the ICT prototype, the constant multiples shown in Eqs. (1) and (2) are chosen to be $\alpha = 2$ and $\beta = 1.5$. According to Eqs. (7) and (8), the basic speed-ratios of the planetary gear trains $A$ and $B$ will be $i_{0A} = 1/3$ and $i_{0B} = 0.5$, respectively. By referring to Eqs. (5)-(8), the teeth numbers of the gears used in the ICT prototype are chosen as listed in Table 1.

By substituting the teeth number, the basic speed-ratio and the constant multiple, Eqs. (27) and (28) can be obtained as follows:

$$T_{output} = 2.4T_{input} - 1.5T_{controller} \qquad (29)$$

$$T_{free-transmission} = \frac{2}{3}T_{input} \qquad (30)$$

### B. *Experimental Results of ICT Prototype and Discussion*

Figure 5 illustrates the configuration of experimental test-bed of the ICT prototype, and Fig. 6 shows the test-bed in the experiment of static torque. Table 2 shows the obtained experimental data, and Fig. 7 is a plot about the calculating values and the least square approximation of $(T_{output} + 1.5T_{controller})/T_{input}$ and $T_{free-transmission}/T_{input}$. The theoretical values of $(T_{output} + 1.5T_{controller})/T_{input}$ and $T_{free-transmission}/T_{input}$ are 2.4 and 2/3 according to Eqs. (29) and (30). The results of the least square approximation, calculated by Microsoft Excel, about the experimental values are 2.38 and 0.64. By comparing the approximate equality between the theoretical and experimental values of $(T_{output} + 1.5T_{controller})/T_{input}$ and $T_{free-transmission}/T_{input}$, the match of these values can be concluded. Therefore, the correctness of the derived equations has been verified.

TABLE 1. TEETH NUMBERS OF GEARS USED IN ICT PROTOTYPE

| Gear | *cmg1D* | *cmg2D* | *cmg3D* | *cmg1E* | *cmg2E* | *cmg3E* | *ps1A* |
|---|---|---|---|---|---|---|---|
| Teeth number | 50 | 80 | 40 | 50 | 60 | 60 | 45 |
| Gear | *ps2A* | *pp1A* | *pp2A* | *ps1B* | *ps2B* | *pp1B* | *pp2B* |
| Teeth number | 30 | 15 | 30 | 40 | 30 | 20 | 30 |

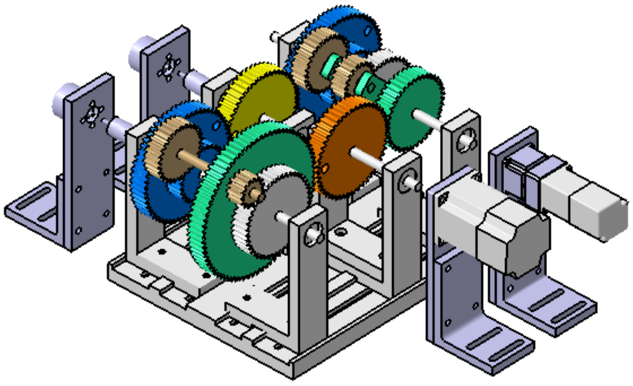

Fig. 4. Prototype of the parallel type ICT

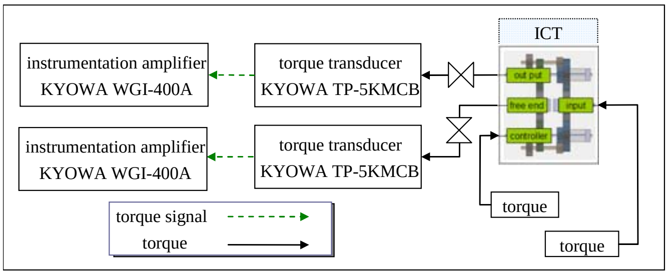

Fig. 5. Configuration of test-bed

TABLE 2. TORQUE VALUES (Nm)

| $T_{input}$ | $T_{controllert}$ | $T_{output}$ | $T_{free-transmission}$ | A | B |
|---|---|---|---|---|---|
| 2.04 | 2.04 | 1.72 | 1.2 | 2.34 | 0.59 |
| 3.51 | 2.04 | 4.08 | 1.55 | 2.03 | 0.44 |
| 4.98 | 2.04 | 7.7 | 2.61 | 2.16 | 0.52 |
| 6.45 | 2.04 | 10.99 | 3.37 | 2.18 | 0.52 |
| 2.04 | 3.51 | 0.7 | 1.91 | 2.92 | 0.94 |
| 3.51 | 3.51 | 2.56 | 2.03 | 2.23 | 0.58 |
| 4.98 | 3.51 | 5.56 | 2.76 | 2.17 | 0.55 |
| 6.45 | 3.51 | 9.43 | 3.91 | 2.28 | 0.61 |
| 2.04 | 4.98 | -1.54 | 1.8 | 2.91 | 0.88 |
| 3.51 | 4.98 | 1.2 | 2.48 | 2.47 | 0.71 |
| 4.98 | 4.98 | 3.16 | 2.63 | 2.13 | 0.53 |
| 6.45 | 4.98 | 6.12 | 3.32 | 2.11 | 0.51 |
| 2.04 | 6.45 | -3.57 | 1.86 | 2.99 | 0.91 |
| 3.51 | 6.45 | 0.03 | 2.95 | 2.76 | 0.84 |
| 4.98 | 6.45 | 2.22 | 3.31 | 2.39 | 0.66 |
| 6.45 | 6.45 | 3.49 | 3.17 | 2.04 | 0.49 |

$$\text{A}=\frac{T_{output} + 1.5T_{controller}}{T_{input}}, \text{ B}=\frac{T_{free-transmission}}{T_{input}}$$

## V. CONCLUSIONS

In this study, the power flows and torque analyses of the parallel type ICT are investigated and their analytical equations are derived. A prototype of the parallel type ICT is implemented for demonstration and the correctness of the derived equations has been investigated and verified. Because the parallel type ICT can produce a required output angular velocity, which is independently manipulated by a controller and not affected by the input angular velocity, it could be applied to variable speed wind turbines and automatic transmission systems. The further researches about the performance and application of the parallel type ICT are also proceeding.

Fig. 6.  A test-bed of ICT prototype

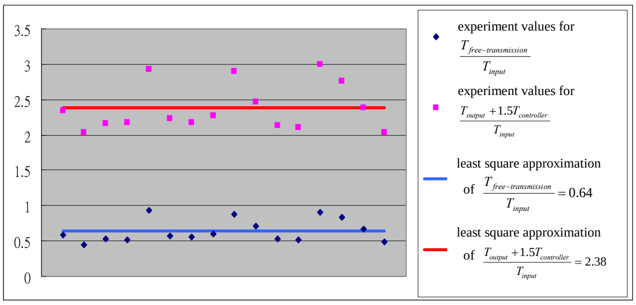

Fig. 7.  Plot of experimental values

## ACKNOWLEDGMENT

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