

An Application of the IMSC on a Non-linear Flexible Structure: Numerical Analysis and Experimental Validation

G. Cazzulani, F. Resta, F. Ripamonti

Abstract— The independent modal control to suppress the vibration of nonlinear flexible structures is applied in this paper. Technological improvements in the mechanical field showed during the recent years have led to high-performance systems with low weight and, as a consequence, high flexibility and low damping. Here active control quickly bettered the traditional passive damping systems. The structure investigated in this paper is a multi-body flexible boom moved by hydraulic actuators. The nonlinear system dynamic was numerically modeled and a control strategy, based on the use of the same actuators, was developed. Finally a test rig was created to experimentally validate the proposed approach.

Index Terms— Independent modal space control, nonlinear control, vibration suppression

I. INTRODUCTION

In recent years vibration control has been increasingly used not only in aerospace research but also in many mechanical application fields. The need for weight reduction to improve system performance led to structures with high flexibility and low damping. These structures suffer fatigue and instability issues raising, as a consequence, a number of safety questions. Traditional external passive control methods are generally more invasive, introducing mass into the structure, and less effective in a large range of frequencies. On the other hand, active control is an attractive solution, especially considering the extensive development of calculator hardware and the consequent cost reduction. Among the different control techniques proposed in recent decades, modal control offers many advantages thanks to its immediacy and connection to the dynamic design of the system. Moreover the improvement of sensor and actuator technology and innovative algorithms allows the spillover limits due to the truncated modes and unmodelled dynamic to be partly overcome.

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In literature modal control was introduced by Balas [1] and Meirovitch [2] between the 1970s and 1980s. Balas' studies deal with vibration suppression in large spacecraft structures, applying the modal expansion theorem and modal control. Some years later Meirovitch proposed an independent modal space control (IMSC) using the modal filter technique for estimating the modal coordinates [3]. The method was further improved by Baz and Poh [4], who suggested a modified independent modal space control (MIMSC) for optimal gain calculation adopting piezoelectric actuators.

Anyway Lin and Chu [5] demonstrated that modal control, for a general dynamic system with complex mode shapes, does not assure stability even for the controlled modes. The spillover problems were partially cushioned by the use of distributed sensor and actuators (Lee and Moon [6]) and some applications of IMSC for the control of flexible linkage mechanism (Zhang [7] and Changjian [8]) can be found. In 2001, Inman too [9] discussed the spillover issue associated with modal control, concluding that modern technology makes the problem manageable.

Moreover, modal control can be employed in conjunction with FEM for model definition, as shown by Skidmore [10] and Khulief [11]. They studied an active control scheme for vibration suppression in a single beam/cable structure, considering also its large motion.

This paper deals with independent modal control on a multi-body (MB) non-linear flexible boom. In particular suppression of the vibration caused by the large motion of the structure was investigated. Because only a few modes actively participate in describing the system's dynamic behavior, modal control is particularly useful.

The modal model required for the control law synthesis can be defined both experimentally, through a modal parameter identification campaign, or numerically, by means of FEM. In the proposed work a FEM approach was followed. The model not only allowed control/observer gain to be defined through pole placement techniques, but also permitted numerical simulations. At the same time a test rig was created to validate the defined control law. Finally a comparison between the behaviour with and without control is proposed.

II. THE SYSTEM

The boom is composed of a number of segments, connected to each other with revolving joints (Fig. 1). Each segment's movement is generated by an hydraulic actuator

through a kinematic chain assumed to be rigid.



Fig. 1. The test rig

Owing to the length and the low flexional stiffness of the section, the structure presents high flexibility producing, in normal operating conditions, low frequency vibrations, generally associated with a low damping ratio.

A numerical model, described in [12], was developed as a tool both for defining the control logic and for simulating boom behaviour. The boom kinematics were solved using the “floating frame of reference” formulation [13] to describe the large motion and the Finite Element Method (FEM) to model the segment flexibility. Using the Lagrange formulation, the boom’s non-linear equation of motion was obtained

$$[\mathbf{M}(\mathbf{z})]\ddot{\mathbf{z}} = f(\mathbf{z}, \dot{\mathbf{z}}) + [\mathbf{\Lambda}_{act}]^T \mathbf{F}_{act}(t) \quad (1)$$

where

- \mathbf{z} represent the independent variable vector, containing all the segment rotations and nodal displacements
- $[\mathbf{M}(\mathbf{z})]$ represents the inertial contribution;
- $f(\mathbf{z}, \dot{\mathbf{z}})$ contains all the non-linear damping, elastic and gravitational terms
- \mathbf{F}_{act} represents the large motion actuator forces and $[\mathbf{\Lambda}_{act}]$ represents the kinematic relationship between the actuator length and the independent coordinates vector \mathbf{z} .

Eq. (1) is a non-linear equation because all the matrices depend on the motion variables and in particular on the segment configuration.

To study the vibration problem and obtain system eigenvalues, the non-linear equation can be evaluated and linearized in each boom configuration. In a generic configuration of the system “ $\mathbf{z} = \mathbf{z}_i$ ”, the (1) becomes

$$[\mathbf{M}_i]\delta\ddot{\mathbf{z}}_i + [\mathbf{R}_i]\delta\dot{\mathbf{z}}_i + [\mathbf{K}_i]\delta\mathbf{z}_i = \mathbf{f}_g(\mathbf{z}_i) + [\mathbf{\Lambda}_i]^T \mathbf{F}_{act} \quad (2)$$

where

- $\delta\mathbf{z}_i$ is defined as $\delta\mathbf{z}_i = \mathbf{z} - \mathbf{z}_i$
- $[\mathbf{M}_i]$ and $[\mathbf{K}_i]$ respectively represent the inertial and elastic matrices, considering both the structural and gravitational terms and the actuator contribution;
- the damping term $[\mathbf{R}_i]$ is assumed to be proportional to the elastic and inertial ones. The proportional coefficients are estimated from experimental data;
- \mathbf{f}_g represents the constant contribution of the gravitational term.

Owing to the low segment rotation speed, centrifugal and Coriolis terms were assumed as negligible.

The following paragraphs present and numerically/experimentally investigates a control logic for reducing the structural vibrations.

III. ACTIVE CONTROL

In operating conditions, the system described earlier is subjected to a generic large motion which causes, because of the high flexibility of the segments, significant vibration levels. To suppress these vibrations ($\delta\mathbf{z}_i$), this paper proposes applying a control action using the same actuators used to move the individual segments. In many practical applications, in fact, the actuator number and position is assigned and cannot be modified.

The vibration control logic adopted is based on an independent modal approach [2,14,15]. Following this approach, the control action is calculated starting from the system vibratory state and a suitably defined gain matrix. In particular the vibratory state is described by a set of modal coordinates representing the system dynamics in the frequency range of interest. The modal coordinates, which for a generic application cannot be measured directly (unless we use distributed sensors [6]), have to be estimated by a modal observer. Below, the individual steps are analyzed in depth.

The control synthesis can be calculated starting from (2), introducing the control law vector \mathbf{u}_c and without considering the gravitational and external forces terms because these contributions don't modify the value of the poles of the system

$$[\mathbf{M}_i]\delta\ddot{\mathbf{z}}_i + [\mathbf{R}_i]\delta\dot{\mathbf{z}}_i + [\mathbf{K}_i]\delta\mathbf{z}_i = [\mathbf{\Lambda}_i]^T \mathbf{u}_c \quad (3)$$

A. Independent modal control

The aim of the control force \mathbf{u}_c is to increase the system’s damping ratio in order to reduce vibrations by imposing new values λ_c for the system’s original eigenvalues λ . In particular the real part of the system’s eigenvalues was increased, while the imaginary part remained unchanged so as to avoid having to use significant control forces and imposing additional mechanical fatigue stress on the boom material.

As previously described, in the present work the motion

equation was obtained using FEM discretization of the links. This means that the variable vector \underline{z} contains, as mentioned in par. 2, all the nodal displacement of the structure, and its dimension is not compatible with a full-state feedback formulation. Moreover, it should be considered that the structure dynamic is ruled only by the first modes, because high frequency modes usually have higher damping ratios and they can hardly be excited. For this reason a reduced modal system needs to be defined to describe boom motion using a limited set of coordinates, taking into account the well-known problem of spillover.

Defining the complete modal coordinates vector as \underline{q}_{tot} and the m reduced modal coordinates vector \underline{q}_z , the following coordinate change can be performed

$$\delta \underline{z}_i = [\boldsymbol{\varphi}_{i,tot}] \underline{q}_{tot} \approx [\boldsymbol{\varphi}_i] \underline{q}_z \quad (4)$$

where $[\boldsymbol{\varphi}_{i,tot}]$ represents the $n \times n$ eigenvector matrix of $[\mathbf{M}]^{-1}[\mathbf{K}]$, while $[\boldsymbol{\varphi}_i]$ is an $n \times m$ partition of $[\boldsymbol{\varphi}_{i,tot}]$ containing only the eigenvectors of the modes considered in the control formulation.

Applying the (4) to the (3), the motion equation becomes a set of independent modal equations

$$[\mathbf{m}_i] \ddot{\underline{q}}_z + [\mathbf{r}_i] \dot{\underline{q}}_z + [\mathbf{k}_i] \underline{q}_z = [\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T \underline{u}_c \quad (5)$$

where $[\mathbf{m}_i]$, $[\mathbf{r}_i]$ and $[\mathbf{k}_i]$ are $m \times m$ diagonal matrices, obtained by

$$[\mathbf{m}_i] = [\boldsymbol{\varphi}_i]^T [\mathbf{M}_i] [\boldsymbol{\varphi}_i] \quad (6)$$

should be noted that (5) and (6) are only valid for systems with real modes. Anyway the examined system damping is very small and its modes can be assumed real.

To apply the state-space control approach to the reduced modal system, the (5) can be written in state-space form

$$\begin{aligned} \dot{\underline{q}} = & \begin{bmatrix} -[\mathbf{m}_i]^{-1}[\mathbf{k}_i] & -[\mathbf{m}_i]^{-1}[\mathbf{r}_i] \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \underline{q} \\ & + \begin{bmatrix} [\mathbf{m}_i]^{-1}[\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T \\ \mathbf{0} \end{bmatrix} \underline{u}_c = [\mathbf{A}_q] \underline{q} + [\mathbf{B}_q] \underline{u}_c \end{aligned} \quad (7)$$

where

$$\underline{q} = \begin{Bmatrix} \dot{\underline{q}}_z \\ \underline{q}_z \end{Bmatrix} \quad (8)$$

The control force is defined as

$$\underline{u}_c = -[\mathbf{G}_c] \underline{q} \quad (9)$$

where, applying the independent modal control approach,

the control gain matrix is calculated as [12]

$$[\mathbf{G}_c] = \begin{bmatrix} [\bar{\mathbf{B}}_q]^{-1} & \mathbf{0} \end{bmatrix} \left([\mathbf{A}_q] - [\mathbf{U}] [\boldsymbol{\lambda}_c] [\mathbf{U}]^{-1} \right) \quad (10)$$

where

- $[\boldsymbol{\lambda}_c]$ is a diagonal matrix containing the eigenvector imposed for the controlled system
- $[\mathbf{U}]$ is the right eigenvector matrix of $[\mathbf{A}_q]$
- $[\bar{\mathbf{B}}_q]$ represents the upper part of $[\mathbf{B}_q]$ and must be invertible; as shown in (7), the lower part is equal to zero when the system is described by 2nd order differential equations

In this way, the (7) becomes

$$\dot{\underline{q}} = [\mathbf{A}_q] \underline{q} - [\mathbf{B}_q] [\mathbf{G}_c] \underline{q} = [\mathbf{A}_{q,c}] \underline{q} \quad (11)$$

The disadvantage of the state-space approach is that the gain matrix values lose physical meaning. For this reason, the modal control law proposed has been calculated imposing that the term $[\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T \underline{u}_c$ of (5) doesn't couple the system modes and leads to an increase of the system damping ratio

$$[\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T \underline{u}_c = -[\mathbf{g}_v] \dot{\underline{q}}_z - [\mathbf{g}_p] \underline{q}_z \quad (12)$$

where $[\mathbf{g}_p]$ and $[\mathbf{g}_v]$ have a precise physical meaning, because they represent respectively the increase in stiffness and damping ratio brought about by the control. To ensure independent modal control they must be diagonal, so that the control law provides an independent stiffness and damping contribution on each mode, keeping the same eigenvectors as the uncontrolled system. This condition can be obtained only if the number of actuators is equal to the number of considered modal coordinates. In addition, the system must be controllable, so any row or column of $[\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T$ must be non-zero. This means that at least one control force must have a non-zero contribution on every mode and each control force must act on at least one mode. The control force can be calculated as

$$\begin{aligned} \underline{u}_c = & - \left([\boldsymbol{\varphi}_i]^T [\boldsymbol{\Lambda}_i]^T \right)^{-1} \left([\mathbf{g}_v] \dot{\underline{q}}_z + [\mathbf{g}_p] \underline{q}_z \right) \\ = & - \begin{bmatrix} [\mathbf{G}_v] & [\mathbf{G}_p] \end{bmatrix} \begin{Bmatrix} \dot{\underline{q}}_z \\ \underline{q}_z \end{Bmatrix} \end{aligned} \quad (13)$$

Selecting $[\mathbf{g}_v]$ and $[\mathbf{g}_p]$, the desired system eigenvalues can be set for the controlled system. In particular $[\mathbf{g}_p]$ can be set to zero to keep the same natural frequencies as the uncontrolled system.

B. The modal observer

Assuming that the exact modal coordinates are known,

the method proposed guarantees a completely decoupled modal control and avoids spillover problems, even if the number of controlled modes is smaller than the number of structure modes.

However, in most flexible structure control applications the modal coordinates are unknown and so have to be estimated using a modal observer or filter [3]. In this paper a modal observer has been considered (Fig. 2).

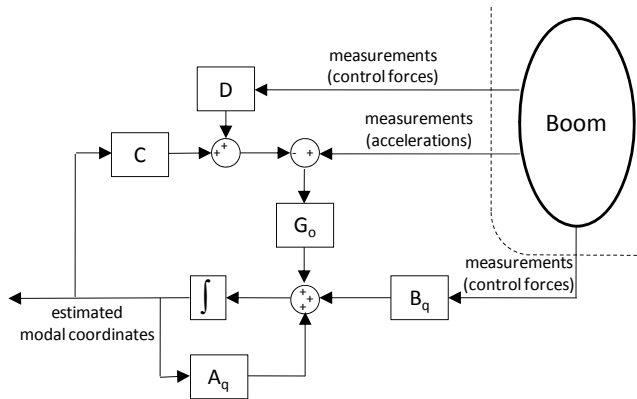


Fig. 2. The scheme of the modal observer

According to the (7), the observer equation can be written as

$$\dot{\hat{\mathbf{q}}} = [\mathbf{A}_q] \hat{\mathbf{q}} + [\mathbf{B}_q] \mathbf{u}_c + [\mathbf{K}_o] (\mathbf{y} - \hat{\mathbf{y}}) \quad (14)$$

where

- $\hat{\mathbf{q}}$ is the estimator of the system modal coordinate vector;
- $[\mathbf{K}_o]$ is the observer gain matrix, which will be defined later;
- \mathbf{y} and $\hat{\mathbf{y}}$ respectively represent the measurements and estimated measurements vectors. \mathbf{y} is an observer input, while $\hat{\mathbf{y}}$ is defined by

$$\hat{\mathbf{y}} = [\mathbf{C}] \hat{\mathbf{q}} + [\mathbf{D}] \mathbf{u}_c \quad (15)$$

The values of $[\mathbf{C}]$ and $[\mathbf{D}]$, which link the estimated measurements vector to the observer states and control forces, depend on sensor type (accelerometer, position sensors, strain gauges, etc.) and position.

Considering the (15), and neglecting the contribution of $[\mathbf{D}]$, the observer equation (14) becomes

$$\begin{aligned} \dot{\hat{\mathbf{q}}} &= ([\mathbf{A}_q] - [\mathbf{K}_o][\mathbf{C}]) \hat{\mathbf{q}} + [\mathbf{B}_q] \mathbf{u}_c + [\mathbf{K}_o] \mathbf{y} \\ &= [\mathbf{A}_o] \hat{\mathbf{q}} + [\mathbf{B}_q] \mathbf{u}_c + [\mathbf{K}_o] \mathbf{y} \end{aligned} \quad (16)$$

Now it's possible to define the last term of the observer, the gain matrix $[\mathbf{K}_o]$, which is calculated using the pole placement method.

IV. NUMERICAL AND EXPERIMENTAL ANALYSIS

In the previous paragraph the vibration control logic was defined. To test the adopted solution, a numerical and experimental campaign was carried out. The test rig, introduced in par. 2, was instrumented with

- 3 load cells, to read total actuator forces;
- 3 LVDT sensors, to obtain boom configuration;
- 1 accelerometer used as the observer input and the boom vibration indicator, located at the end of the last segment.

As said in par. 2, the motion of the system is described by a non linear equation. For this reason all control matrices need to be updated at every step to make the control logic rigorous. This updating can be easily reproduced in the numerical simulations, where there are no limits due to real-time issues.

However, the experimental application requires real-time calculation of control matrices and normal microprocessors do not have enough power or speed to handle the required time scale, so an approximation must be applied. In any case, considering that the rotation speed of the segment (and so the matrix variation) is lower with respect to the controller dynamic, this approximation doesn't compromise control efficiency and two possible approaches can be adopted.

The first consists in calculating control matrices every time steps, using the control board in a multi-tasking mode. A faster method calculates the control forces every time step while the other method calculates them at longer intervals.

The second approach is to pre-calculate the matrices for a discrete set of boom configurations. In this way, at every time step, the software simply needs to extract the nearest configuration matrices using linear interpolation. The actual boom configuration, as mentioned, can be determined from the actuator LVDT output. The limit of this solution is the memory available on the control board.

Considering the small amount of memory required by the present application, this second method was preferred and implemented. The controlled system and observer pole are set as follows:

- the imaginary part of each controlled and pole is set equal to the imaginary part of the uncontrolled system. This is because we are not interested in modifying the system frequencies since this is not the aim of the present work and would lead to high control forces. As mentioned, a control force increase could lead to high mechanical stress and fatigue problems. Instead, the imaginary parts of the observer poles are set twice those of the uncontrolled system, in order to make the observer dynamics faster than the system dynamics;
- the real part of each controlled and observed pole is placed respectively equal to 15% and 30% of the corresponding imaginary one, in order to set the controlled system damping ratio higher than that of the uncontrolled boom.

For comparison, the possibility of using a co-located control force was analyzed. This solution is the simplest one, because it only requires the measurement of the actuator length and is the most natural solution for adding damping to the system. The resulting control force, consists

of a co-located derivative control, that increases the total damping of the system by adding a damping contribution on the boom actuators. This solution is simpler than modal control. In fact it requires low computational effort and doesn't compromise system stability. However, this solution doesn't provide a large increase in the damping ratio. In fact this derivative action only works on actuator vibrations, without considering the vibrations of the segments due to their flexibility. As a consequence, even after optimization, only a small damping ratio increase is achieved.

A. Numerical analysis

This paragraph presents some numerical results. All the simulations refer to the numerical model of the test rig that was used for the experiments. All the results (with and without vibration control) refer to the same large movement of the boom (Fig. 3).

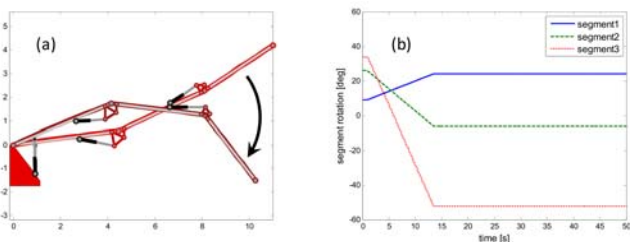


Fig. 3. Large motion reference for the boom segments: starting and final configuration (a) and rotation reference (b)

Fig. 4 compares the system vibrations with and without modal control, highlighting the increase in damping. The acceleration of the end of the third segment is considered as indicating control performance. On the left, the figure shows the acceleration time history during the large motion described in Fig. 3. The time history also includes static acceleration due to gravity. On the right it shows, on a logarithmic scale, the acceleration spectrum with the application of an external force on the first segment end. This simulation was carried out in the final configuration of the large motion shown in Fig. 3. The external force consists of a frequency sweep up to 7 Hz, that involves the first three natural frequencies of the structure. The figure shows that modal control causes a large reduction of vibrations near the controlled natural frequencies. However, system response increases between these.

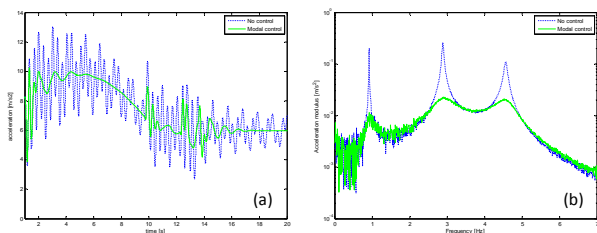


Fig. 4. Numerical acceleration of the end of the third segment: time history (a) and spectrum (b)

In any case, as Table 1 also shows, modal control can ensure a large increment in the boom damping ratio.

TABLE I

COMPARISON BETWEEN THE DAMPING RATIOS OF THE FIRST THREE MODES OF THE BOOM IN HORIZONTAL CONFIGURATION WITH AND WITHOUT CONTROL

Damping ratios [%]	No control	Modal control
Mode 1	0.19	14.4
Mode 2	0.42	13.9
Mode 3	0.73	13.1

B. Experimental results

This paragraph presents the results obtained on the test rig (Fig. 1). As in the numerical case, the acceleration of the end of the third segment is considered to estimate control performance.

Fig. 5 shows the experimental results, obtained reproducing the same conditions as in the numerical simulations (movement, control parameter, measurement instruments). A good agreement with the numerical data was obtained.

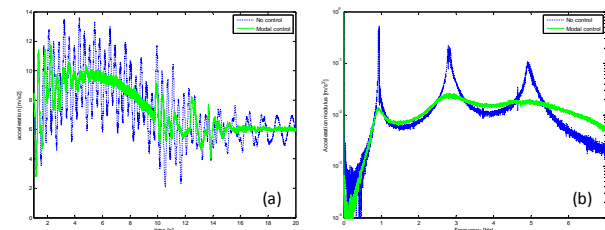


Fig. 5. Experimental acceleration of the end of the third segment: time history (a) and spectrum (b)

In order to complete the numerical-experimental analysis, in Fig. 6 a comparison is shown.

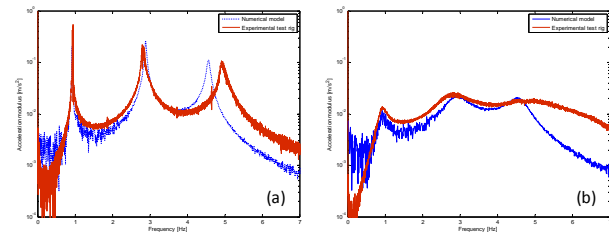


Fig. 6. Numerical-experimental spectra comparison without (a) and with (b) modal control

V. CONCLUSIONS

In this paper a control strategy was implemented to suppress the vibrations induced in a non-linear highly-flexible boom by its large motion. For this kind of structure the modal solution remains an attractive one for modelling the system dynamics with a reduced number of degrees of freedom in a focused range of frequencies. As a consequence the non-linear active control logic adopted is based on the independent modal control theory, illustrated using the 2nd order equation typical of mechanical systems. In the same way, a modal observer was implemented to estimate the non-measurable modal coordinates. The control force is applied to the system through the same hydraulic actuators responsible for the boom's large motion.

First this methodology was tested using numerical

simulation, showing that an increase of the damping ratio of up to about 15% could be achieved without affecting the large motion and reducing material stress during operation.

Finally a test rig was created and the methodology was experimentally validated. The experimental results show a very good agreement with the numerical ones and the modal control action provides an high reduction of test rig vibrations.

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