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Contact Force Optimization For Stable Grasp Of Multifingire Robotic Grippers

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Abstract— Grasping is concerned with characterizing and achieving the conditions that will ensure that a robot gripper holds an object securely, preventing, for example, any motion due to external forces. A system where in a desired object is gripped by the fingers of a robot (or human) and is generally called a grasp. Most of past researches are restricted to tip prehension grasp (object gripped by fingertips only). In the present work, a grasp is used often to mean the grasped object itself, and this is also true with the literature. I the Theory of grasping internal forces, formulated in the form of equal and opposite pairs of forces acting along the lines of contact, can be used to selectively orient the net force vector. A grasp situation, which satisfies this condition, is a stable grasp. This Paper deals with the optimization to obtain the most stable grasp for a nominated set of contact points and loading on an object. The equilibrating forces have been calculated on the basis of algorithms developed. The values of friction angles are optimized so as to satisfy the condition of stable grasp. The unit cube considered for these calculations was assumed to be loaded under its own weight (no moment appears due to the weight of the body because of symmetry; there are no external moments either) and had its body diagonal coincident with the z axis. The body diagonal of the cube is now shifted from the z-axis and the cube is subjected to what is known as quasi-static motion. The study concludes that the stable grasp for a nominated set of contact points and loading condition is obtained at maximum friction angles and minimum contact points.

Index Terms—Algorithm, Equilibrating, forces friction angles, quasi-static motion, Robotic Grasping.

I. INTRODUCTION

Over the last two decades, grasping has showed a somewhat marginal topic to an important field of robotics research. This increasing interest in grasping is partly due to the evolution of industrial automation towards flexible automation. The transition from large batch size to medium and small sizes has led to the replacement of special purpose devices with more general purpose and effectors enabling the manipulation of a broader class of objects. At the same time, more attention has been given to fine manipulation and assembly.

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This has pointed out the needs for tools able to increase the robot's manipulative capacity with fine position and force control. Therefore, as end effector become more flexible, control becomes more complex and a better understanding of grasping turns out to be a challenging issue [1].

Regarding the grasping and manipulation by a robot hand, many papers have been published [2]-[6]. Authors have reported various multi-fingered hands for manipulating objects skills fully. They have also done analytical studies of grasping and manipulation by robot hand. Numerous factors are available to determine the effective grasp of an object. Researchers [7]-[10] in robotics have tried to analyze, what constitutes good grasps. Nine analytic measures were shown, for describing a grasp, compliance, connectivity, force closure, from closer, grasp isotropy, internal forces, manipulability, resistance to slipping and stability [11]. Multitude of properties that an articulated grasp must possess in order to able to perform everyday tasks similar to those performed by human hands has been discussed [12].Even after these studies, mechanics of grasping, manipulation & grasp properties have not yet fully been understood so far [13]. This understanding is important for designing robot hands and for developing grasping and manipulation algorithms.

II. METHODOLOGY

Minimums of three fingers are needed to grasp an object. In carrying out a sequence of motion a particular finger may reach a joint limit or start slipping. It then becomes necessary to relieve that finger with an unused finger. Hence dexterous manipulability requires at least four fingers [14]. Even simple operations in real life, like turning a coke can about its axis of symmetry require phasing in and out of the fingers. A algorithm is developed for Computation of contact force at the contact point, when to phase out a finger.

The algorithm developed will be used to numerically optimize the values of the various friction angles (θ_i) and k_{ij} so as to satisfy the stable grasp requirement. However, a numerical computation of the friction angles not only offers solidity to the theory behind the entire analysis but also reveals a few interesting trends. The unit cube considered for these calculations was assumed to be loaded under its own weight (no moment appears due to the weight of the body because of symmetry; there are no external moments either) and had its body diagonal coincident with the *z* axis. The body diagonal of the cube is now shifted from the *z*-axis and the cube is subjected to what is known as *quasi-static motion*. That is, the cube is now set in motion and moves about the $0.707\ \hat{\mathbf{i}} - 0.707\ \hat{\mathbf{j}}$ axis from -15° to +15°, in *intervals* of 1°. The interaction forces and angles are recomputed for every such interval and the trend is plotted.

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III. FORMULATION OF THE MINIMAX PROBLEM

A. The point contact

The development in this section is algebraic. Whenever possible, a parallel geometric interpretation of the result will be shown. Vectorially the force system can be represented for a three-point contact by

$$P_1 = F_1 + k_1u_1 - k_2u_2$$
$$P_2 = F_2 - k_3u_3 - k_2u_2 \qquad \text{----------- (1)}$$
$$P_3 = F_3 - k_1u_1 - k_3u_3$$

Where the vector P_i are the net forces and the vectors F_i are the equilibrating forces at point of contact 'i'. The u_j are the unit direction vectors along which the equilibrating forces are to be applied and k_j is the associated scalar factor. Correspondingly the friction angle θ_i at a point is obtained from the relationship

$$Cos\ \theta = \frac{n_i P_i}{\|P_i\|} \qquad \text{--------- (2)}$$

Where n_i is the unit normal to the surface at contact point i.

The limiting friction problem can be formulated as that of finding a set of internal forces such that the angle θ_I does not exceed the maximum allowable friction angle. A grasp situation, which satisfies this condition, is a stable grasp. The development below produces a set of equations, solution of which yields the most stable grasp possible using a given set of contact points. This achieved by maximizing the minimum of the three friction angles for a set of three contact points. The grasp plane is the plane containing the three points of contact.

Consider

$$Cos\theta = \frac{n_1 \cdot (F_1 + k_1u_1 - k_2u_2)}{\|F_1 + k_1u_1 - k_2u_2\|} \qquad \text{---(3)}$$

For the following section, we shall assume that the inward drawn contact normal and the equilibrating forces are on the same side of the grasp plane for all the contact locations. If this condition is violated, zero friction angles cannot be achieved by manipulating the interaction force field.

B. A Unique Minimum value of Friction Angle (θ_I)

As stated above that the object we chose to work on was a unit cube. The notations used in equation system (1) can be modified slightly to include subscripts which clarify the directions of the vectors involved. We let the subscript '*ij*' denote that a vector is directed from point *I* to point *j*. When used with the *k* terms (which are scalars) it denotes the direction in which the interaction forces pertinent to that point are present. We then have,

$$\mathbf{P}_1 = \mathbf{F}_1 + k_{12}\,\mathbf{u}_{12} + k_{13}\,\mathbf{u}_{13}$$
$$\mathbf{P}_2 = \mathbf{F}_2 + k_{23}\,\mathbf{u}_{23} + k_{21}\,\mathbf{u}_{21} \qquad \text{---------(4)}$$
$$\mathbf{P}_3 = \mathbf{F}_3 + k_{31}\,\mathbf{u}_{31} + k_{32}\,\mathbf{u}_{32}$$

The interaction forces in this form still constitute a null solution as $\mathbf{u}_{ij} = - \mathbf{u}_{ji}$. Again, as in equation kij = k_{ji}.

In the previous section it has been shown analytically that there is a unique value of k_{12} and k_{13} for which $\cos\theta_1$ equals unity. Using equation (2) and (3) a MATLAB code was written (listed in Appendix-A) and a mesh surface was plotted (figure 2) for a hundred values of k_{12} and k_{13}. The plot shows the existence of those values.

The unit cube considered for these calculations had its body diagonal coincident with the *z* axis. The mathematical manipulations involved in achieving such an orientation require the use of rotation matrices. The rotation matrix expression, which achieves the necessary coordinate transformations, about the axis $0.707\ \hat{\mathbf{i}} - 0.707\ \hat{\mathbf{j}}$ is given by,

$${}^{A}\mathbf{R}_{B} = \begin{bmatrix} r_x^2 V\phi + C\phi & r_x r_y V\phi - r_z S\phi & r_x r_z V\phi + r_y S\phi \\ r_x r_y V\phi + r_z S\phi & r_y^2 V\phi + C\phi & r_y r_z V\phi - r_x S\phi \\ r_x r_z V\phi - r_y S\phi & r_y r_z V\phi + r_x S\phi & r_z^2 V\phi + C\phi \end{bmatrix} \qquad \text{----(5)}$$

Where ϕ is the angle that the cube has to be rotated by to align its body diagonal with the *z*-axis. The term $V\phi$ = versϕ=1-cosϕ . $C\phi$ and $S\phi$ are the cosine and sine of ϕ respectively. In terms of the object frame, the cube has its *z* axis coincident with one of the edges of the cube. From considerations of geometry, the angle ϕ can thus be found to be $\cos^{-1}(0.5773) = 54.73°$. The terms r_x, r_y, and r_z are the direction cosines of the axis of rotation. In this case, they are respectively 0.707, 0.707, and 0.

C. Cube Motion: The Special Case of Complete Symmetry

One of the ways to grasp a cube with 4-fingers is in a way such that 4 fingers are on four adjacent faces of the cube subject to the condition that no three faces are intersection. However, this type of a grip limits the size of the cube which can be handled. So, a cliff event grip was chosen – one with three fingers, each touching one of the intersecting faces at the corner. It is obvious that with such an arrangement, we will have to hold the cube vertically else it will fall down

The cube when oriented with the body diagonal coincident with the *z* axis presents a case of special interest. Let the angle of rotation (for the quasi static motion mentioned above) be represented byα. When the body diagonal is coincident with the *z* axis, α equals zero. The cube in this condition is symmetrical in all respects. For simplicity of calculation, center-points of the cube faces are chosen as the points of contact. This further lends to the symmetry of the case. For all the calculations the cube is assumed to be loaded under its own weight. The weight vector for all cases is assumed to be directed along the –z axis. The weight vector will pass through the center of gravity. Since the reference for all the calculations takes the centroid as the origin, for any α the weight vector will create a moment about the centroid, the lever arm being the difference of the position vectors of the center of gravity and the centroid.

The case when the cube has one of its edges coincident with the *z* axis does seem to offer the advantages of symmetry as well as simplicity. However, when an optimization routine is allowed to run on this case the interaction forces tend to infinity. This seems anomalous at first, but at the same time it is not hard to see why. The local equilibrating forces balance the weight of the cube. Also the surface normal is completely orthogonal to the applied weight. It would indeed require infinite force in the direction of the surface normal (which lie on the grasp plane) so that the resultant force is applied at zero friction angles. This case does not inherently possess sufficient constraints to run an optimization routine successfully.

The case of the body diagonal coincident with the *z-axis* is thus considered. At $\alpha = 0$, the *z* axis (and thus the weight vector) passes through the centroid as well as the center of gravity. Moment due to the weight vector is therefore zero. As it is assumed that no other external moment is acting on the cube the total moment is zero. Equation (7) expression for equilibrating force, involves calculation of quantities such as screw pitch. These values are divided by the applied torque. The algorithm can fail as we are attempting to divide a quantity by zero. The escape to this is provided by considering the case intuitively. In the absence of any torque the local equilibrating forces have to balance only the weight (force applied) of the cube. As the points are symmetrically located, each contact point sees the same fractional quantity of force as the other two points. The equilibrating force for each point of contact in this case is quite simply the one third of the applied force (weight).

Figure 1: 15 degree rotation along x axis

Even more interestingly, for the case of $\alpha = 0$, it is possible to predict what the interaction forces will be. Since this case exhibits complete symmetry, each point sees the same loading conditions. A force configuration adopted for one contact point will mean that the same force configuration is true for the remaining two contact points. It can thus be concluded that for the case of $\alpha = 0$, the angles of contact for all the three points of contact will be the same. This implies that all the k_{ij} will be equal.

The concept of symmetry can be extended further to include the assumption that the three points of contact lie on three mutually perpendicular faces of the cube. The triangle that the three contact points make is then equilateral. Further the grasp plane is perpendicular to the *z* axis. Consider one contact point. Since all the k_{ij} are equal the analysis is valid for all the three points. The resultant interaction force vector will have magnitude $2 \cdot k \cos 30° = k \cdot \sqrt{3}$. Let the weight of the cube be *w*. For reasons discussed above, the equilibrating forces at each contact point are *w*/3. The minimax problem states that it is desirable to minimize the friction angle. For an angle of zero, the applied contact force P will be in the direction of the surface normal. The surface normal itself is inclined to the *z* axis (and thus the weight vector) by an angle $\phi = 57.4°$ approximately. Hence the cosine component of P will balance the fraction of weight seen by it and will thus equal the equilibrating force at that point. The sine component of P (which is parallel to the grasp plane) will equal the interaction force as calculated above. We thus have the following relationships:

$$F \sin\phi = k\sqrt{3} \quad \text{-------(6)}$$
$$F \cos\phi = w/3 \quad \text{-------(7)}$$

where k_{ij} is replaced by k since all k_{ij} are equal. Dividing the two equations we get,

$$k\sqrt{3} = \frac{w \tan\phi}{3}$$

or,

$$k = \frac{w \tan\phi}{3\sqrt{3}} \quad \text{------(8)}$$

For the case $w = 5$, we get $k = 1.3608$.

The algorithm for calculating the two force fields and optimizing the friction angle was implemented on MATLAB. The optimization routine used was called 'fminu' and was an unconstrained nonlinear optimization library routine. The subroutine required an initial value of the variable to be optimized, as one of the arguments. The value for *k* from equation 8 is an apt value for usage in this unbounded case. It further provides a neat check for the results of the algorithm.

IV. RESULTS

Results obtained by running MATLAB Program and further plotted in figure 2.For the quasi-static motion described earlier, α is allowed to take values form -15° to +15°. The equilibrating forces and interaction forces, as well as the optimization for θ_i are recomputed for each interval of 1°. The results thus obtained were plotted as a graph of the minimized maximum θ_i along with the values of all k_{ij} againstα.

Axis	*Values*
x	k_{12}
y	k_{13}
z	θ_1

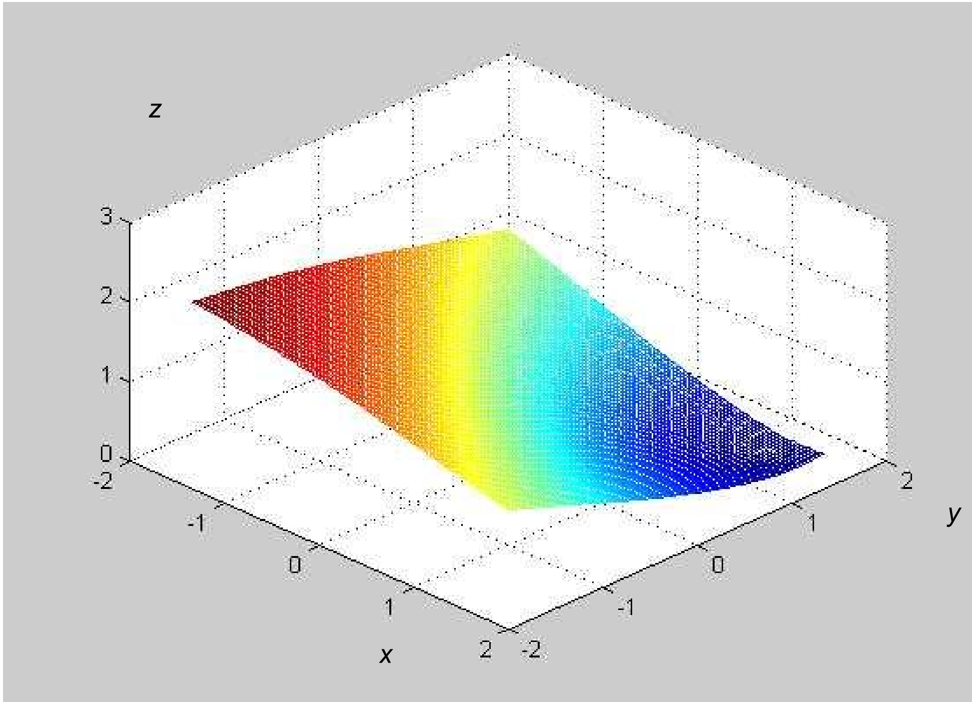

Figure 2- Mesh plot of θ_1 against various values of k_{12} and k_{13}.

V. CONCLUSIONS

The findings show that the values of θ_i closely follow each other without any specific tolerance/constraints. The closeness of the values of θ_i suggests that achieving equality is a distinct possibility. The advantage that accrue from having a case where $\theta_1 = \theta_2 = \theta_3$ is twofold. Firstly, it allows to eliminating two variables from the set of k_{ij}. As a result, an indeterminate system of six unknowns in three equations now becomes a *deterministic* system that calculates various contact forces. Secondly, the system presents a polynomial solution for those constraints whose efficient evaluation methods are known. Optimum stable grasp for a nominated set of contact points and loading condition is obtained at maximum friction angles and minimum contact points. A numerical computation of the friction angles offers solidity to the theory behind the entire analysis [15]. The study may further be extended to include constraints.

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