

# Comparative Study of the Phenomenon of Propagation of Elastic Waves in Conduits

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**Abstract**—The transient phenomena of elastic waves in conduits represent a complicated problem in the field of hydraulics particularly in the large hydraulic installations like the hydroelectric stations and the pumping stations where dimensions of the pipe are very important and where the phenomenon of the water hammer can cause enormous damage. In our study we focused on one of causes of the wave propagation, which is the case of sudden closure of valves in the gravity adduction pipes, using both the most effective numerical methods for discretizing and solving the problem; the finite difference method using the WHAMO program and the method of characteristics with software AFT Impulse. The results are very interesting especially with regard to the gravity piping system with variables geometric characteristics (section changes and bifurcation) where pressure fluctuations vary dangerously and particularly in the cases of the convergence and the unsynchronized closure of the valves.

**Index Terms**—Water hammer, Transient, Pressure, Flow, Numerical Simulation.

## I. INTRODUCTION

When sudden changes in flow occur, the energy associated with the flowing water is suddenly transformed into pressure at that location. This excess pressure is known as hydraulic shock or water hammer and is greater with large changes in velocity. Characteristics of the pipe such as the materials used in construction, the wall thickness, and the temperature of the pipe all affect the elastic properties of the pipe and how it will respond to surge pressures or the propagation of pressure waves in conduits.

The field who studies this case is the hydraulic transient where the analytical solutions are not possible. Allievi [1] developed classical solutions by both analytical and graphical solution. Graphical solutions mentioned above had some practical application in pipe design before the advent of computer. Streeter [2] developed a numerical model by using a constant value of turbulent friction factor. Wiggert and Sundquist [3] solved the pipeline transients using fixed grids projecting the characteristics from outside the fundamental grid size. Watt and al [4] have solved for rise of pressure by MOC for only 1.2 seconds and the transient friction have not been considered. Chaudhry and Hussaini [5] solved the water hammer equations by MacCormack, Lambda, and Gabutti explicit FD schemes.

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Pezzinga [6] worked to evaluate the transient flow resistance by MOC. Hence the study of the previous works denotes that there are various numerical models, which include Method of characteristics (MOC) and Finite Difference (FD) presented by different investigators to obtain the transient pressure and discharge in water hammer situations.

The approach to this problem in our study often requires using a numerical simulation; two numerical methods to compare and better understand the phenomenon in all details (magnitude, its behavior towards the conduits, its influence on hydraulic systems, etc.). Through numerical simulation, a realistic approximation of the shape and magnitude of this phenomenon was produced.

The purpose of this study is to reduce the effects generated by the complex phenomenon of water hammer in hydraulic systems and especially those equipped with large-size pipes by knowing the previous details with an efficient manipulation of valves.

## II. DISTURBANCE OF EQUILIBRIUM, EFFECT ON PRESSURES

Whether a conduit in which water flows in steady state, the pressure level is lower than the hydrostatic pressure under conditions of no flow. For a temporal modification of the flow by acceleration or deceleration, and considering the principles of Newtonian mechanics [7] there will be a transfer of energy between kinetic and potential energies to try to reach a new equilibrium taking into account losses and without transgressing the principle of conservation of energy. Assuming that in a fluid, the potential energy translates into pressure energy and the water is slightly compressible and that the pipe admits of elastic deformations, our analysis focused on particular cases.

## III. THE EQUATIONS OF WAVES PROPAGATION IN CONDUITS

The Saint Venant equations formed by the continuity equation and momentum equation [8] are the basic equations governing transient phenomena in conduits:

$$\begin{cases} \frac{\partial H}{\partial t} + \frac{C^2}{gA} \frac{\partial Q}{\partial x} = 0 \\ \frac{\partial Q}{\partial t} + gA \frac{\partial H}{\partial x} + \frac{\lambda Q|Q|}{2DA} = 0 \end{cases} \quad (1)$$

With Q the discharge and H the pressure head, the friction

term is represented by  $\frac{\lambda Q|Q|}{2DA}$ . The celerity of pressure wave

which is equal to the speed of sound in the medium [water - conduit] is expressed by the relation:

$$C = \sqrt{\frac{1}{\rho \left( \frac{1}{E_{eau}} + \frac{D}{e E_c} \right)}} \quad (2)$$

The essential factors involved in the propagation velocity of pressure wave are the compressibility of water  $E_{eau}$  and the elasticity of the material constituting the pipe  $E_c$ .

#### IV. NUMERICAL METHODS FOR THE RESOLUTION OF WAVE EQUATIONS

##### A. Finite Difference Method

In this study we used a structured mesh of Cartesian type [9], introducing  $N+1$  node spaced with a step  $\Delta x$ , as time is divided into intervals of constant time step  $\Delta t$ . We will have:  $x_i = i\Delta x$  (for node) where:  $0 \leq i \leq N+1$  and  $t^n = n\Delta t$  (for time) for  $n \geq 0$ .

So we look for solutions of the system (1) at all points into interval  $[0, L]$  and for time  $t \geq 0$ . ( $L$  is the length of the pipe). For  $i=0$ ,  $x_i=0$  and for  $i=N+1$ ,  $x_i=L$ .

The approximation scheme used is the four-point implicit scheme with Dirichlet boundary condition. The general form of the equations of system (1) becomes [10]:

$$\begin{cases} H_{i+1}^n + H_{i+1}^{n+1} + \alpha_i(Q_{i+1}^{n+1} - Q_i^{n+1}) = \beta_i \\ -H_{i+1}^n + H_{i+1}^{n+1} + \gamma_i(Q_i^{n+1} + Q_{i+1}^{n+1}) = \delta_i \end{cases} \quad (3)$$

Where:  $\alpha_i, \beta_i, \gamma_i, \delta_i$  are the coefficients simplify.

##### B. Method of Characteristics

For example to calculate  $H_{I+1}$  and  $Q_{I+1}$  for using the following formulas [11]:

$$H_{i+1} = \frac{WP + WM}{2} \quad Q_{i+1} = \frac{WP - WM}{2R} \quad (4)$$

With:  $WP = H_I + Q_I[R - T|Q_I|]$  and

$$WM = H_{I+2} - Q_{I+2}[R - T|Q_{I+2}|] \quad (5)$$

WP is a reference to the "characteristic plus" (W+) and WM is a reference to "characteristic minus" (W-).

Where:  $R = \frac{C}{gA}$  and  $T = B(x_{I+1} - x_I)$  Knowing that:

$$B = \frac{\lambda}{2gDA^2}$$

#### V. NUMERICAL SIMULATIONS

The external boundary conditions were set for solving partial differential equations are [12]:

- In the reservoir  $H_i$  const where  $H_i = H_{res} - \text{losses}$ ;
- Constant discharge or null  $Q = Q_{cl} = 0$  at the valve (cl as boundary condition) for our case of closing the valve.

Characteristics of valves used: valve type 1 (gate) specifies that the valve is a disk type gate valve; diameter 3048mm, with for  $t = 0s$  the valve is 100% open, for  $t = 0.1 s$  the valve is 80% open, for  $t = 0.2 s$  the valve is at 60% and so on.

The closing time of the valve for divergence and convergence is  $T = 0.6$  seconds ( $T < 2L / C = 2.37$  seconds, quick closing valve).

##### Model study 1

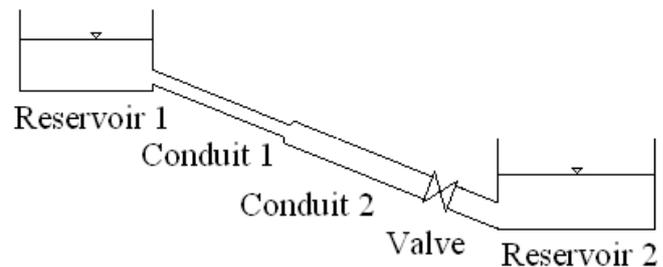


Fig. 1. Case 1: a divergent.

##### Using the WHAMO program

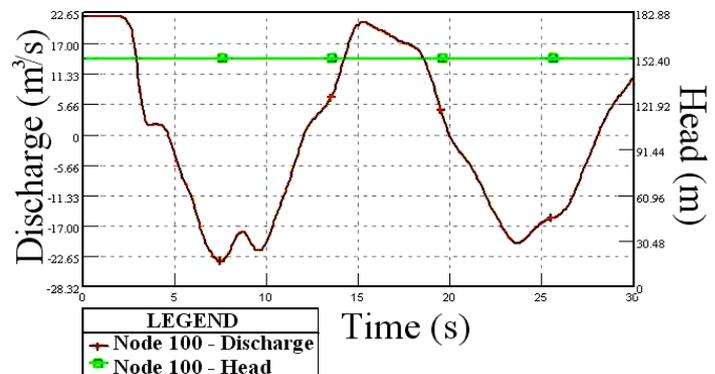


Fig. 2. Evolution of Q and H at the reservoir.

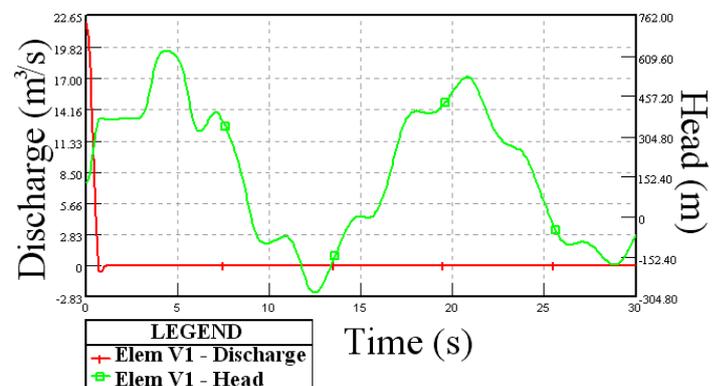


Fig. 3. Evolution of Q and H at the valve.

Using the software AFT Impulse

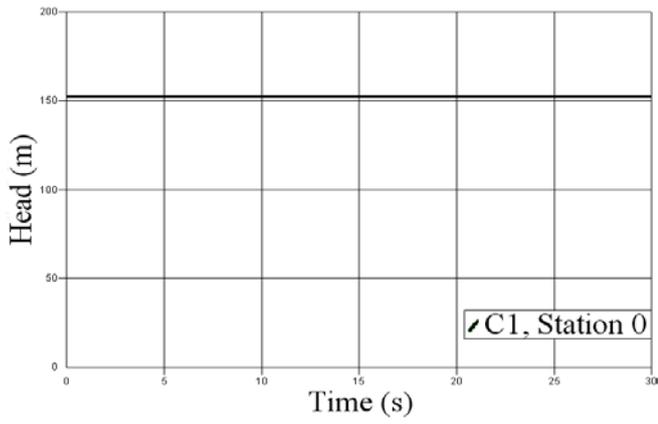


Fig. 4. Evolution of H at the reservoir.

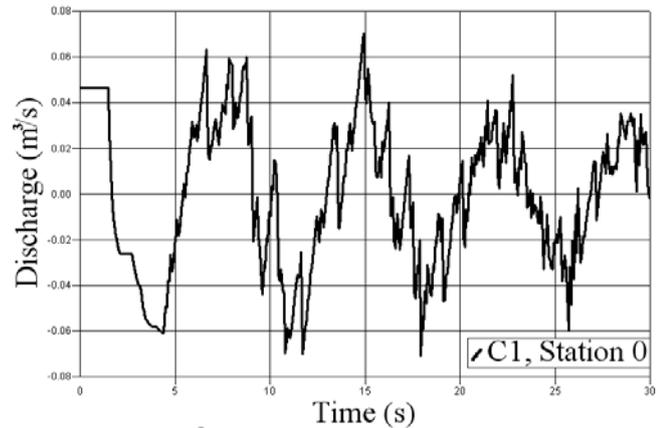


Fig. 5. Evolution of Q at the reservoir.

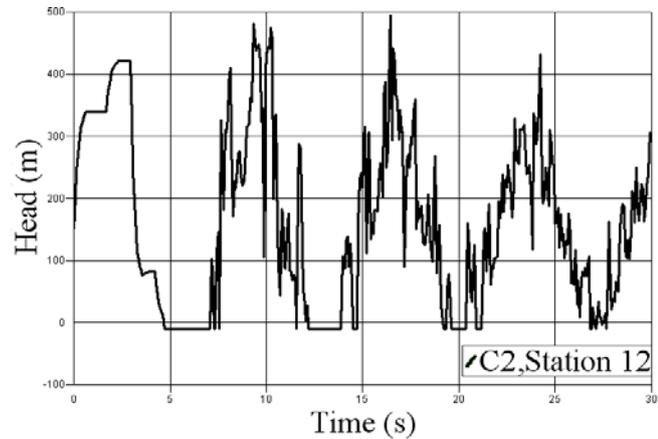


Fig. 6. Evolution of H at the valve.

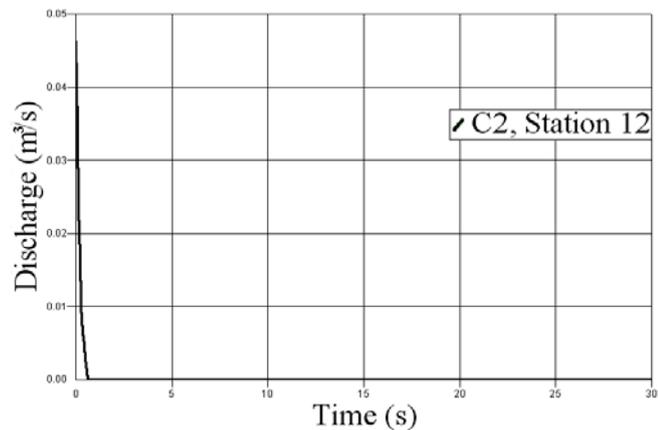


Fig. 7. Evolution of Q at the valve.

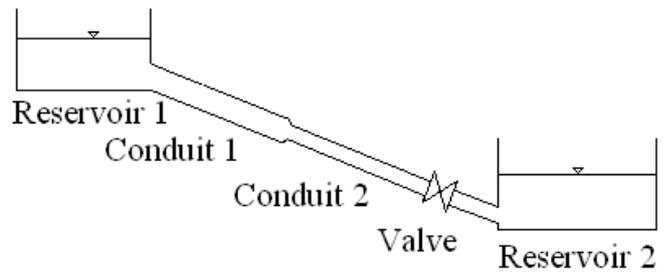


Fig. 8. Case 2: a convergent.

Using the WHAMO program

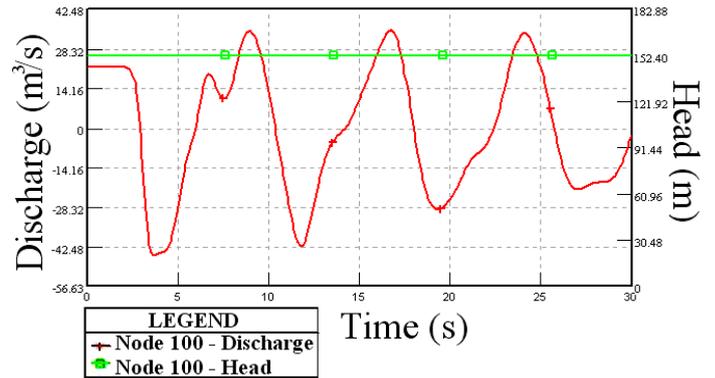


Fig. 9. Evolution of Q and H at the reservoir.

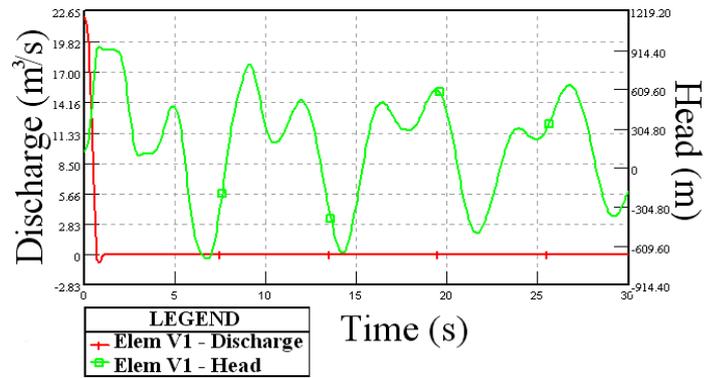


Fig. 10. Evolution of Q and H at the valve.

Using the software AFT Impulse

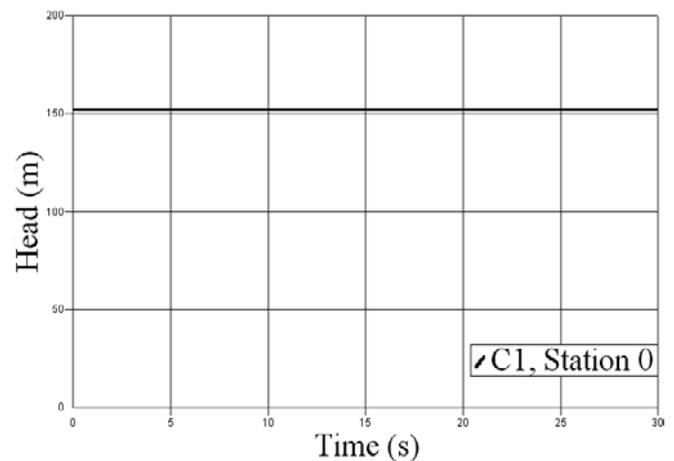


Fig. 11. Evolution of H at the reservoir.

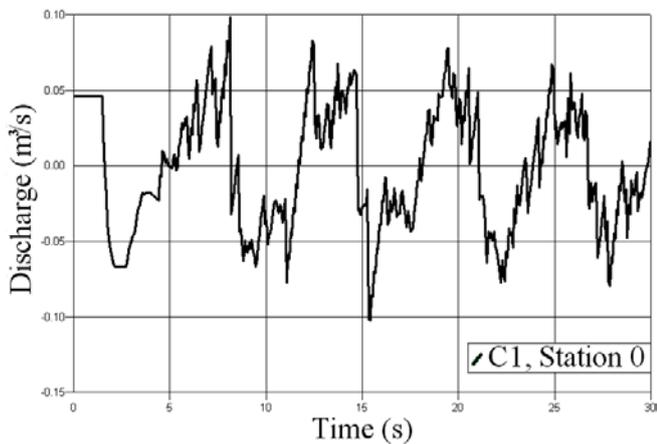


Fig. 12. Evolution of Q at the reservoir.

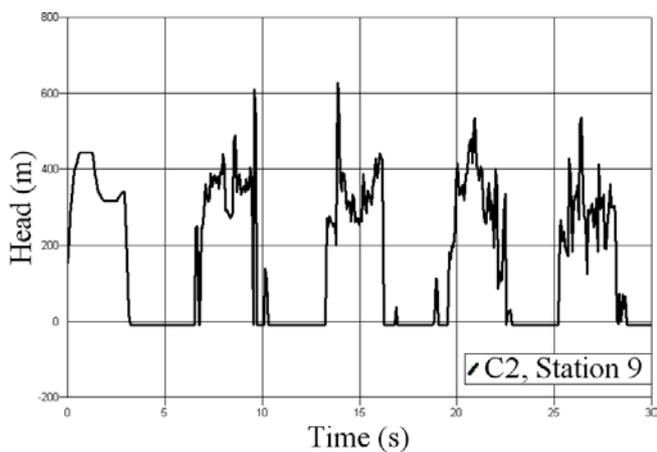


Fig. 13. Evolution of H at the valve.

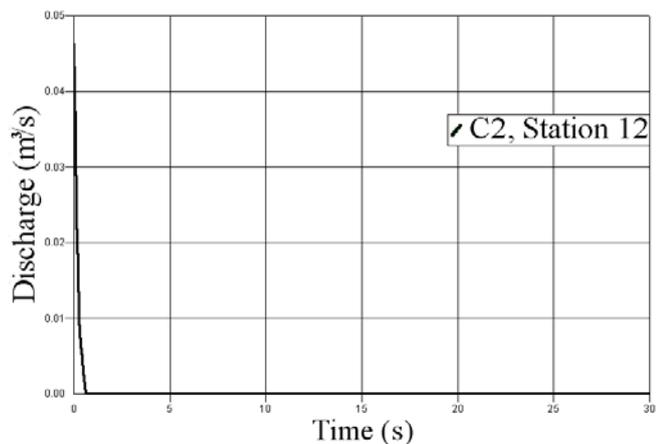


Fig. 14. Evolution of Q at the valve.

Model study 2

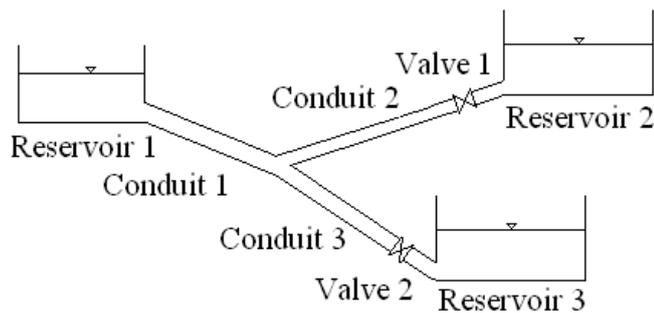


Fig. 15. Schematic of the second model.

The closing time of the valves for synchronized closure  $T_1 = T_2 = 0.6s$ , for unsynchronized closure  $T_1 = 0.6s$  and  $T_2 = 3s$ .

Case of the synchronized closure of the valves (WHAMO program)

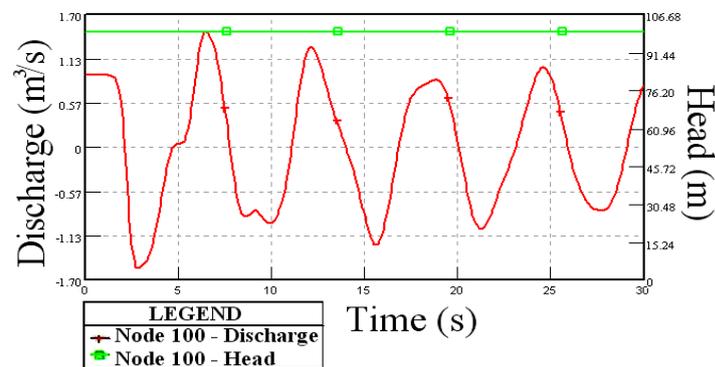


Fig. 16. Evolution of Q and H at the reservoir.

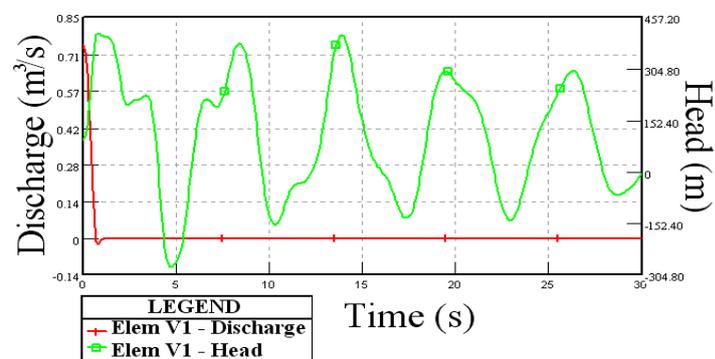


Fig. 17. Evolution of Q and H at the valve1.

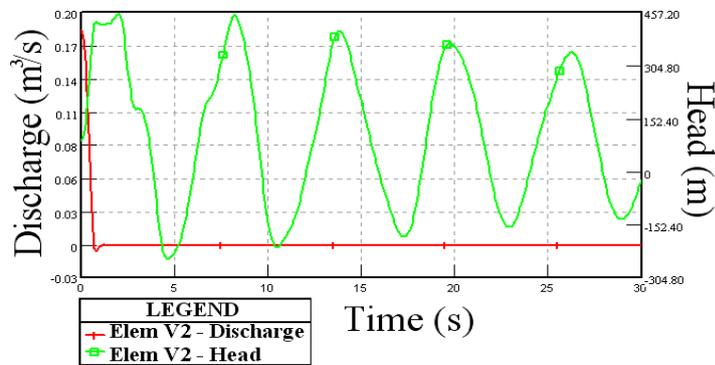


Fig. 18. Evolution of Q and H at the valve2.

Case of the unsynchronized closure of the valves (WHAMO program)

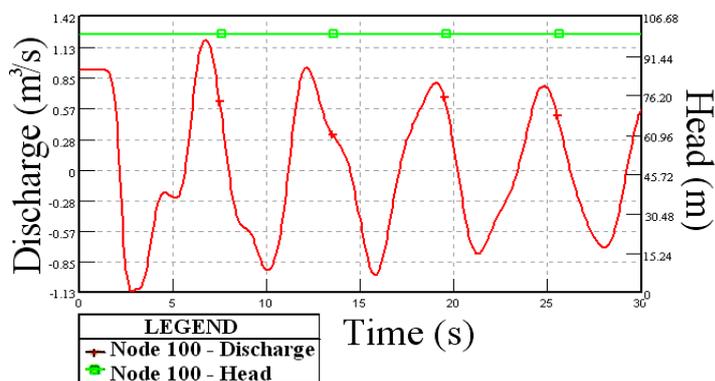


Fig. 19. Evolution of Q and H at the reservoir.

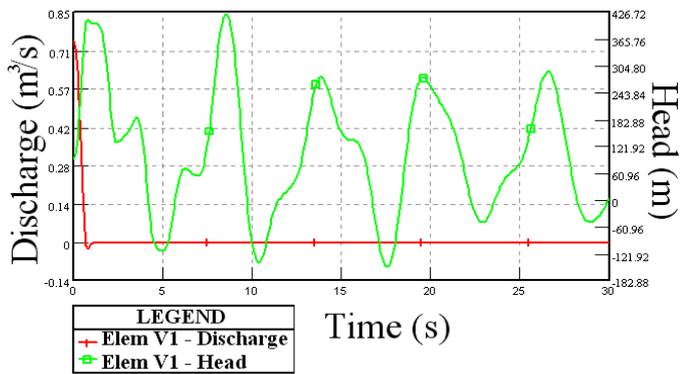


Fig. 20. Evolution of Q and H at the valve1.

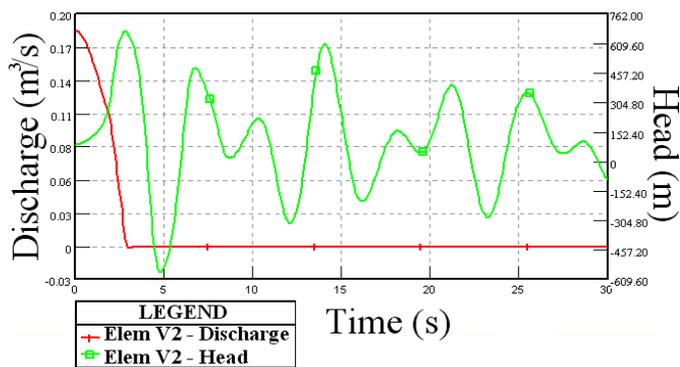


Fig. 21. Evolution of Q and H at the valve2.

*Case of the synchronized closure of the valves (with the software AFT Impulse)*

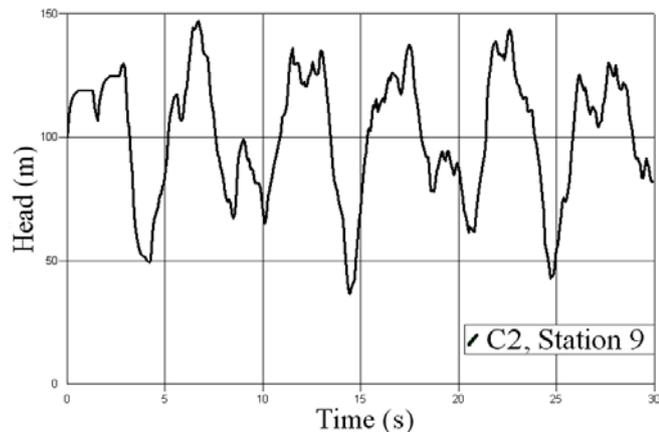


Fig. 22. Evolution of H at the valve1.

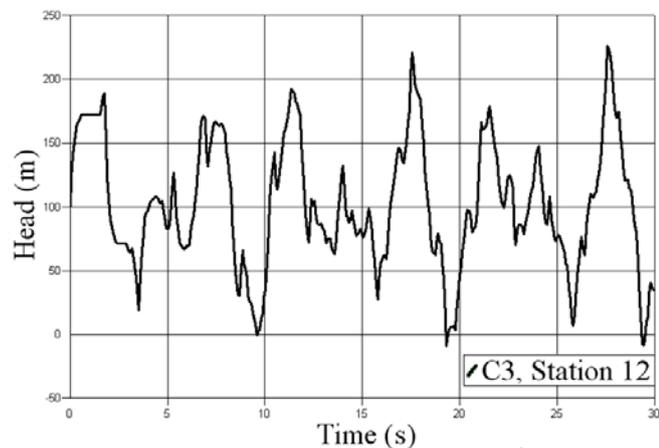


Fig. 23. Evolution of H at the valve 2.

*Case of the unsynchronized closure of the valves (with the software AFT Impulse)*

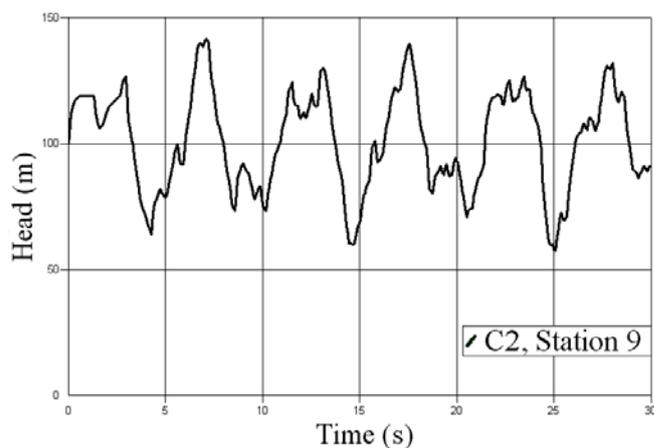


Fig. 24. Evolution of H at the valve1.

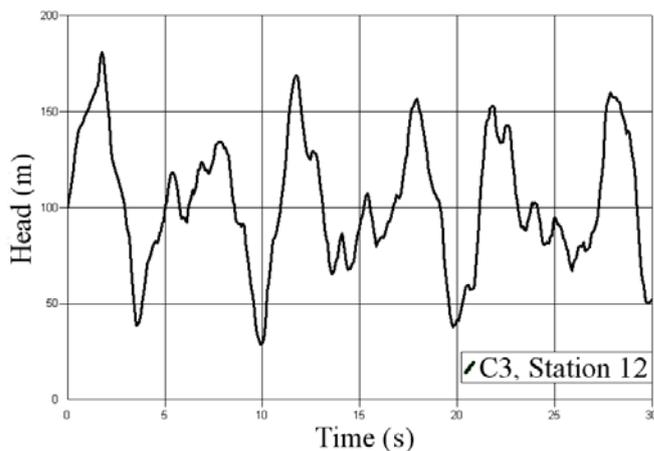


Fig. 25. Evolution of H at the valve 2.

## VI. INTERPRETATIONS OF RESULTS

*A. The difference between the divergence and the convergence*

The changes in maneuvers of the valve acting on the discharge fluctuations are very important in the case of convergence more than in the case of divergence;

The changes of head fluctuations are also very important in the case of convergence more than in the case of divergence.

*B. The difference between the synchronized and the unsynchronized closure of the valves*

Between the synchronized and the unsynchronized closure there is a difference in their influence on the fluctuations of the head and discharge over time in different points of our model, in the reservoir to the bifurcation; fluctuations of head and discharge reach maximum and minimum values in the case of the synchronized closure, across the valves can be said that the amplitude of the head is important in the case of the unsynchronized closure (Fig. 26 and Fig. 27) and finally between bifurcation and

valves; fluctuations of head and discharge vary between synchronized and unsynchronized closure.

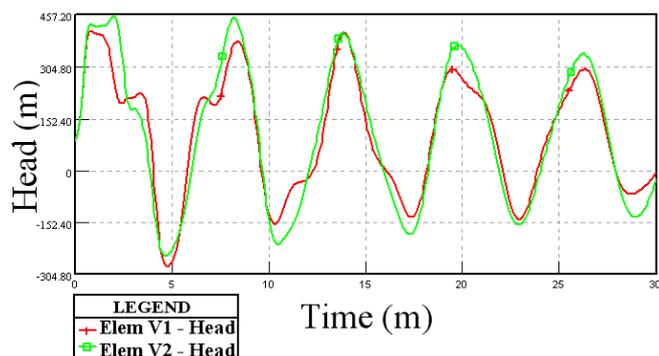


Fig. 26. Evolution of H in both valves in the synchronized closure.

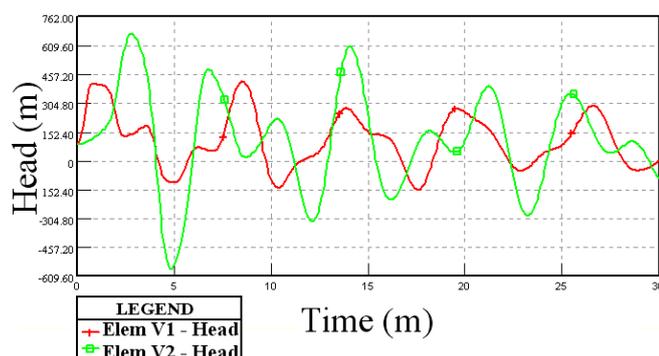


Fig. 27. Evolution of H in both valves in the unsynchronized closure.

## VII. CONCLUSION

In the gravity adduction pipes the influence of the propagation of elastic waves or the water hammer is greater in the case of pipes has variable characteristics (section changes with a divergence, a convergence or a bifurcation). These characteristics must be taken into account in practical cases since they cause rapid changes in the pressure head and discharge of initial flow in the hydraulic system passing from the steady case to the transient causing considerable damage (bursting of pipes, destruction of hydroelectric equipments like the waterproof joints, the valves, the turbines or the pumps).

This study was therefore particularly interested in these types of conduit with reference to hydraulic system in order to understand the transient phenomena that take place following the sudden closure of valves.

So for a hydraulic installation we must replace pipes with a convergence by other with a divergence and for existence of bifurcation we must use the synchronized closures of the valves rather than unsynchronized closures.

Knowing that we can't neutralize completely or permanently the problem of water hammer but we can reduce the devastating effects.

In addition it was based on the case of monophasic unidimensional flow with temperature and density of the liquid const.

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