Pure Bending Determination of Stress-Strain Curves for an Aluminum Alloy

D. Torres-Franco, G. Urriolagoitia-Sosa, G. Urriolagoitia-Calderón,
L. H. Hernández-Gómez, A. Molina-Ballinas, C. Torres-Torres, B. Romero-Ángeles

Abstract. - **The uniaxial tensile or compressive test provides a fundamental indication of strength and stiffness properties for a given material. The primary concept to characterize a material by axial testing is changing. Currently, there is a pure bending experimental method by which it is possible to obtain simultaneously tensile and compressive stress-strain curves. In this paper, it is presented a pure bending method to determine the stress-strain behavior of a material. Results were obtained on final manufactured specimens from Aluminum alloy 6063-T5. The advantages of this method are related mainly with its versatility and easy implementation. Besides, it demonstrates feasibility to determine previous loading history in a component. Additionally it does not need extensive mechanical preparation of specimens. The method presented is based on calculations from the value of the bending moment and the corresponding value of the strain produced in the upper and lower surfaces of the specimen. Bending experimental results were compared against axial testing. Results and the affinity derived from the balance in stress-strain curves are satisfactory.**

Index Terms—**Analytical methods, pure bending test and stress-strain curves**

I. Introduction

THE stress-strain curves are the best way to describe the mechanical behavior of materials. The most significant property of a solid is its mechanical stability. In other words the material resistance is strongly related with shape and size, by the application of an external agent [1].

The mechanical properties describe how a material is affected by internal as well as external agents, including forces in tension, compression, impact, fatigue, etc [2].

The mechanical properties are very important for the selection of materials with requirements that arise from a particular use. Very suitable materials for certain applications can be completely useless for others jobs. Therefore, before selecting a material, it is important to consider what will be expected from the component at service. So, it is important to determine which mechanical properties are required at service, so that the material can present an appropriate response [3].

The materials may be characterized through tests that are standardized. Axial testing to obtain the mechanical properties usually can be referred as tension and compression tests. However, in the engineering field it has been opted for a different alternative in order to determine the material behavior. In this sense, bending tests provide multiple benefits; three of the most important aspects of pure bending tests are (1) the simultaneous determination of the tensile and compressive curves; (2) the magnitude and effect of prior loading history into the material, and finally; (3) it is virtually unnecessary to produce a specific or standardized specimens to perform the test [2], [4].

The use of a bending test to determine mechanical properties was investigated for the first time by *Herbert* in 1910, who published a study showing results in terms of tensile and compressive stress strain curves of a cast Iron material [5]. A few years later, similar experimental techniques were performed by *Nadai* [6] and *Marin* [3].

By 1981, *Laws* [7] published a research on a method by which he calculated stress-strain curves through the data of a bending test for a composite material. Results were compared against those obtained from an axial tensile test. One year later *Mayville* and *Finnie* [8] conducted a study on a bending method. They found the stress strain tensile and compressive behavior for three different materials that were previously pre-strained. In recent years, *Urriolagoitia-Sosa et al*. [9] published a new bending method. It is proposed an equation to obtain simultaneously the tensile and compressive behavior in beams with strain hardening and Bauschinger effect. This procedure allows to determine the effect on the induced residual stress field. Additionally, *Schajer* and *An* [1] developed an inverse calculation for the simultaneous determination of stress strain curves from bending test data. They used the inverse calculation to reduce the possible error percentage found by other researchers.

This paper presents an analytical method that is based on the bending test for providing stress strain curves related with both directions of loading (tension and compression). The components tested are pieces directly used by the automotive industry in Mexico. The results (stress-strain relationship) obtained through this new analytical approach are compared with experimental axial tensile data and compression test evidence, which validate the proposed method.

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D. Torres-Franco, G. Urriolagoitia-Sosa, G. Urriolagoitia-Calderon, L.H. Hernández-Gómez, A. Molina-Ballinas, C. Torres-Torres and B. Romero-Ángeles are with the National Polytechnic Institute in Section of Postgraduate Studies and Research, School of Mechanical and Electrical Engineering, building 5, 2nd floor, Adolfo Lopez Mateos Professional Unit “Zacatenco”, Col. Lindavista, C.P. 07738, Mexico D.F., Mexico, phone: 5729-6000; Ext.: 54740 (e-mail: david_torres20@ hotmail.com, guiurri@hotmail.com, urrio332@hotmail.com, luishector56@hotmail.com, yased_83@hotmail.com).

II. METHODOLOGY

A. *Theoretical Details* [9]

This new proposed method is based on the assumption that the tensile and compressive stress strain curves of a material have an asymmetric distribution. Furthermore, it is considered that the specimen has a cross section with wide *b* and height *h*, which is subjected to pure bending moment. Also, the cross section plane remains flat before and after the bending [10]. The corresponding strains and stresses at the top and bottom surfaces of the specimens are given by ε_t, ε_c, σ_t and σ_c respectively. The instantaneous location of the neutral axis of the beam for the hypothetical state is assumed to be at a distance h_t and h_c from the tensile and compression surfaces. The total depth *h*, is equal to the sum of h_t and h_c. A linear strain distribution across the depth of the specimen is considered following *Timoshenko´s* assumption. Under pure bending condition, the neutral axis is located by imposing the equilibrium condition that the total axial force vanishes. The second condition required for the solution is the equilibrium of internal and external moments.

Let the strain range ε_t, be described by a series of *m* strain increments $\Delta\varepsilon_{tj}$, *j* = 1 , *m* and let the corresponding matching strain increment for ε_c be $\Delta\varepsilon_{cj}$. The strains and strain increments for given moment levels are obtained from test data. At any given instance of loading, and considering axial equilibrium and areas under the stress strain curve on the tensile and compressive sides, the equation for the total number of increments is [11]:

$$\sum_{j=1}^{m} \sigma_{tj}\, \Delta\varepsilon_{tj} = \sum_{j=1}^{m} \sigma_{cj}\, \Delta\varepsilon_{cj} \tag{1}$$

It is possible to write the equation for equilibrium of internal and external moments as:

$$M_m = \frac{bh_{tm}^2}{\varepsilon_{tm}^2}\sum_{j=1}^{m}\sigma_{tj}\varepsilon_{tj}\Delta\varepsilon_{tj} + \frac{bh_{cm}^2}{\varepsilon_{cm}^2}\sum_{j=1}^{m}\sigma_{cj}\varepsilon_{cj}\Delta\varepsilon_{cj} \tag{2}$$

From the linearity of strain distribution across the depth it follows that:

$$\frac{bh_t^2}{\varepsilon_t^2} = \frac{bh_c^2}{\varepsilon_c^2} = \frac{bh^2}{\left(\varepsilon_t + \varepsilon_c\right)^2} \tag{3}$$

Therefore (2) can be rewritten as:

$$M_m = \frac{bh^2}{(\varepsilon_{tm} + \varepsilon_{cm})^2}\sum_{j=1}^{m}\left[\sigma_{cj}\varepsilon_{cj}\Delta\varepsilon_{cj} + \sigma_{tj}\varepsilon_{tj}\Delta\varepsilon_{tj}\right] \tag{4}$$

From (1), it can be deduced that for each increment *j*

$$\sigma_{tj}\Delta\varepsilon_{tj} = \sigma_{cj}\Delta\varepsilon_{cj} \tag{5}$$

This implies equality of total strain energy stored or dissipated on the tensile side as on the compressive sides over the increment of *j*. Therefore (4) can be written as:

$$M_m = \frac{bh^2}{(\varepsilon_{tm} + \varepsilon_{cm})^2}\sum_{j=1}^{m}\left[\sigma_{tj}\Delta\varepsilon_{tj}(\varepsilon_{cj} + \varepsilon_{tj})\right] \tag{6}$$

Assuming that the stresses and strain at increments 1 to *k*-1 have been determined, the tensile stress at any increment *k* can be obtained from (6) as:

$$\sigma_{kt} = \left[\overline{M}_k - \sum_{j=1}^{k-1}\bar{\varepsilon}_j\sigma_{tj}\Delta\varepsilon_{tj}\right] / \left[\,\bar{\varepsilon}_k\Delta\varepsilon_{kt}\right] \tag{7}$$

where,

$$\overline{M}_k = M_k \;/\left[\frac{bh^2}{(\varepsilon_{tm} + \varepsilon_{cm})^2}\right] \tag{8}$$

and $\bar{\varepsilon}_k = (\varepsilon_{ck} + \varepsilon_{tk})$. Alternatively we can write it as;

$$\overline{M}_m = \sum_{j=1}^{m}\left[\sigma_{tj}\Delta\varepsilon_{tj}(\varepsilon_{cj} + \varepsilon_{tj})\right] \tag{9}$$

and by considering any two consecutive increments *k* - 1 and *k*, and generalizing for tensile and compression stresses, we can write (8) as;

$$\sigma_{\alpha k} = \frac{\overline{M}_k - \overline{M}_{k-1}}{\bar{\varepsilon}_k\Delta\varepsilon_{\alpha k}} \tag{10}$$

where $\alpha = t$ for tension stress and $\alpha = c$ for compression stress. The (9) gives the average stresses at any increment *k* = 1….*m*. The tension and compression stresses in this work were obtained using (10) [12] – [13].

B. *Experimental Procedure and Material*

The material used for the experiments was an alloy of Aluminum 6063-T5 [14]. Eight specimens of square cross section (12.7 mm x 12.7 mm) with a total length of 250 mm were used. Bending tests were developed on a four points configuration [10]. The specimen is part of a bar used as lock lever for car´s door and it was decided to simplify the problem by transforming the geometry of the component into a beam.

A heat treatment procedure of annealing was used to free any previous load history into the specimens. The specimens were placed into an oven and heated at 250° c for 25 minutes. Finally, the samples were slowly cooled down inside the oven.

All the specimens were instrumented with strain gauges (EA-06-060LZ-120 [15]) at the top and bottom surfaces (Figure 1). The strain gauges are extremely helpful to obtain the strain data caused by the application of the moment into the specimen. The bending moment produces a simultaneous tensile and compressive deformation into the beam. The experimental bending procedure was performed in order to collect information about deformations and to calculate the stress to characterize the material. The best manner to produce a pure bending moment by an experimental test is by the use of a four point bending rig (Figure 2).

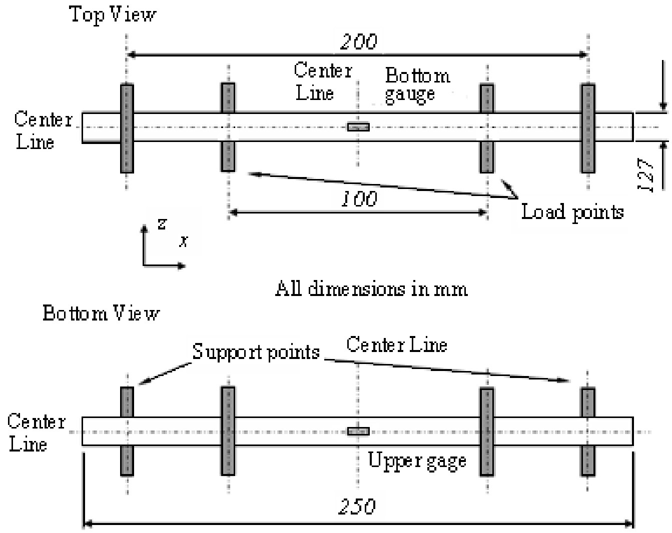

Fig. 1. The position of the gauges on the beam in Aluminum alloy.

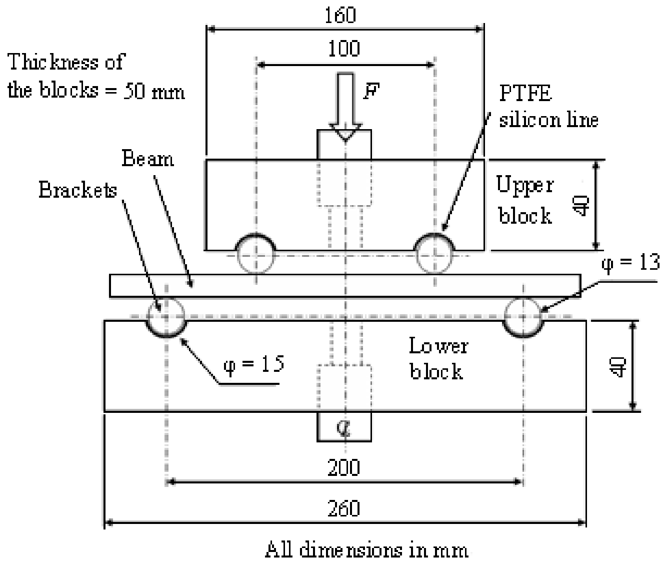

Fig. 2. System of four points of loading

In order to characterize the material, the theoretical development for the bending method was based on the assumption that a pure bending moment configuration will not produce shear stress into the specimen and the bending moment will be always constant at the central section of the beam. These considerations allow describe tensile and compressive stress properties from strain data.

Figure 2, shows the configuration of the four-point bending rig and the set up of the specimen. It is important to stand out that the four point rig has several circular blocks made from steel, which are positioned in a semicircle indentation produced in the main blocks of the rig. The principal purpose of these cylinders is to allow free bending movement of the specimen, so no pulling will be produced at the time to apply the bending moment. Additionally, in order to reduce the possible friction between the elements, a strip of silicone *PTFE*, 2 mm thick was added between the semicircle indentation and the cylindrical block.

C. Results and discussions

Before bending tests were carried out, 3 tensile and 3 compressive specimens were prepared from the same batch of 6063-T5 aluminum alloy material. The specimens were annealed, and tests were performed by following standards [16] – [17]. The average tensile result obtained from a tensile test in a form of a stress strain curve is illustrated in Figure 3. It is important to state, that because the material was annealed, its behavior was similar in tension as in compression, so it was decided do not present the compressive stress strain curve.

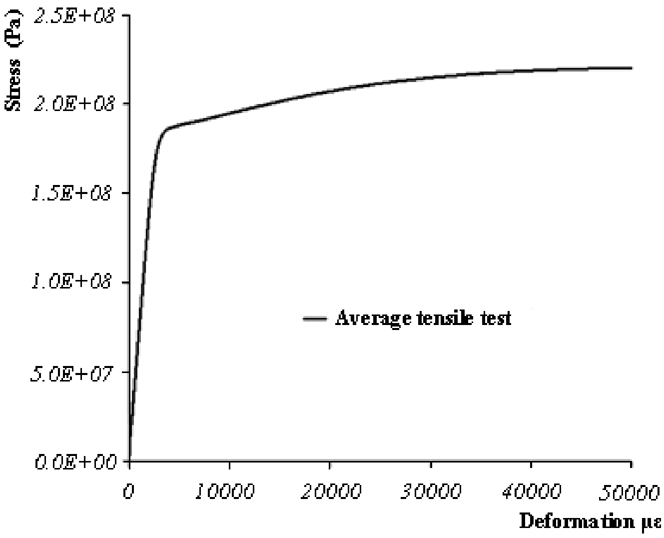

Fig. 3. Tensile results for an annealed Aluminum alloy specimens

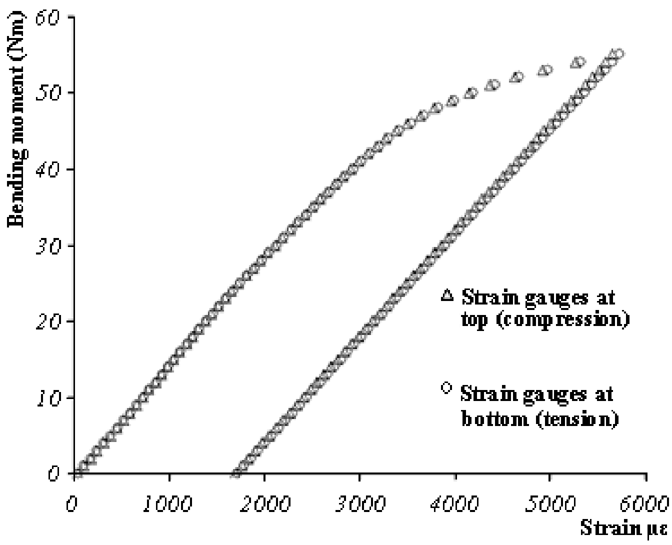

Fig. 4. Four point bending procedure, strain data against moment

The bending experimental procedure was performed in a hydraulic axial machine. For each test a bending moment of 55 Nm was applied. The strain data obtained by the moment effect from the bending procedure presented in this research can be observed in Figure 4.

From the strain data collected in the bending test, it can be observed that the strain data from the top and bottom surfaces are almost identical. From this observation it can be concluded, that the bending procedure was performed correctly. The annealing process erased the stress strain prior loading history in the material and that the stress strain curve of the component is virtually identical in tension and compression. It is important to mention that the results of these cases are the average of five bending tests.

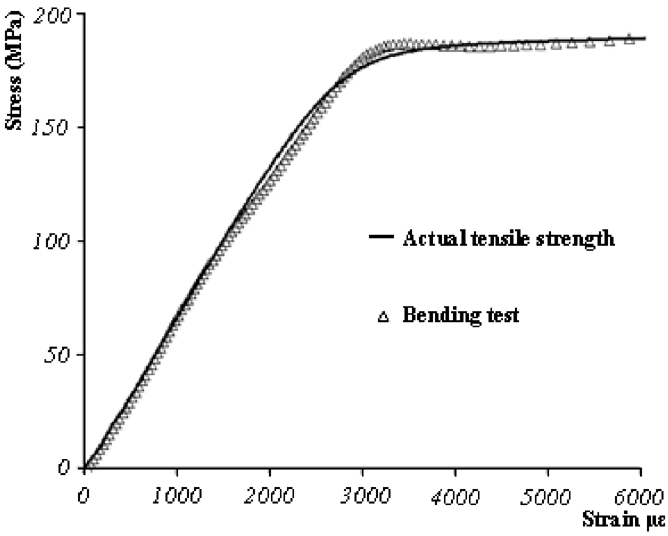

Fig. 5. Comparison of stress strain results for a 6063-T5 Aluminum alloy

Figure 5 presents the results of both tests (axial tension and four point bending), showing the relation between the stress-strain curves using the bending method presented in this paper against the axial tensile test. It is clear that the results are almost identical between both methods. However, there is a small difference in the yield region between stress-strain curves, being slightly greater the yield strength found by the bending procedure. *Urriolagoitias-Sosa et. al.* [2], [12] have explained this effect as a consequence of the abrupt change in the stress strain behavior and the mathematical procedure used by the equation system applied for this case. Nevertheless, it is clear that the bending procedure has provided an efficient methodology to characterize the material, and several mechanical properties can be obtained as the axial tensile test.

Bending test is considered as a new method to determine the mechanical properties in a material, but it has not been normalized yet. However, it is a test that provides some advantages and it has not been properly exploited, because most of laboratories engaged in mechanical testing consider that the tensile test is the most efficient and correct one, which in most of the circumstances this is not so.

Nonetheless, this new bending method can get simultaneously the properties of a material in tension and compression. Also it can be used to determine possible prior loading history (homogenous loading, strain hardening and Bauschinger effect) and non homogeneous loading (residual stresses) in both directions for determining anisotropic behavior. In this sense, both tensile and compressive axial testing ought to be performed to determine any prior loading history, but in most cases the cause cannot be determined, because the preparation of the normalized specimens will add a loading history to the component.

The new analytical method for determination of the simultaneous behavior in tension and compression by a stress-strain curve can reduce the number of tests to be made to fully characterize the material; moreover resources could be saved by the application of the bending procedure.

For the bending procedure, it is also necessary to perform at least three testing procedures for the characterization of the material. In any case, the entire tensile and compressive measurements must be performed. Additional specimens that have a flat geometry can be tested without adding any manufacturing process.

III. Conclusion

An analytical method for calculation tension and compression stress-strain curves exhibited during the characterization of annealed material bending is presented. The resulting stress-strain curves obtained from the bending method applied to beam specimens closely agree when they are compared against the axial experimental procedure for all cases. This means that the evaluation of the bending method with the proposed conditions was generally correctly. This method can be used in any case where the behavior of the stress-strain relation is an important issue. It is worth remarking that this technique could also be applied to a material with previous history. However, anticlastic effects could be a problem, since it is not possible to determine the ultimate stress with the approach of the bending test. The ultimate stress may be easily achieved in an axial test, due to the action of the load, where the specimen starts to be narrowed, so this can be measured directly. It would be interesting to found how could be determined the ultimate resistance by using the bending approach.

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