# Designing of Sixlink Mechanism Schemes with the Changeable Contour Taking into Account Forces Transfer

Y.S.Temirbekov, S. Baimukhanov

*Abstract* — Movement transfer for special positions of four link mechanism was estimated by absolute value of a sine of the angle of transfer. The similar approach was also applied in case of flat six link mechanism of II class, and in general, multilink mechanisms of II class. However, transfer corners are insufficiently for six link mechanisms with the changeable closed contour for an estimation of movement transfer ability. Here indicators for an estimation of movement transfer ability and forces of six link mechanism with the changeable closed contour are received.

*Index Terms* — Designing, the mechanism, transfer of forces, transfer of movement, criterion, the kinematics, the inverted movement.

## I. INTRODUCTION

Movement transfer for special positions of four link mechanism was estimated by absolute value of a sine of the angle of transfer. The similar approach was also applied in case of flat six link mechanism of II class, and in general, multilink mechanisms of II class. However, transfer corners are insufficiently for six link mechanisms with the changeable closed contour for an estimation of movement transfer ability. Here indicators for an estimation of movement transfer ability and forces of six link mechanism with the changeable closed contour are received.

#### II. MODEL MECHANISMS

Let's consider six link mechanisms with a changeable contour (fig. 1), its closedness equations are:

$$\begin{cases} \overline{L}_{1} + \overline{L}_{2} + \overline{L}_{3} + \overline{L}_{CP} + \overline{r} = \overline{L}_{0} \\ \overline{L}_{1} + \overline{L}_{2A} + \overline{L}_{4} + \overline{L}_{EP} + \overline{r} = \overline{L}_{0} \end{cases}$$
(1)



Fig.1 Sixlink mechanism

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If we tale the leading link as 1, then the projections (1) on the axis OXY shall look like:

If we give an input parameter " $\varphi_1$  " a variation  $\delta \varphi_1$ , we shall obtain:

$$\begin{cases} L_2 \cos \varphi_2 + L_3 \cos \varphi_3 + r = -L_1 \cos \varphi_1 + L_{0_x} - L_{(CP)_x} \\ L_2 \sin \varphi_2 + L_3 \sin \varphi_3 = -L_1 \sin \varphi_1 + L_{0_y} - L_{(CP)_y} \\ L_{2A} \cos \varphi_{2A} + L_4 \cos \varphi_4 + r = -L_1 \cos \varphi_1 + L_{0_x} - L_{(EP)_x} \\ L_{2A} \sin \varphi_{2A} + L_4 \sin \varphi_4 = -L_1 \sin \varphi_1 + L_{0_y} - L_{(EP)_y} \end{cases}$$

When the determinant isn't equal to zero, the value of small variations of angular displacement  $\delta \varphi_2$ ,  $\delta \varphi_3$ ,  $\delta \varphi_4$ ,  $\delta r$  is defined by final values. It is possible to calculate values  $\delta \varphi_2$ ,  $\delta \varphi_3$ ,  $\delta \varphi_4$ ,  $\delta r$  in the case when the absolute value of the determinant is sufficiently great compared with the order  $O(\delta \varphi_1)$  of the variation  $\delta \varphi_1$  value, but when this value is less than  $O(\delta \varphi_1)$ , it is necessary to apply calculation formulas with the variables above the 2nd order of vanishing. In such position even small change of the input variable " $\varphi_1$ " leads to significant angular displacement and consequently reduces the movement transfer ability.

Let's consider the mechanism where the value is  $\Delta = 0$ , in this case:

$$L_{3}L_{4}[L_{2}\sin(\varphi_{2}-\varphi_{3})\cos\varphi_{4}-L_{2A}\sin(\varphi_{2A}-\varphi_{4})\cos\varphi_{3}]=0$$
(2)

Let's assume that links are rigid and weightless and the mechanism is loaded with driving moment  $M_D$  and drag force Fc. Lets split the mechanism into separate links, then the balance conditions shall look as (Fig.2):

Link 1, 2: 
$$R_{01}^x + R_{21}^x = 0$$
,  $R_{01}^y + R_{21}^y = 0$ ,  
 $-R_{21}^x L_1 \sin \varphi_1 + R_{21}^y L_1 \cos \varphi_1 + M_D = 0$   
 $R_{12}^x + R_{32}^x + R_{42}^x = 0$ ,  $R_{12}^y + R_{32}^y + R_{42}^y = 0$ ,  $-R_{32}^x L_2 \sin \varphi_2 + 1$ 

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$$+R_{32}^{y}L_{2}\cos\varphi_{2}-R_{42}^{x}L_{2A}\sin\varphi_{2A}+R_{42}^{y}L_{2A}\cos\varphi_{2A}=0$$

Link 3:  $R_{23}^x + R_{53}^x = 0$ ,  $R_{23}^y + R_{53}^y = 0$ ,  $-R_{53}^x L_3 \sin \varphi_3 + R_{53}^y L_3 \cos \varphi_3 = 0$ 

Link 4, 5: 
$$R_{24}^{x} + R_{54}^{x} = 0$$
,  $R_{24}^{y} + R_{54}^{y} = 0$ ,  
 $-R_{54}^{x}L_{4}\sin\varphi_{4} + R_{54}^{y}L_{4}\cos\varphi_{4} = 0$   
 $R_{35}^{x} + R_{45}^{x} + F_{c} = 0$ ,  $R_{35}^{y} + R_{45}^{y} + N_{p} = 0$ ,  $-R_{35}^{x}L_{5}\sin\varphi_{5} + R_{35}^{y}L_{5}\cos\varphi_{5} - R_{45}^{x}L_{5A}\sin\varphi_{5A} + R_{45}^{y}L_{5A}\cos\varphi_{5A} + M_{p} = 0$ 

With provision that  $R_{ij}^x = -R_{ji}^x$  is  $R_{ij}^y = -R_{ji}^y$ (*i*, *j* = 1,..., 5), we have:  $R_{23}^x = -R_{53}^x$ ,  $R_{23}^y = -R_{53}^y$ ,  $R_{24}^x = -R_{54}^x$ ,  $R_{24}^y = -R_{54}^y$ .

Then the closed six equation system shall look as follows:

$$\begin{aligned} R_{23}^{x}L_{2}\sin\varphi_{2} - R_{23}^{y}L_{2}\cos\varphi_{2} + R_{24}^{x}L_{2A}\sin\varphi_{2A} - \\ - R_{24}^{y}L_{2A}\cos\varphi_{2A} &= 0, \\ R_{23}^{x}L_{3}\sin\varphi_{3} - R_{23}^{y}L_{3}\cos\varphi_{3} &= 0, \\ R_{24}^{x}L_{4}\sin\varphi_{4} - R_{24}^{y}L_{4}\cos\varphi_{4} &= 0, \\ - R_{23}^{x}L_{5}\sin\varphi_{5} + R_{23}^{y}L_{5}\cos\varphi_{5} + R_{24}^{x}L_{5A}\sin\varphi_{5A} - \\ - R_{24}^{y}L_{5A}\cos\varphi_{5} &= 0, \\ R_{23}^{x} + R_{24}^{x} + F_{c} &= 0, R_{23}^{y} + R_{24}^{y} + N_{p} &= 0 \end{aligned}$$
(3)

Let's enter normal and tangential components of reaction forces in B and D. Let's consider equilibrium condition of units B and D:

$$R_{24}^{x} + R_{B}^{n} \cos \varphi_{4} + R_{B}^{\tau} \cos(\varphi_{4} + 90^{\circ}) = 0 \quad or$$

$$R_{24}^{x} = -R_{B}^{n} \cos \varphi_{4} + R_{B}^{\tau} \sin \varphi_{4} \qquad (4)$$

$$R_{24}^{y} + R_{B}^{n} \sin \varphi_{4} + R_{B}^{\tau} \sin(\varphi_{4} + 90^{\circ}) = 0 \quad or$$

$$R_{24}^{y} = -R_{B}^{n} \sin \varphi_{4} + R_{B}^{\tau} \cos \varphi_{4}$$

$$R_{23}^{x} + R_{D}^{n} \cos \varphi_{3} + R_{D}^{r} \cos(\varphi_{3} + 90^{\circ}) = 0 \quad or$$

$$R_{23}^{x} = -R_{D}^{n} \cos \varphi_{3} + R_{D}^{r} \sin \varphi_{3}$$

$$R_{23}^{y} + R_{D}^{n} \sin \varphi_{3} + R_{D}^{r} \sin(\varphi_{3} + 90^{\circ}) = 0 \quad or$$

$$R_{23}^{y} = -R_{D}^{n} \sin \varphi_{3} + R_{D}^{r} \cos \varphi_{3}$$
(5)

Let's consider balance of tangential reactions of forces, affecting links 3 and 4 and force moments with respect to hinges C and E accordingly:

$$R_B^{\tau} + R_C^{\tau} = 0, \ R_D^{\tau} + R_E^{\tau} = 0, \ R_B^{\tau} L_4 = 0, \ R_D^{\tau} L_3 = 0,$$
  
Whence it follows, that  $R_B^{\tau} = R_C^{\tau} = 0, \ R_D^{\tau} = R_E^{\tau} = 0$   
and therefore from (4) and (5) we shall obtain:

$$R_{23}^{x} = -R_{D}^{n}\cos\varphi_{3} \qquad R_{24}^{x} = -R_{B}^{n}\cos\varphi_{4} \qquad (6)$$
$$R_{23}^{y} = -R_{D}^{n}\sin\varphi_{3} \qquad R_{24}^{y} = -R_{B}^{n}\cos\varphi_{4}$$

After substitution in (3) of expressions (6), we obtain:

$$\begin{bmatrix} L_{2A}\sin(\varphi_4 - \varphi_{2A}) & L_2\sin(\varphi_3 - \varphi_2) & 0 & 0 \\ L_{5A}\sin(\varphi_4 - \varphi_{5A}) & L_5\sin(\varphi_5 - \varphi_3) & 0 & 1 \\ -\cos\varphi_4 & -\cos\varphi_3 & 0 & 0 \\ -\sin\varphi_4 & -\sin\varphi_3 & 1 & 0 \end{bmatrix} * \begin{bmatrix} R_B^n \\ R_D^n \\ M_P \\ N_P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -F_C \\ 0 \end{bmatrix} (7)$$



Fig.2 Mechanism on separate links

# III. MATHEMATICAL MODEL

Let's consider the positions of the mechanism, where:  $\Delta$  is equal to zero:

$$\Delta = -L_2 \sin(\varphi_3 - \varphi_2) \cos\varphi_4 + L_{2A} \sin(\varphi_4 - \varphi_{2A}) \cos\varphi_3$$
(8)

Comparison of (2) and (8) shows that they agree within the factor  $L_3L_4$ . Thus, the positions where the current response of the kinematic pairs becomes significant coincide with the special positions of the mechanism, that is, the positions where the change of output parameters resulting from negligibly small changes in the entrance angle becomes extremely large. Let's determine the relationship between driving torque and useful resistance force by the kinematical

parameters of the mechanism. 
$$R_B^n$$
,  $R_D^n$  are equal:  
 $R_B^n = \frac{-L_2 \sin(\varphi_3 - \varphi_2)F_c}{\Delta}$  and  $R_D^n = \frac{-L_{2A} \sin(\varphi_4 - \varphi_{2A})F_c}{\Delta}$ .

From the equilibrium equations we see that:

$$R_{12}^{x} = -R_{D}^{n}\cos\varphi_{3} - R_{B}^{n}\cos\varphi_{4}, R_{12}^{y} = -R_{D}^{n}\sin\varphi_{3} - R_{B}^{n}\sin\varphi_{4},$$

then we will deduce the relator connecting  $M_D \mu F_c$ 

$$M_{D} = \frac{-L_{2}\sin(\varphi_{3}-\varphi_{2})\sin(\varphi_{1}-\varphi_{4}) + L_{2A}\sin(\varphi_{4}-\varphi_{2A})\sin(\varphi_{1}-\varphi_{3})}{L_{2}\sin(\varphi_{3}-\varphi_{2})\cos\varphi_{4} - L_{2A}\sin(\varphi_{4}-\varphi_{2A})\cos\varphi_{3}} * L_{1}F_{2}$$
(9)

In the areas close to special positions negligibly small changes in the entrance angle result in significant increase of constraint reactions and this complicates to maintain the movement of the mechanism within theoretical accuracy. When the mechanism reaches the special position, the determinant  $\Delta$  becomes equal to zero. Let's normalize the functional part of the determinant and name it as prescribed motion transmission indicator  $\tau_1$ . On the basis of  $\tau_1$  we estimate ability of motion transmission:

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$$\tau_1 = \frac{-L_2 \sin(\varphi_3 - \varphi_2) \cos \varphi_4 + L_{2A} \sin(\varphi_4 - \varphi_{2A}) \cos \varphi_3}{L_2 + L_{2A}}$$

 $\tau_1(-1 \le \tau_1 \le 1)$  only in special positions becomes equal to zero and change of its sign indicates that the mechanism passes from one junction assembling to another one.

Let's now take sliding bar 5 as a driving link exposed to force  $F_D$ , and  $M_C$  – the resistance moment affecting link 1, and consider the result mechanism with the changeable closed contour. Then similarly to previous example, we shall deduce the formula connecting  $F_D \mu M_C$ :

$$F_{D} = \frac{L_{2}\sin(\varphi_{3} - \varphi_{2})\cos\varphi_{4} + L_{2A}\sin(\varphi_{4} - \varphi_{2A})\cos\varphi_{3}}{-L_{2}\sin(\varphi_{2} - \varphi_{3})\sin(\varphi_{1} - \varphi_{4}) + L_{2A}\sin(\varphi_{1} - \varphi_{3})\sin(\varphi_{4} - \varphi_{2A})} * \frac{M_{c}}{L_{1}}$$

On the basis of  $\tau_2$  we estimate ability of motion transmission:

$$\tau_2 = (-L_2 \sin(\varphi_3 - \varphi_2) \cos(\varphi_1 - \varphi_4 + L_{2A} \sin(\varphi_1 - \varphi_3) \sin(\varphi_4 - \varphi_{2A}))/(L_2 + L_{2A})$$

As is obvious, there is a direct relationship between the kinematical and force parameters of the considered mechanisms. This relationship is based on the fact that the expression of the coefficient matrix for the vector equations of mechanism closure for the variation of the generalized coordinate and the expression of matrix of coefficients of the mechanism equilibrium equations agree within the factor. This confirms the conclusions of the works [1,3].

In the article [4] to estimate the efficiency of mechanisms on the stage of their kinematic synthesis two criteria for the motion transmission are used, the first criterion  $(K_1)$  - characterizes the relationship between the torques (driving and resistance) at the inlet and outlet, the second criterion  $(K_2)$  - characterizes the relative level of reactions in the joints of the mechanism:

$$K_1 = M_D/M_C$$
 or  $K_1 = M_D/(F_Cd)$ ,  
 $K_2 = R/(M_C/d)$  or  $K_2 = R/F_C$ ;

where  $M_D$  - module driving torque;  $M_C$ ,  $F_C$  - the torque and useful resistance force module; R - modulo maximum reaction in the mechanism joints; d - a linear size. The value of  $M_C$  or  $F_C$  is considered as preset, and  $M_D$  and R values shall be determined by static analysis. At the same time assumptions like: weightless, rigid links; ideal constraints; and no other active forces, are accepted. Criteria  $K_1 \ \mu \ K_2$ take into account the essential elements of the real picture of the force transmission in the mechanism. The force transmission criteria are used in kinematic analysis and synthesis of the mechanism. Force transmission criteria play the same role as the angles of pressure used to evaluate the efficiency of class II mechanisms.

Let's demonstrate that the criteria  $K_1$  and  $K_2$  are interconnected. According to the formula (9) we have (Fig. 1):

$$\frac{M_D}{F_c L_1} = \frac{L_2 \sin(\varphi_3 - \varphi_2) \sin(\varphi_1 - \varphi_4) + L_{2A} \sin(\varphi_4 - \varphi_{2A}) \sin(\varphi_1 - \varphi_3)}{L_2 \sin(\varphi_3 - \varphi_2) \cos\varphi_4 - L_{2A} \sin(\varphi_4 - \varphi_{2A}) \cos\varphi_3}$$

Expression in the denominator of the right part of (10) is  $\Delta$  (according to the formula 9), its zero values correspond to the maximal values of the reactions. Then from (10) it is obvious that the positions where the reactions become maximum (criterion K<sub>2</sub>), coincide with the positions where the M<sub>D</sub> becomes extremely high (criterion K<sub>1</sub>). This suggests that they are constituents of more general criterion of the mechanism motion and forces transmission, deduced by Jacobian decomposition of the independent vector contour closedness equation. The above is described in the article [3]. Normalized Part (10) is the value of preprescribed motion and forces  $\tau$ . Value of transmission of motion  $\tau$  (-1  $\leq \tau \leq$  1) only in special situations becomes zero. By its value it is also possible to estimate its distance from the boundaries of the mechanism domain of existence.

The modular approach [3,5] is widely applied for kinematical synthesis of mechanisms, when:

a) the initial kinematical chain (IKC) with several degrees of mobility makes the prescribed motions;

b) Introduction in the certain way of the closing kinematic chains (CKC) results in formation of the mechanisms. As CKC a dyad or a binary link, with one zero and one negative degree of mobility is usually considered.

Restrictions are applied on pressure (transmission) angles in kinematic pairs [3,5] for acquisition of efficient mechanisms. However for mechanisms with the changeable closed contour the estimation of their ability to transmit the prescribed motion along with the transmission angles is insufficient as it is shown in the article [3]. In that article the account for transmission of the prescribed motion at synthesis is reduced to the problem of three-criterion optimization, quite complicated from the practical point of view.

Up to now the issues of synthesis and kinematic analysis have been considered separately and therefore various problems connected with of existence and identification of different mechanism assemblage required additional resolution. Since it is known that at the mechanism synthesis, so-called "branching defect" often appears, and then one part of N predetermined position is approximated by one assembly of mechanism, and the other part is approximated by the other type of the mechanism assembly, owing to that the resulting mechanism does not perform the prescribed motion.

We propose to use the below mechanism synthesis algorithm allowing transferring the process of synthesis to practical optimization (as is shown in example in the Fig.3below), this algorithm is based on well-known approaches and allows solving the problem of mechanism scheme efficiency already at the stage of mechanism synthesis:

1. The initial kinematic chain (IKC) is selected - OABC;

2. IKC is forced to execute the prescribed motions  $\varphi_1$  and S and by introduction of closing kinematic chains (CKC) - ADEC the mechanism is formed (the cyclic quadratic point  $D(m_2^*n_2^*)$  in the plane  $Cm_5n_5$  is found in inverse motion of the moving plain  $Cm_5n_5$  relative to the moving plain  $Am_2n_2$  by the procedure of minimization of the objective function [6]);

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Fig.3 To algorithm synthesis

3. CKC, with consideration of the points of adjunction D and E found in a course of optimization, is forced to carry out the prescribed motions;

4. For each of N set positions of IKC - OABC the value of motion and force transmission indicator  $\tau$  is estimated by calculation of the variable distances between points of adjunction D and E.

5. If  $\tau$  values are not close to zero and don't change a sign for the entire cycle of the prescribed motion of IKC, then the resulting kinematic scheme of the mechanism works in one assemblage and with some approximation reproduces the prescribed motions.

Efficiency of algorithm and the computer software developed on its basis are confirmed by schemes of mechanisms, for which the inventor's certificates are received [7, 8]. The account of the motion and forces transmission indicator  $\tau$  transfer at a stage of computer realization of the kinematic synthesis algorithm does not guarantee the efficiency of the mechanism after its design development. However, it creates the necessary background for this purpose. At unfavorable values of  $\tau$  it is impossible to compensate weak points of the kinematic scheme by means of design solutions and to create the efficient mechanism.

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