Thermal Convection of a Water-Based Nanofluid in an Enclosure with an Oscillating Wall

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Abstract - The effects of oscillatory flows on convective heat transfer in an enclosure filled with a water based nanofluid (water with Al_2O_3 particles) are investigated. The fluid motion is driven by the periodic vibration of the enclosure left wall. The vertical walls of the enclosure are adiabatic while the horizontal walls are heated differentially. A control-volume method based, explicit time-marching Flux-Corrected Transport (FCT) Algorithm is used to simulate the transport phenomena in the enclosure. The oscillatory fluid motion significantly enhances overall heat transfer from the bottom wall compared to pure conduction.

Index Terms - Nanofluids, oscillatory flow, heat transfer

I. INTRODUCTION

Nanofluids are the promising heat transfer media. Heat transfer in nanofluids has received increasing attention recently because of its possible application to a number of practical devices. However, to our knowledge, the oscillatory flows and associated thermal transport phenomena in nanofluids are not investigated. The term of ‘nanofluid’ was first used by Choi [1] for suspension of base fluid and nano particles. Heris et al. [2] performed laminar flow experiments by using water based nanofluids with Al_2O_3 nano particles. Although the thermal conductivity increases less than 15 percent, the increase in heat transfer coefficient reaches 40 percent. Yang et al. [3] and Ding et al. [4] also conducted heat transfer experiments with nanofluids. Ma et al. [56] carried out experiments in water-diamond filled oscillating heat pipes. They reported the significant increase of the heat transport capability of the oscillating heat pipe when it was charged with the nanofluid.

Xuan and Li [6] analytically computed Nusselt number with isothermal wall boundary condition for a nanofluid flowing at constant velocity. Maiga et al. [7] investigated the forced convection heat transfer under constant and uniform heat flux boundary condition for water-γAl_2O_3 and ethylene glycol-γAl_2O_3 nanofluids. Xuan and Roetzel [8] proposed a general function for Nusselt number in nanofluids. Maiga et al. [9] numerically investigated the heat transfer enhancement by nanofluids.

Maiga et al. [10] studied the hydrodynamic and thermal behavior of nanofluids carrying Al_2O_3 nanoparticles in a tube under constant heat flux boundary condition for turbulent flows. Heris et al. [11] predicted the laminar forced convection heat transfer for nanofluids in tubes under isothermal wall boundary condition. Behzadmehr et al. [12] studied the turbulent forced convection in nanofluids under constant heat flux boundary condition. Mirmasoumi and Behzadmehr [13] also considered the effect of nano particle size on laminar mixed convection in a horizontal pipe. Mirmasoumi and Behzadmehr [14] investigated the laminar mixed convection heat transfer in a tube carrying water-γAl_2O_3 nanofluid.

Khanafer et al. [15] numerically investigated the natural convective thermal transport in a rectangular enclosure filled with water-Cu nanofluid. Jou and Tzeng [16] reported the enhancement of average Nusselt number by utilizing nanofluids. Hwang et al. [17] studied the natural convection in a rectangular enclosure heated from below and reported a significant decrease of average Nusselt number with increasing nano particle size. Oztop and Abu-Nada [18] numerically studied the natural convection in a differentially heated cavity with partially heated vertical wall. Ogut [19] investigated the natural convection in a water based nanofluid filled inclined square enclosure subject to partial heating from the side walls, numerically. A limited review of forced convective thermal transport in nanofluids is given by Daungthongsuk ve Wongwises [20]. Kakac and Pramuanjaroenkij reviewed the enhancement of forced convective heat transfer with nanofluids [21].

The above survey indicates that the investigations on the oscillatory flow of nanofluids and related thermal transport phenomena have received relatively less attention. In the present manuscript, we focus on the effects of oscillatory flow on convective heat transfer in a rectangular shallow enclosure filled with water-γAl_2O_3. The fluid motion is driven by the periodic vibration of the enclosure left wall. We perform a parametric study in order to evaluate the effect of wall displacement amplitude, particle volume concentration, and wall temperature difference on the transient thermal convection in the enclosure. The results of the present investigation may be used in the design of various heat removal applications utilizing oscillatory flow of nanofluids. The findings of this work may be helpful in the design of oscillating heat pipes carrying nanofluids.

II. PROBLEM DEFINITION

The water-γAl_2O_3 nanofluid-filled two dimensional shallow rectangular enclosure is schematically shown in

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Figure 1. The vertical walls of the enclosure are adiabatic while the horizontal walls are heated differentially. The left wall of enclosure is vibrating with a frequency that induces standing waves along the enclosure. The displacement of left wall is given by,

$$X(t) = X_{max}\sin\omega t \qquad (1)$$

Here, X_{max} is the maximum displacement of left wall. The vibration frequency of the left wall is chosen as f = 7850 Hz. The length of the enclosure is half wavelength $L=\lambda/2=100$ mm, and the height of the enclosure is H=10 mm. For the left wall maximum displacement two different values are considered, X_{max1} = 20 nm and X_{max2} = 25 nm. Relatively small left wall displacement amplitude (20 nm and 25 nm) and temperature difference of horizontal walls $(T_B - T_T) \leq 0.2$ K are chosen since water-γAl_2O_3 in a rigid container (as considered here), would sustain large changes in pressure.

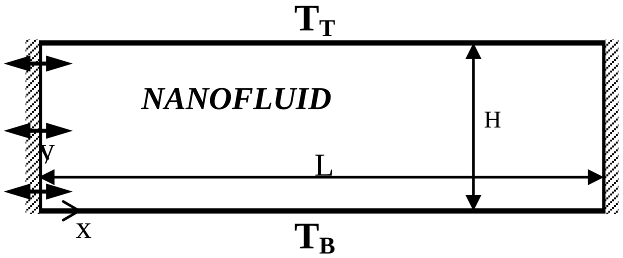

Figure 1. Problem schematic

III. MATHEMATICAL MODEL

A. Transport Equations

The fully compressible form of the Navier-Stokes and energy equations in 2-D Cartesian coordinate system are employed to model the transport phenomena in the enclosure considered:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \qquad (2)$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y} \qquad (3)$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{xy}}{\partial x} \qquad (4)$$

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x}\left[(E+p)u\right]+\frac{\partial}{\partial y}\left[(E+p)v\right]=$$
$$\frac{\partial}{\partial x}\left[u\tau_{xx}+v\tau_{xy}\right]+\frac{\partial}{\partial y}\left[u\tau_{xy}+v\tau_{yy}\right]-\frac{\partial q_x}{\partial x}-\frac{\partial q_y}{\partial y} \qquad (5)$$

The effect of gravity is not considered in the present numerical simulations. Here

$$E=\rho C_v T+\frac{1}{2}\rho(u^2+v^2) \qquad (6)$$

C_v is the specific heat at constant volume and the equation of state is represented as $p = f(\rho, T)$.

The specific heat of the nanofluid is computed from [21];

$$C_v=(1-\varphi)C_f+\varphi C_n \qquad (7)$$

where φ is the particle volume concentration. The components of the stress tensor τ are:

$$\tau_{xx}=\frac{4}{3}\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\frac{\partial v}{\partial y} \qquad \tau_{yy}=\frac{4}{3}\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\frac{\partial u}{\partial x}$$
$$\tau_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \qquad (8)$$

where μ is the shear viscosity. For the viscosity, we used the expression suggested by Brinkman [22];

$$\mu=\mu_f\frac{1}{(1-\varphi)^{2.5}} \qquad (9)$$

The components of the heat-flux vector are written as:

$$q_x=-k\frac{\partial T}{\partial x} \qquad q_y=-k\frac{\partial T}{\partial y} \qquad (10)$$

where k is thermal conductivity and T is temperature. In order to calculate the nanofluid thermal conductivity, we employed Maxwell's [23] equation:

$$k=k_f\frac{k_n+2k_f+2(k_n-k_f)\varphi}{k_n+2k_f-2(k_n-k_f)\varphi} \qquad (11)$$

B. Equation of State for Water Based Nanofluids

In this study, we used the equation of state for liquid water proposed by Jeffery and Austin [24]. This approximation is mainly based on the conclusion of Nie et al. [25] on the compressibility of nanofluids. In their investigation, Nie et al. [25] reported that for low volume fractions, the effect of nano particles on the compressibility of the base fluid is insignificant. The propagation of the pressure waves in the enclosure is the most critical flow behavior in the present problem. Thus, we neglected the contribution of the nano particles on the base fluid density and adopted an highly accurate equation of state [24] of water (base fluid) in the present numerical simulations. In the present equation of state, the pressure value at a given temperature and density is calculated by:

$$p = p_{EOS} + 2p_{HB} \qquad (12)$$

where p_{EOS} refers to the pressure calculated from the modified van der Waals equation of state, and p_{HB} provides the contribution from the hydrogen bonds. These terms are given as:

$$\frac{p_{EOS}}{\rho RT}=1-b^{*}\rho-\frac{a_{VW}\rho}{RT}+\frac{\alpha\rho}{1-\lambda b\rho} \qquad (13)$$

$$p_{HB} = -\rho^2 \left(\frac{\partial A_{HB}}{\partial \rho} \right)_T \qquad (14)$$

where R is the specific gas constant, b^*, a_{VW}, α and λ are constants, and b is a temperature dependent coefficient. A_{HB} refers to the Helmholtz free energy. The average deviation of the present equation of state from the experimentally determined p-V-T data is 0.507% in the range of $0<T<1200°C$, $0.1<p<2000$ bar, $0.16<\rho<1025$ kg/m^3.

A general thermodynamic relation suggests that the pressure variations in a pure substance can be expressed in terms of the isothermal compressibility (κ_T) and the pressure variations;

$$\kappa_T = -\rho \left. \frac{\partial (1/\rho)}{\partial P} \right|_T \qquad (15)$$

This relation is useful for estimating the compression of the water-γAl_2O_3 media in the simulated conditions.

C. Numerical Model

An explicit finite volume approach is used to solve the discretized form of the governing equations. The convective terms are discretized using a Flux-Corrected Transport (FCT) algorithm while the diffusion terms are discretized by a central-difference scheme. The FCT approach for solving the convective transport terms allows higher order accuracy and the ability to control numerical diffusion within the finite-difference grid [26]. Time-step splitting approach is used to include the convection, diffusion and compressibility effects.

D. Initial and Boundary Conditions

The boundary conditions of the problem under investigation are:

$u(0,y,t) = \omega X_{max} \cos\omega t \qquad v(0,y,t) = 0$

$u(L,y,t) = v(L,y,t) = 0$

$u(x,0,t) = v(x,0,t) = 0$

$u(x,H,t) = v(x,H,t) = 0$

$$\frac{\partial T}{\partial x}(0,y,t) = \frac{\partial T}{\partial x}(L,y,t) = 0$$

$T(x,0,t) = T_B \qquad T(x,H,t) = T_T$

The nanofluid media confined in the enclosure is initially considered to be quiescent. Initially, the container is slightly pressurized (5 bar) with a temperature at 300 K.

The boundary conditions for the higher order FCT-based solutions of the Navier-Stokes equations require a rigorous formulation. No-slip boundary conditions are used for velocity on all walls. The wall boundary conditions for density are updated using the formulation developed by Poinsot and Lele [27] based on characteristic wave relations. The use of this method avoids over-specification of variables and incorrect extrapolations from interior point values. The wall pressure is updated using the equation of state for the nanofluid, described earlier.

IV. RESULTS

The results of the numerical simulation of transient convective heat transfer in a water-γAl_2O_3 nanofluid filled differentially heated enclosure are presented here. Typically 100×20 uniformly spaced mesh points are used for the present simulations. The simulations performed using denser computational grids did not change the results significantly and 100×20 mesh was found to be sufficient for the present computations. Table 1 lists the cases studied in order to determine the influence of oscillatory flow on convective heat transfer in the differentially heated enclosure. The particle volume concentration, the maximum displacement amplitude of the left wall and the temperature difference of the horizontal walls are presented in the table.

Table 1: Summary of the cases simulated

Case	***φ***	**X_{max} (nm)**	***$T_B - T_T$ (K)***
1	% 1	20	0.1
2			0.2
3		25	0.1
4			0.2
5	% 5	20	0.1
6			0.2
7		25	0.1
8			0.2

Figure 2 shows the temporal variation of the pressure in the nanofluid near t = 1 s. at the immediate vicinity of the left wall center point for Case 1 and Case 3. The pressure fluctuations reach pseudo-steady values by this time. The approximate value of the isothermal compressibility of water at 300 K is $\kappa_T \cong 4.5\times10^{-10}$ Pa^{-1}. For the density variations computed for Case 1, Eq. (15) estimates a corresponding pressure variation of about 155 kPa. The computationally predicted difference between the pressure maximum and minimums (165 kPa) shown in Figure 2 is close to this estimate. The pressure variation is rather large due to low compressibility of water-γAl_2O_3. With increasing left wall displacement amplitude, stronger pressure waves (190 kPa) are observed in the nanofluid for Case 3.

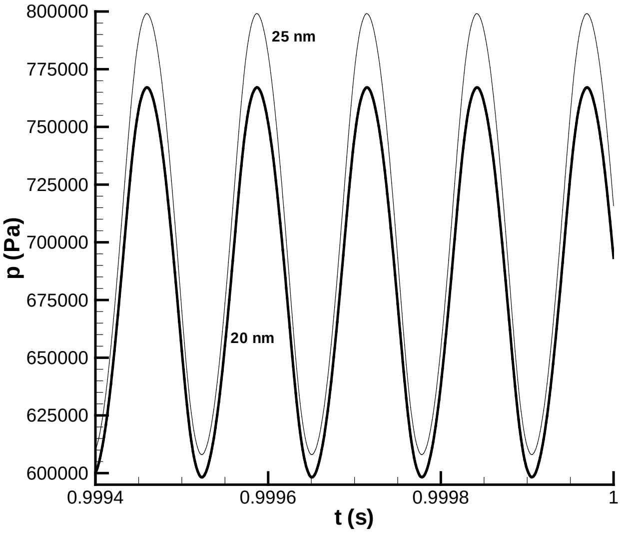

Figure 2. Temporal variation of the left wall pressure near t=1 s for Case 1 and 3.

Figure 3 depicts the temporal variation of the x-component of the instantaneous velocity at the midpoint of the enclosure for Case 1 and 3. The oscillatory nature of the flow field is clearly observed. The flow is nearly periodic. Larger left wall displacement results faster fluid motion in the container.

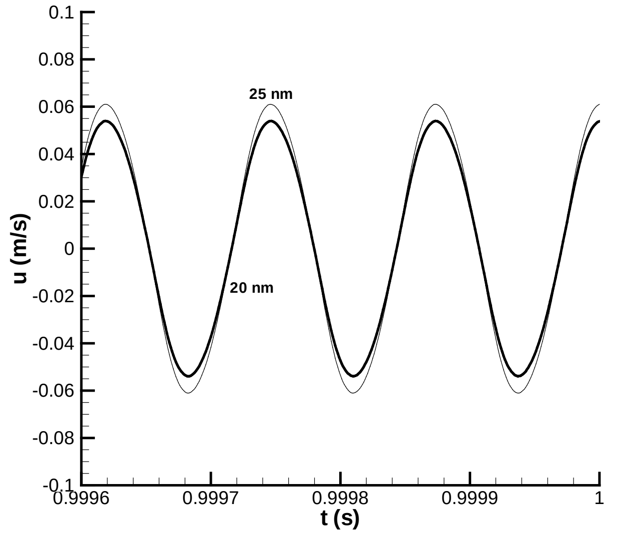

Figure 3. Temporal variation of the u- velocity at the midpoint of the enclosure for Case 1 and 3

The instantaneous velocity distribution in the enclosure at t=1 s for Case 1 is given in Figure 4. The flow field is fairly symmetric with respect to the horizontal mid-plane. The maximum velocity in the enclosure reaches 0.053 m/s.

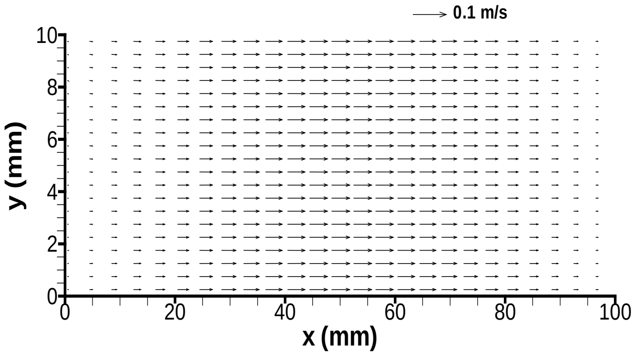

Figure 4. Instantaneous flow field in the enclosure at t = 1 s for Case 1

Figure 5 depicts the time averaged flow field in the enclosure. This secondary, steady flow structure is computed based on the average mass transport velocities defined as;

$$u_{st} = \frac{\langle \rho u \rangle}{\langle \rho \rangle} \qquad v_{st} = \frac{\langle \rho v \rangle}{\langle \rho \rangle} \tag{17}$$

Here time averaging is applied during one vibration cycle of the left wall.

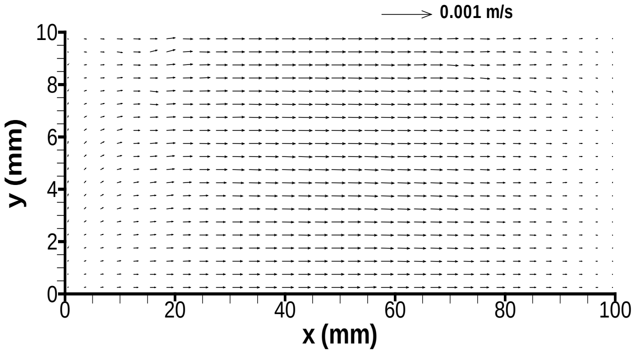

Figure 5. Cycle averaged flow field in the enclosure at t = 1 s for Case 1

The time averaged flow velocities are two order of magnitude less than the primary oscillatory flow velocities.

The spatial variation of the cycle averaged temperature along the vertical mid-plane of the enclosure for Case 1 is plotted in Figure 6 at t=1 s. The transient pure conduction solution of water-γAl_2O_3 (with % 1 volume concentration) nanofluid at the same instant for stationary left wall case (no acoustic perturbation) is also included in the figure. As the figure indicates, in the presence of oscillatory fluid motion (Case 1) the bottom wall temperature gradient is much higher.

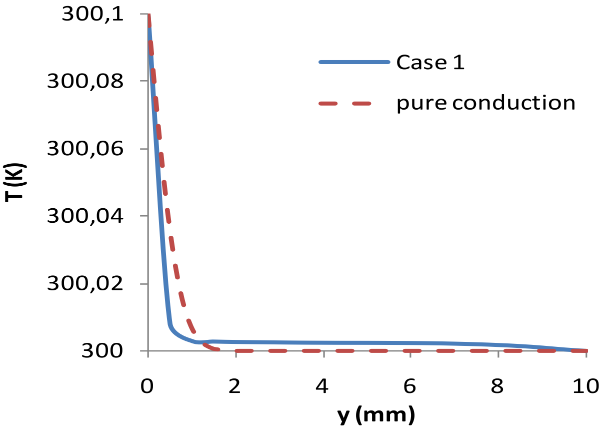

Figure 6. Cycle averaged temperature profiles at x = L/2 for Case 1, t = 1 s

Table 2 lists the average Nusselt numbers for the transient convective heat transfer from the enclosure bottom wall for all cases simulated. The average Nusselt number along the bottom wall is given by,

$$\overline{\mathrm{Nu}} = -\frac{H}{L}\frac{1}{T_B - T_T}\int_0^L \frac{dT}{dy}dx \qquad (18)$$

Nusselt numbers computed for the transient pure conduction solutions for each case are included in this table. As the values indicates, the oscillatory flow significantly enhances the heat transfer when compared to pure diffusion. The enhancement is more pronounced for the low nano particle volume fractions.

Table 2: Nusselt numbers for the cases simulated

Case	**φ**	**X_{max} (nm)**	$\overline{\mathrm{Nu}}$	**Nu_{cond}**
1	% 1	20	18.36	14.6
2			18.65	
3		25	18.19	
4			18.58	
5	% 5	20	18.25	14.7
6			18.60	
7		25	18.03	
8			18.55	

V. CONCLUSIONS

The effects of oscillatory fluid motion on the convective heat transfer in a differentially heated shallow enclosure (heated from the bottom wall) filled with a nanofluid are computationally investigated. The nanofluid utilized was water-γAl_2O_3. Based on the results reported in the manuscript, the following observations can be made.

- The oscillatory flow significantly augments the heat transfer when compared to pure conduction.
- The augmentation is more evident in the case of low nano particle volume concentration.

Further investigation is required in order to fully characterize the influence of the oscillatory flow strength and pattern on the convective heat transfer in enclosures carrying nanofluids.

REFERENCES

[1] S. U. S. Choi, “Enhancing Thermal Conductivity of Fluids with Nanoparticles”, *The Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition*, San Francisco, California, USA, November 12-17, 1995.

[2] S. Z. Heris, G. Etemad, and M. N. Esfahany, “Experimental Investigation of Oxide Nanofluids Laminar Flow Convection Heat Transfer”, *International Communications in Heat and Mass Transfer*, Vol. 33, pp. 529-535, 2006.

[3] Y. Yang, Z. G. Zhang, E. A. Grulke, W. B. Anderson and G. Wu, “Heat Transfer Properties of Nanoparticle-in-Fluid Dispersions (Nanofluids) in Laminar Flow”, *International Journal of Heat and Mass Transfer*, Vol. 48, pp. 1107-1116, 2005.

[4] Y. Ding, H. Alias, D. Wen, and R. A. Williams, “Heat Transfer of Aqueous Suspensions of Carbon Nanotubes (CNT nanofluids)”, *International Journal of Heat and Mass Transfer*, Vol. 49, pp. 240-250, 2006.

[5] H. B. Ma, C. Wilson, Q. Yu, K. Park, U. S. Choi and M. Tirumala, “An experimental investigation of heat transport capability in a nanofluid oscillating heat pipe”, *Journal of Heat Transfer*, Vol. 128, pp. 1213-1216, 2006.

[6] Y. Xuan and Q. Li, “Heat Transfer Enhancement of Nanofluids”, *International Journal of Heat and Fluid Flow*, Vol. 21, pp. 58-64, 2000.

[7] S. E. B. Maïga, C. T. Nguyen, N. Galanis and G. Roy, “Heat Transfer Behaviours of Nanofluids in a Uniformly Heated Tube”, *Superlattices and Microstructures*, Vol. 35, pp. 543-557, 2004.

[8] Y. Xuan and W. Roetzel, “Conceptions for Heat Transfer Correlation of Nanofluids”, *International Journal of Heat and Mass Transfer*, Vol. 43, pp. 3701-3707, 2000.

[9] S. E. B. Maïga, S. J. Palm, C. T. Nguyen, G. Roy and N. Galanis, “Heat Transfer Enhancement by Using Nanofluids in Forced Convection Flows”, *International Journal of Heat and Fluid Flow*, Vol. 26, pp. 530-546, 2005.

[10] S. E. B. Maïga, C. T. Nguyen, N. Galanis, G. Roy, T. Maré and M. Coqueux, “Heat Transfer Enhancement in Turbulent Tube Flow Using Al2O3 Nanoparticle Suspension”, *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 16, pp. 275-292, 2006.

[11] S. Z. Heris, M. N. Esfahany and G. Etemad, “Numerical Investigation of Nanofluid Laminar Convection Heat Transfer through a Circular Tube”, *Numerical Heat Transfer, Part A*, Vol. 52, pp. 1043-1058, 2007.

[12] A. Behzadmehr, M. Saffar-Avval and N. Galanis, “Prediction of Turbulent Forced Convection of a Nanofluid in a Tube with Uniform Heat Flux Using a Two Phase Approach“, *International Journal of Heat and Fluid Flow*, Vol. 28, pp. 211-219, 2007.

[13] S. Mirmasoumi and A. Behzadmehr, “Effect of Nanoparticles Mean Diameter on Mixed Convection Heat Transfer of a Nanofluid in a Horizontal Tube”, *International Journal of Heat and Fluid Flow*, Vol. 29, pp. 557-566, 2008.

[14] S. Mirmasoumi and A. Behzadmehr, “Numerical Study of Laminar Mixed Convection of a Nanofluid in a Horizontal Tube Using Two-Phase Mixture Model”, *Applied Thermal Engineering*, Vol. 28, pp. 717-727, 2008.

[15] K. Khanafer, K. Vafai and M. Lightstone, “Buoyancy-Driven Heat Transfer Enhancement in a Two-Dimensional Enclosure Utilizing Nanofluids”, *International Journal of Heat Mass Transfer*, Vol. 46, pp. 3639–3653, 2003.

[16] R. Y. Jou, and S. C. Tzeng, “Numerical Research of Nature Convective Heat Transfer Enhancement Filled with Nanofluids in Rectangular Enclosures”, *International Communications in Heat and Mass Transfer*, Vol. 33, pp. 727–736, 2006.

[17] K. S. Hwang, J. H. Lee and S. P. Jang, “Buoyancy-driven Heat Transfer of Water-based Al2O3 Nanofluids in a Rectangular Cavity”, *International Journal of Heat and Mass Transfer*, Vol. 50,

pp. 4003–4010, 2007.

[18] H. F. Oztop and E. Abu-Nada, “Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids”, *International Journal of Heat and Fluid Flow*, Vol. 29, pp. 1326-1336, 2008.

[19] E. B. Ogut, “Natural convection of water-based nanofluids in an inclined enclosure with a heat source”, *International Journal of Thermal Sciences*, Vol. 48, pp. 2063-2073, 2009.

[20] W. Daungthongsuk and S. Wongwises, “A Critical Review of Convective Heat Transfer of Nanofluids”, *Renewable & Sustainable Energy Reviews*, Vol. 11, pp. 797-817, 2007.

[21] S. Kakac and A. Pramuanjaroenkij, “Review of convective heat transfer enhancement with nanofluids”, *International Journal of Heat and Mass Transfer*, Vol. 52, pp. 3187-3196, 2009.

[22] H. C. Brinkman, “The viscosity of concentrated suspensions and solutions”, *Journal of Chemical Physics*, Vol. 20, pp. 571-581, 1952.

[23] J. C. Maxwell, “Colours in Metal Glasses and in Metallic Films”, *Philosophical Transactions of the Royal Society A*, Vol. 203, pp. 385–420, 1904.

[24] C. A. Jeffery and P. H. Austin, “A new analytic equation of state for liquid water”, *Journal of Chemical Physics*, Vol. 110, pp. 484-496, 1999.

[25] C. Nie, W. H. Marlow and Y. A. Hassan, “Discussion of proposed mechanisms of thermal conductivity enhancement in nanofluids”, *International Journal of Heat and Mass Transfer*, Vol. 51, pp. 1342–1348, 2008.

[26] E. S. Oran and J. P. Boris, “Numerical Simulation of Reactive Flows”, *Cambridge University Press*, Cambridge, England, 2000.

[27] T. J. Poinsot and S. K. Lele, “Boundary conditions for direct simulations of compressible viscous flows”, *Journal of Computational Physics*, Vol. 101, pp. 104–129, 1992.