A New Knee Prosthesis Design Based on Human Lower Limb Cinematic Analysis

Cristian Copilusi, Mihnea Marin and Ligia Rusu

***Abstract*— In this paper a human lower limb cinematic analysis is realized, by using a computerized method with a flexible character, easy to implement onto computer. Through this method we obtain the generalized coordinates motion laws of the knee cinematic joint from the equivalent model of the human lower limb. These laws are useful for a new human knee prosthesis design.**

***Index Terms*—cinematic, knee prosthesis, human lower limb.**

I. Introduction

MANY researchers have been developed different methods for study the human lower limb cinematic [1], [2], [3], [4].

The cinematic methods aim is to study different motion types of the human lower limb in order to improve the athletes' performances or to design new human lower limb prosthesis.

The cinematic analysis of a mechanical model consists in solving two important problems: cinematic direct problem and cinematic inverse problem.

In the cinematic direct problem's frame, the displacements from cinematic joint are known, and it will be determined the positions – orientation, speeds and accelerations of the mechanism elements or some characteristic points onto the analyzed mechanism.

In the cinematic inverse problem, the cinematic parameters for some characteristic points motion are known, and it will be determined the cinematic parameters of the cinematic joints relative motions.

With these, in the cinematic analysis context, we identify many problems such as:

- Positional problem;
- Speed problem;
- Accelerations problem.

Each of these problems presents a direct or inverse aspect.

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C. Copilusi is with the Faculty of Mechanics, University of Craiova. Calea Bucuresti no. 113. Romania (corresponding author to provide phone: +04 0747222771; e-mail: cristache03@yahoo.co.uk).

M. Marin, is with the Faculty of Mechanics, University of Craiova. Calea Bucuresti no. 113. Romania (e-mail: mih_marin@yahoo.com).

L. Rusu is with the Faculty of Educational Physics and Sport. University of Craiova. Brestei street. Romania (e-mail: ligiarusu@hotmail.com).

II. Human Lower Limb Cinematic Analysis

The method used in this paper has a flexible character and assures an interface for dynamic analysis especially for finite element modeling of spatial and planar mobile mechanical systems [5].

For the cinematic analysis the cinematic model presented in figure 1, will be considered. The cinematic model analysis will be performed only for walking activity, for a single gait. The cinematic parameters variation laws were obtained by processing with the MAPLE software aid the mathematical models which are defining the human lower limb experimentally cinematic analysis.

From a structural viewpoint, the cinematic chain it consists in 8 rotation joints.

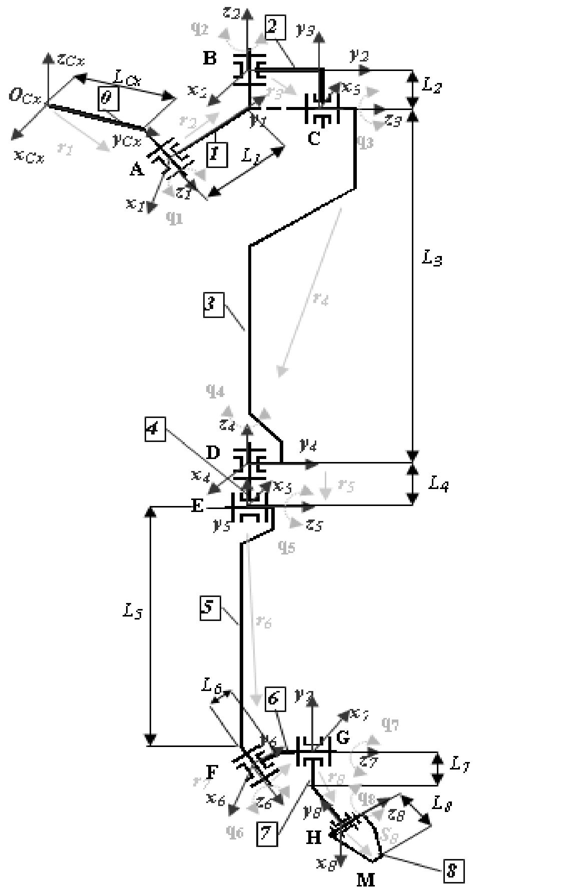

Fig. 1. The cinematic model equivalent with the human lower limb

The $\overline{r_i}$ position vectors in the T_{i-1} reference coordinate system are:

$$r_1 = [0, L_{cx}, 0]_{cx}^T; r_3 = [0, 0, -L_2]_2^T;$$
$$r_2 = [0, L_1, 0]_1^T; r_4 = [0, -L_3, 0]_3^T;$$
$$r_5 = [0, 0, -L_4]_4^T; r_6 = [0, -L_5, 0]_5^T; \quad (1)$$
$$r_7 = [0, L_6, 0]_6^T; r_8 = [0, -L_7, 0]_7^T;$$
$$S_8 = [0, -L_8, 0]_8^T.$$

The connectivity order will be:**C_x - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8**

A. Position calculus

The position vectors are:

$$\overrightarrow{r_1} = \{r_1^x, r_1^y, r_1^z\} = \{r_1\}^T \cdot \{\overrightarrow{W_{cx}}\} \quad (2)$$
$$\overrightarrow{r_2} = \{r_2^x, r_2^y, r_2^z\} = \{r_2\}^T \cdot \{\overrightarrow{W_1}\} \quad (3)$$
$$\overrightarrow{r_3} = \{r_3^x, r_3^y, r_3^z\} = \{r_3\}^T \cdot \{\overrightarrow{W_2}\} \quad (4)$$
$$\overrightarrow{r_4} = \{r_4^x, r_4^y, r_4^z\} = \{r_4\}^T \cdot \{\overrightarrow{W_3}\} \quad (5)$$
$$\overrightarrow{r_5} = \{r_5^x, r_5^y, r_5^z\} = \{r_5\}^T \cdot \{\overrightarrow{W_4}\} \quad (6)$$
$$\overrightarrow{r_6} = \{r_6^x, r_6^y, r_6^z\} = \{r_6\}^T \cdot \{\overrightarrow{W_5}\} \quad (7)$$
$$\overrightarrow{r_7} = \{r_7^x, r_7^y, r_7^z\} = \{r_7\}^T \cdot \{\overrightarrow{W_8}\} \quad (8)$$
$$\overrightarrow{r_8} = \{r_8^x, r_8^y, r_8^z\} = \{r_8\}^T \cdot \{\overrightarrow{W_7}\} \quad (9)$$
$$\overrightarrow{S_8} = \{S_8^x, S_8^y, S_8^z\} = \{S_8\}^T \cdot \{\overrightarrow{W_8}\} \quad (10)$$

Where:

$$\{r_1\}^T = [0, L_{cx}, 0]_{cx}; \{r_2\}^T = [0, L_1, 0]_1;$$
$$\{r_3\}^T = [0, 0, -L_2]_2; \{r_4\}^T = [0, -L_3, 0]_3;$$
$$\{r_5\}^T = [0, 0, -L_4]_4; \{r_6\}^T = [0, -L_5, 0]_5; \quad (11)$$
$$\{r_7\}^T = [0, L_6, 0]_6; \{r_8\}^T = [0, -L_7, 0]_7;$$
$$\{S_8\}^T = [0, -L_8, 0]_8.$$

The $\overrightarrow{r_M^{Cx}}$ vector, has the following expression:

$$\overrightarrow{r_M^{Cx}} = \overrightarrow{r_1} + \overrightarrow{r_2} + \overrightarrow{r_3} + \overrightarrow{r_4} + \overrightarrow{r_5} + \overrightarrow{r_6} + \overrightarrow{r_7} + \overrightarrow{r_8} + \overrightarrow{S_8} \quad (12)$$

Changing the versors base at crossing from a reference coordinate system to another (introducing the coordinate transformation matrices):

$$\{\overrightarrow{W_1}\} = [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (13)$$
$$\{\overrightarrow{W_2}\} = [A_{12}] \cdot \{\overrightarrow{W_1}\} = [A_{Cx2}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (14)$$
$$\{\overrightarrow{W_3}\} = [A_{23}] \cdot \{\overrightarrow{W_2}\} = [A_{Cx3}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (15)$$
$$\{\overrightarrow{W_4}\} = [A_{34}] \cdot \{\overrightarrow{W_3}\} = [A_{Cx4}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (16)$$
$$\{\overrightarrow{W_5}\} = [A_{45}] \cdot \{\overrightarrow{W_4}\} = [A_{Cx5}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (17)$$
$$\{\overrightarrow{W_6}\} = [A_{56}] \cdot \{\overrightarrow{W_5}\} = [A_{Cx6}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (18)$$
$$\{\overrightarrow{W_7}\} = [A_{67}] \cdot \{\overrightarrow{W_6}\} = [A_{Cx7}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (19)$$
$$\{\overrightarrow{W_8}\} = [A_{78}] \cdot \{\overrightarrow{W_7}\} = [A_{Cx8}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (20)$$

By analyzing the (13) ... (20) relations we observe that:

$$[A_{Cx2}] = [A_{12}] \cdot [A_{Cx1}] \quad (21)$$
$$[A_{Cx3}] = [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \quad (22)$$
$$[A_{Cx4}] = [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] = [A_{34}] \cdot [A_{Cx3}] \quad (23)$$
$$[A_{Cx5}] = [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] = = [A_{45}] \cdot [A_{Cx4}] \quad (24)$$
$$[A_{Cx6}] = [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] = = [A_{56}] \cdot [A_{Cx5}] \quad (25)$$
$$[A_{Cx7}] = [A_{67}] \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot \cdot [A_{12}] \cdot [A_{Cx1}] = [A_{67}] \cdot [A_{Cx6}] \quad (26)$$
$$[A_{Cx8}] = [A_{78}] \cdot [A_{67}] \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] = [A_{78}] \cdot [A_{Cx7}] \quad (27)$$

Based on (21)... (27) relations we identify the coordinates transformation matrices for each cinematic joints, with $\alpha_{i,i+1} = 90^\circ$, and $i = \overline{1,8}$.

Point: **A, B, C, D, E, F, G, H and M** positions in rapport with T_{cx} coordinate system, bounded to the coxae bone, will be identified through relations:

$$\{\overrightarrow{r_A^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} \quad (28)$$
$$\{\overrightarrow{r_B^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (29)$$
$$\{\overrightarrow{r_C^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (30)$$
$$\{\overrightarrow{r_D^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_4\}^T \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (31)$$
$$\{\overrightarrow{r_E^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + \{r_4\}^T \cdot [A_{23}] \cdot [A_{12}] \cdot \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + \{r_5\}^T \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (32)$$
$$\{\overrightarrow{r_F^{T_{Cx}}}\} = \{r_1\}^T \cdot \{\overrightarrow{W_{Cx}}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + \{r_4\}^T \cdot [A_{23}] \cdot [A_{12}] \cdot \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + \{r_5\}^T \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} + + \{r_6\}^T \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overrightarrow{W_{Cx}}\} \quad (33)$$

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$$\begin{aligned}\left\{\overleftarrow{r}_G^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_2\}^T\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_6\}^T\cdot\\ &\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_7\}^T\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{34}$$

$$\begin{aligned}\left\{\overleftarrow{r}_H^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_2\}^T\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot\\ &\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_6\}^T\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_7\}^T\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot\\ &\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_8\}^T\cdot[A_{67}]\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot\\ &\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{35}$$

$$\begin{aligned}\left\{\overleftarrow{r}_M^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_2\}^T\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot\\ &\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_6\}^T\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_7\}^T\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot\\ &\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_8\}^T\cdot[A_{67}]\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot\\ &\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{S_8\}^T\cdot[A_{78}]\cdot[A_{67}]\cdot\\ &\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{36}$$

B. Speed calculus

We follow to determine the **M** point speed in rapport with T_{cx} reference system. For this we differentiate successively the (28) ... (36) relations, but for achieve this calculus is necessary to build the anti symmetric matrices for each joint, like this form:

$$\left[\tilde{\omega}_{Cxi}\right]=\begin{bmatrix}0 & \omega_{Cxi} & -\omega_{Cxi}\\ -\omega_{Cxi} & 0 & \omega_{Cxi}\\ \omega_{Cxj} & -\omega_{Cxj} & 0\end{bmatrix},\ \text{with } i,j=\overline{1,8} \tag{37}$$

For this:

$$\left[\dot{A}_{Cx1}\right]=\left[\tilde{\omega}_{Cx1}\right]\cdot\left[A_{Cx1}\right] \tag{38}$$

$$\left[\dot{A}_{Cx2}\right]=\left[\tilde{\omega}_{12}\right]\cdot\left[A_{Cx3}\right] \tag{39}$$

$$\left[\dot{A}_{Cx3}\right]=\left[\tilde{\omega}_{23}\right]\cdot\left[A_{Cx3}\right] \tag{40}$$

$$\left[\dot{A}_{Cx4}\right]=\left[\tilde{\omega}_{34}\right]\cdot\left[A_{Cx4}\right] \tag{41}$$

$$\left[\dot{A}_{Cx5}\right]=\left[\tilde{\omega}_{45}\right]\cdot\left[A_{Cx5}\right] \tag{42}$$

$$\left[\dot{A}_{Cx6}\right]=\left[\tilde{\omega}_{56}\right]\cdot\left[A_{Cx7}\right] \tag{43}$$

$$\left[\dot{A}_{Cx7}\right]=\left[\tilde{\omega}_{67}\right]\cdot\left[A_{Cx7}\right] \tag{44}$$

$$\left[\dot{A}_{Cx8}\right]=\left[\tilde{\omega}_{78}\right]\cdot\left[A_{Cx8}\right] \tag{45}$$

For each: **A, B, C, D, E, F, G, H** and **M** point we obtain:

$$\left\{\overleftarrow{v}_A^{T_{cx}}\right\}=0 \tag{46}$$

$$\left\{\overleftarrow{v}_B^{T_{cx}}\right\}=0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\} \tag{47}$$

$$\begin{aligned}\left\{\overleftarrow{v}_C^{T_{cx}}\right\} &= 0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot[A_{Cx2}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{48}$$

$$\begin{aligned}\left\{\overleftarrow{v}_D^{T_{cx}}\right\} &= 0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot[A_{Cx2}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_4\}^T\cdot\left[\tilde{\omega}_{23}\right]\cdot[A_{Cx3}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{49}$$

$$\begin{aligned}\left\{\overleftarrow{v}_E^{T_{cx}}\right\} &= 0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot[A_{Cx2}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot\left[\tilde{\omega}_{23}\right]\cdot\\ &\cdot[A_{Cx3}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot\left[\tilde{\omega}_{34}\right]\cdot[A_{Cx4}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{50}$$

$$\begin{aligned}\left\{\overleftarrow{v}_F^{T_{cx}}\right\} &= 0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot[A_{Cx2}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot\left[\tilde{\omega}_{23}\right]\cdot\\ &\cdot[A_{Cx3}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot\left[\tilde{\omega}_{34}\right]\cdot[A_{Cx4}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_6\}^T\cdot\left[\tilde{\omega}_{45}\right]\cdot[A_{Cx5}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{51}$$

$$\begin{aligned}\left\{\overleftarrow{v}_G^{T_{cx}}\right\} &= 0+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot[A_{Cx1}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\\ &+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot[A_{Cx2}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_4\}^T\cdot\left[\tilde{\omega}_{23}\right]\cdot[A_{Cx3}]\cdot\\ &\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_5\}^T\cdot\left[\tilde{\omega}_{34}\right]\cdot[A_{Cx4}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_6\}^T\cdot\left[\tilde{\omega}_{45}\right]\cdot\\ &\cdot[A_{Cx5}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}+\{r_7\}^T\cdot\left[\tilde{\omega}_{56}\right]\cdot[A_{Cx6}]\cdot\left\{\overleftarrow{W}_{Cx}\right\}\end{aligned} \tag{52}$$

$$\begin{aligned}
\left\{\overleftarrow{v}_H^{Tcx}\right\} &= 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\
&\cdot \left[A_{Cx3}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \\
&\cdot \left[A_{Cx6}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_8\}^T \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\}
\end{aligned} \tag{53}$$

$$\begin{aligned}
\left\{\overleftarrow{v}_M^{Tcx}\right\} &= 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\
&\cdot \left[A_{Cx3}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \\
&\cdot \left[A_{Cx6}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \{r_8\}^T \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{S_8\}^T \cdot \left[\tilde{\omega}_{78}\right] \cdot \left[A_{Cx8}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\}
\end{aligned} \tag{54}$$

C. *Acceleration calculus*

These will be obtained by differentiating successively the (46) ... (54). For **A,** and **B**, we will obtain the accelerations from (55) and (56) relations. Similarly, we obtain the accelerations of the following points: **C, D, E, F, G, H**. The acceleration for **M** point is given by (57) relation.

$$\left\{\overleftarrow{a}_A^{Tcx}\right\} = 0 \tag{55}$$

$$\begin{aligned}
\left\{\overleftarrow{a}_B^{Tcx}\right\} &= 0 + \{r_2\}^T \cdot \left[\dot{\tilde{\omega}}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\}
\end{aligned} \tag{56}$$

$$\begin{aligned}
\left\{\overleftarrow{a}_M^{Tcx}\right\} &= 0 + \{r_2\}^T \cdot \left[\dot{\tilde{\omega}}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_3\}^T \cdot \left[\dot{\tilde{\omega}}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_4\}^T \cdot \left[\dot{\tilde{\omega}}_{23}\right] \cdot \left[A_{Cx3}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \left[\tilde{\omega}_{23}\right] \cdot \left[A_{Cx3}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_5\}^T \cdot \left[\dot{\tilde{\omega}}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_6\}^T \cdot \left[\dot{\tilde{\omega}}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_7\}^T \cdot \left[\dot{\tilde{\omega}}_{56}\right] \cdot \left[A_{Cx6}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \left[\tilde{\omega}_{56}\right] \cdot \left[A_{Cx6}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_8\}^T \cdot \left[\dot{\tilde{\omega}}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{r_8\}^T \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&+ \{S_8\}^T \cdot \left[\dot{\tilde{\omega}}_{78}\right] \cdot \left[A_{Cx8}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\} + \\
&\{S_8\}^T \cdot \left[\tilde{\omega}_{78}\right] \cdot \left[\tilde{\omega}_{78}\right] \cdot \left[A_{Cx8}\right] \cdot \left\{\overleftarrow{W}_{Cx}\right\}
\end{aligned} \tag{57}$$

III. NUMERICAL PROCESSING

For the cinematic analysis we consider known the geometrical elements. The calculus algorithm was elaborated with the MAPLE software's aid. The geometrical elements dimensions are: **L_{cx}=15,744 mm; L_1=12,5 mm; L_2=12,5mm; L_3=385 mm; L_4=5 mm; L_5=325 mm; L_6=10,7 mm; L_7=61,85 mm; L_8=50,42 mm.**

The generalized coordinate system variations from the 8 joints equivalent to the human lower limb are presented in the cinematic diagrams (figures: 2, 3, 4, and 5).

In figure 6, the human locomotion system's 3D virtual model is presented. This was designed by and simulated with the MSC NASTRAN software's aid. For virtual simulation we follow the procedures from [6] and we implement the motion laws resulted from the analysis developed previously (figures 2 3, 4 and 5), on a human subject without locomotion disabilities.

We mention that the motion laws were processed for each joint individually, in such a manner that the virtual model can perform a single gait in the walking activity.

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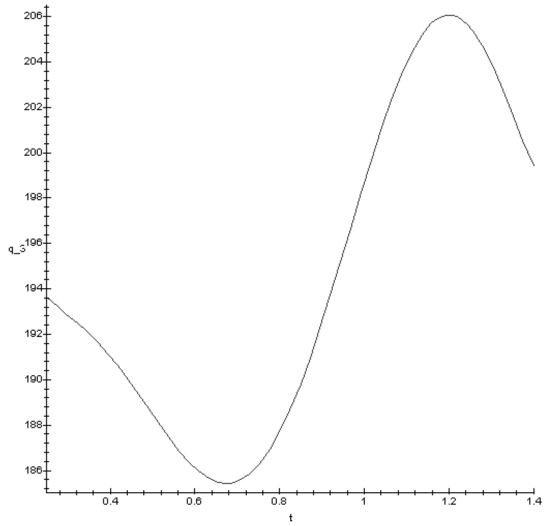

Fig. 2. **q3** variation angle [degrees], corresponding with the equivalent hip joint

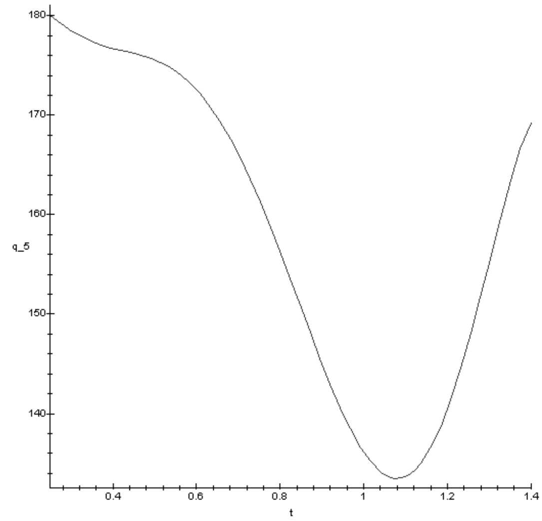

Fig. 3. **q5** variation angle [degrees], corresponding with the equivalent knee joint

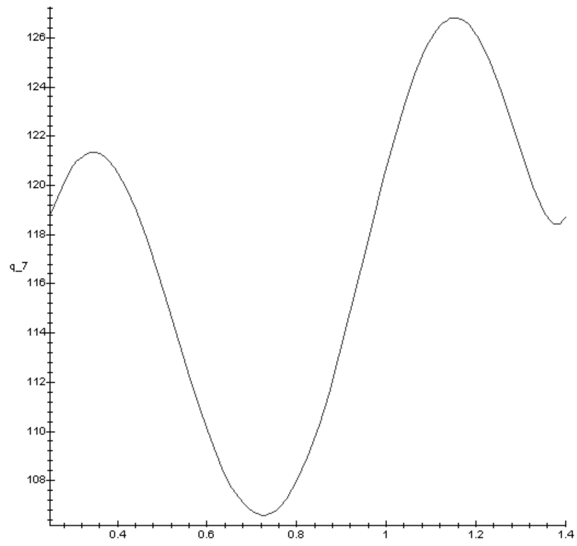

Fig. 4. **q7** variation angle [degrees], corresponding with the equivalent ankle joint

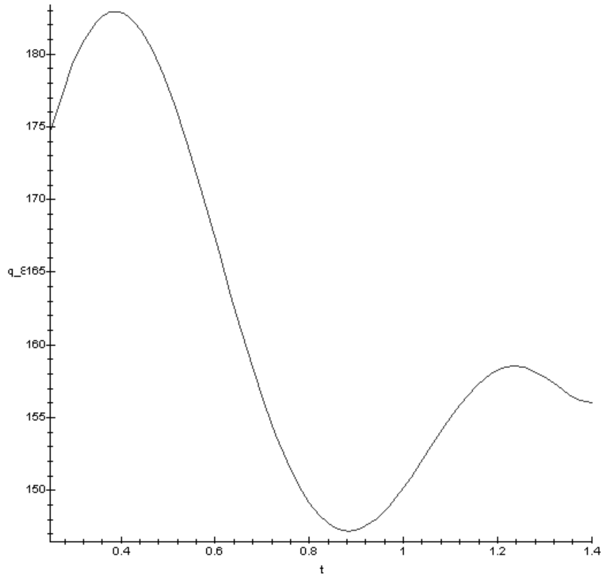

Fig 5. **q8** variation angle [degrees], corresponding with the equivalent foot joint

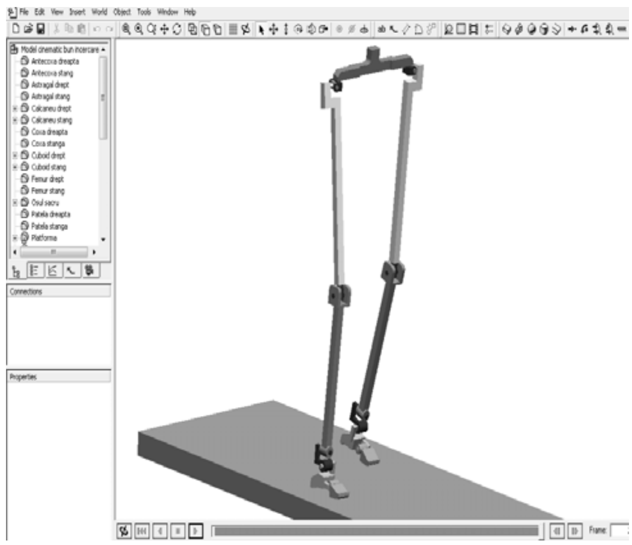

Fig. 6. 3D model, designed and simulated with MSC NASTRAN

IV. CINEMATIC RESULTS APPLICATION ONTO KNEE PROSTHESIS MECHANICAL SYSTEMS

Based on cinematic motion laws imposed for a knee prosthesis mechanism, we design a 3D virtual model of cam mechanism prosthesis, with the CATIA V5 R16 aid. This mechanism is similar with a one designed for human ankle prosthesis [7].

We integrated the FESTO YSR-20-25-C shock absorber in the prosthesis resistance structure, which enables some axial adjustments with a view to establishing the prosthesis alignment. Figure 7 shows the new knee prosthesis design. This is where we identify 1-femur component, 2-cilindrical joint, 3- cam follower, 4- cam, 5- tibia component, 6- FESTO shock absorber, 7-aditional shock absorber mechanism. After simulating the virtual model and validating the cam mechanism through calculation, we executed and adapted this prosthesis in accordance with an amputee's needs and suggestions. In figure 8, we present an aspect from the new prosthesis experimental tests, which were performed with SIMI Motion’s aid.

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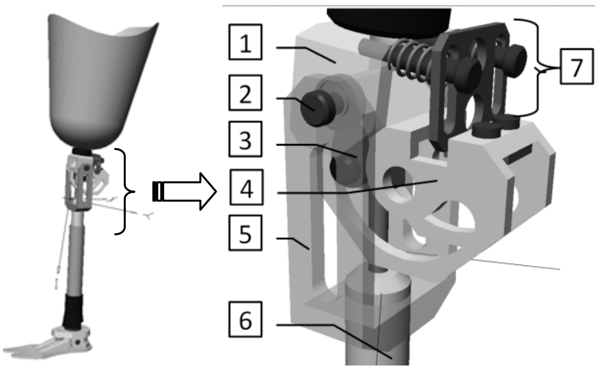

Fig. 7. Virtual model of the prosthesis used in human knee disarticulations

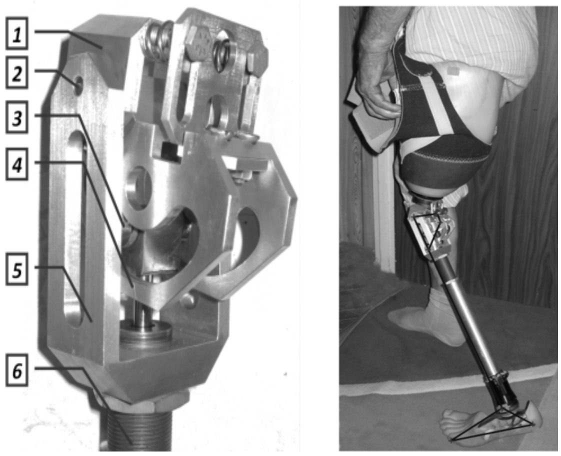

Fig. 8. The new knee prosthesis and an aspect from the new prosthesis experimental tests achieved with SIMI Motion software

V. CONCLUSION

For cinematic modeling we use a method which is based on simple matrices formalism with the possibility to implement on a computer program for the direct or inverse cinematic analysis. This method is valid for planar and spatial cinematic mechanisms with possibility to study the cinematic parameters in the absolute or relative motion mode. For the mathematical models processing corresponding to the cinematic analysis, a program under MAPLE programming language was elaborated.

It was elaborated a cinematic scheme for the human lower limb equivalent mechanism, based on some specialty literature references, but also with proper observations mainly for knee joint. Mathematical model were elaborated for position, speeds and accelerations determination, for some interest points, used for experimental modeling, according with a new prosthesis design for knee joint.

The novelty element which assures the prosthesis models design is represented by cam mechanisms.

Based on an experimental cinematic analysis of these prostheses, by using SIMI Motion software, the angular amplitude developed by this mechanism is appropriate with the one developed by a healthy human subject. So, for the human knee joint replacement mechanism, the angular amplitude for walking activity was 63 degrees (figure 9), and the one developed by a healthy subject was 65 degrees [9], [10].

The prosthesis presented in this paper is cheap, in comparison with the ones manufactured by the specialized prosthetic centers. The knee mechanism functionality validates the cinematic analysis of the human lower limb.

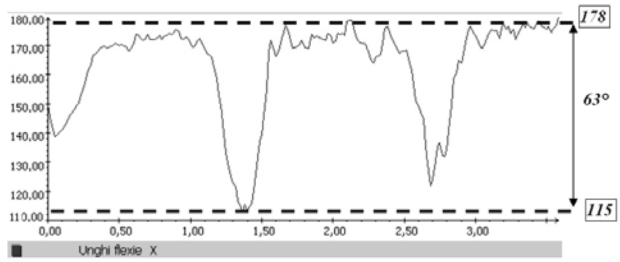

Fig. 9. The new prosthesis flexion/extension angular displacement variation, depending on time

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