State Feedback Based Linear Slip Control Formulation for Vehicular Antilock Braking System

Otis T.C. Nyandoro, Jimoh O. Pedro, Barry Dwolatzky, and Olurotimi A. Dahunsi

Abstract—**This paper presents the formulation of a slip-control model for purposes of performing slip tracking of target slip. Antilock braking system modelling is performed to develop a quarter car vehicle deceleration model for braking without cornering. Input-state based feedback linearisation is applied to the highly non-linear developed antilock braking system model. Input-state feedback linearisation is shown to provide a transformed linear ABS model while ensuring a verifiable stable state transformation. Lie algebra is used to formalise the analysis of the linearising transformation. Simulation results of a quarter car vehicle's braking dynamics demonstrate the validity of the approach along with the key development of a output to state transformation that facilitates the implementation of the linearisation approach as a mechatronic technique to antilock braking system control.**

Index Terms—**antilock braking systems, feedback linearisation with differential geometry, mechatronics, nonlinear dynamics, computational mechanics**

I. Introduction

ANTILOCK Braking Systems (ABS) have been in use in wheeled vehicles for numerous decades with the specific aim of avoiding the locking of wheels during braking and thereby improving braking performance [1]. As research into reducing braking times and distances under various road conditions has developed, a number of approaches have been suggested, most notably the slip control during deceleration [2], [3], [4]. Slip control primarily allows for the maximising of the braking friction coefficient, hence primarily providing the maximum braking force and achieving minimum braking times and/or distances [5].

During the braking of a wheeled vehicle, a braking force is applied to the wheel to reduce the wheel's angular velocity. In turn the reduced linearised wheel velocity leads to the wheel skidding relative to the road/driving surface. This skidding is called slip and while braking, slip varies from a minimum of zero to a maximum of unity [6]. Zero slip corresponds to the case when there is no braking force that is applied so the linearised wheel speed is the same as the car speed relative to the road surface. On the opposite extreme the unity or 100% slip case corresponds to the case when the wheel speed is zero but the car's speed is not zero as the car has not come to rest. The 100% slip case is also called $wheel\ locking$ or full-skid. Antilock braking systems (ABS) are utilised in wheeled vehicles with the specific aim of avoiding the locking of the wheel. The effective braking force while braking increases to a maximum as the slip increases from zero after which the effective braking force decreases as the slip value approaches 100% or the locking value [6]. Pacejka models and various works have demonstrated this explained observation [7], [8], [1].

The goal of an optimal Anti-lock Braking System (ABS) is to quickly reduce the speed of a vehicle from an initial travelling speed towards rest in either the minimum possible time or minimum possible distance, [1], [9], [3], []. Optimal control theory has been successfully applied to ABS to prove the need for slip control of ABS for various types of vehicle braking systems such as pneumatic brakes, electro-pneumatic brakes and electrical brakes in works such as [7], [8]. Various disturbances and ABS uncertainties such as road conditions, initial vehicle speeds, braking actuator dynamics in [10], [11], wheel bearing friction, suspension effects in [7] and wind resistance have also been treated in applying optimal theory and a range of other controllers to ABS in [4], [5], [12], [13], [8], [14], [15], [10] with significant degree of success for ABS which is a typical safety critical system as highlighted in [7],[1], [6].

A major challenge in designing controllers for ABS is the highly non-linear model. Additionally when enhancements such as suspension effects are included in the ABS model the above mentioned works and solutions are not as effective due mainly to the multiple ABS model inputs and also multiple outputs. Yet another challenge brought in by the high level of non-linearity in ABS is when discrete methods are to be incorporated as linear discrete analysis is the default approach for discretisation techniques. Thus this paper's main focus is to present a framework for linearising the highly non-linear ABS model while ensuring stability of the linearising technique as well as handling given system outputs and states. In particular feedback linearisation technique is used to establish a $stable\ linearisation\ technique$ that can avoid often complicated internal stability analysis.

A common disadvantage of feedback linearisation is internal stability, [16], [17]. In the work [6] a stability analysis condition is developed due to the often detailed internal stability condition analysis that is required when performing input-output feedback linearisation. In this paper an output to

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state transformation is developed to circumvent the detailed internal instability analysis while also providing an approach to linearise the highly non-linear ABS model.

The rest of this paper is structured as follows, first a model for ABS is obtained followed by slip control motivation, and the major contribution of formulating a stable linearisation approach for the ABS model. Simulation results for a linearised controller are provided to demonstrate the effectiveness of the linearising technique.

II. Modeling

A. Physical Model

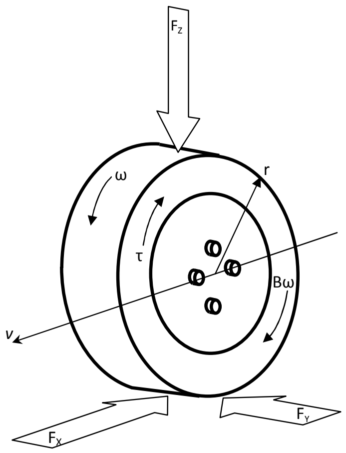

Fig. 1. Quarter car braking model

The general quarter-car model and formulation used in [7], [8], [1], [9], [6] is also utilised in this paper. The quarter-car wheel and braking system is shown in Fig. 1. At any point in time, t, the car has a forward longitudinal velocity, $v(t)$, the wheel has an angular velocity, $\omega(t)$. As the braking force is applied a braking torque τ is applied to the wheel. The wheel will have a component of the car's weight F_z exerted on it. It is assumed that the weight is equally distributed on the four wheels of the vehicle and that each of the four wheels of the car contribute equally to the car's total braking force. It is further assumed in this quarter-car model that cornering forces road roughness related forces are negligible.

B. Mathematical Model

The car while travelling has brakes applied at an initial time $t = t_0 = 0$ and comes to a stop at a final time $t = t_f$. As the brakes are applied the car's longitudinal velocity v is initially $v(t_0)$ and at $t = t_f$ the car's velocity will have come to zero i.e. $v(t_f) = 0$. Application of Newton's law to the quarter-car wheel and braking system shown in Fig.1 gives the governing equations of motion. The vehicle translational dynamics are:

$$M\dot{v} = -\mu(\lambda)F_z - C_x v^2 \qquad (1)$$

where M is the quarter-car's total mass, μ is the longitudinal friction coefficient, F_z is the normal force acting on the vehicle wheel, and C_x is the vehicle aerodynamic friction coefficient. For slip control as explained later μ is a function of the slip ratio λ as detailed later in Fig.2. The wheel rotational dynamics are:

$$J\dot{\omega} = \mu(\lambda)F_z r - B\omega - \tau_b \qquad (2)$$

where J is the moment of inertia of the wheel, r is the wheel radius, B is the wheel bearing friction coefficient, and τ_b is the braking torque. An electro-mechanical set of brakes is used to apply a braking torque, τ_b, on the disk/drum brakes. The weight component of the quarter-car, F_N, is given by:

$$F_N = Mg \qquad (3)$$

where g is the acceleration due to gravity.

By definition the slip ratio λ is:

$$\lambda = \frac{v - r\omega}{v} \qquad (4)$$

Typical relationship between μ and λ is given in Fig.2, and is modeled by the approximate equation, [7]:

$$\mu(\lambda) = 2\mu_0 \frac{\lambda_0 \lambda}{\lambda_0^2 + \lambda^2} \qquad (5)$$

The peak μ value μ_0 occurs at a λ value λ_0 and from Fig.2 for a dry asphalt road surface μ_0 = 0.9 and λ_0 = 0.22 respectively.

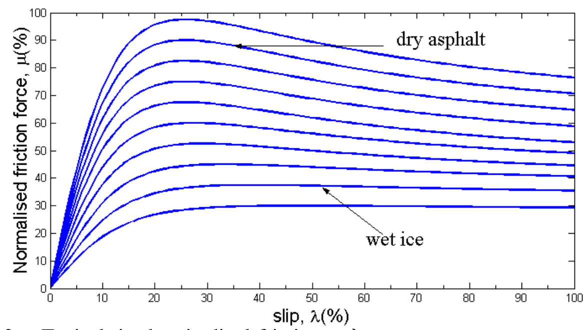

Fig. 2. Typical tire longitudinal friction μ-λ curves

Different road surfaces are modelled by different λ_0 and corresponding μ_0 values as they are unique for each road surface. Since the peak friction coefficient μ_0 is obtained when λ has the value λ_0, i.e. $\mu_0 = \mu(\lambda_0)$ hence the goal of slip control is to generate a braking torque τ_b to maintain the braking slip value always close or equal to its optimal value λ_0.

Suspension effects are assumed to be negligible so the vertical forces applicable to the quarter car model are the respective weight F_Z. Hence

$$F_N = F_Z \qquad (6)$$

The quarter car model is assumed to have negligible cornering forces F_Y and so is free from cornering torque considerations. Additionally, road roughness is thus lumped into the equation (5) and Fig.2 for the various typical road surfaces. The electro-mechanical braking system producing the torque τ_b is assumed to have a very high bandwidth and as such is a very fast actuator whose dynamics are thus considered negligible.

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III. Slip Control Formulation

A. ABS Model State Equations

From (1) we obtain the following non-linear state equations

$$\begin{aligned}\dot{x}_1 &= -\mu(\lambda)F_N/M - \mu(\lambda)k_w x_4/M - C_x x_1^2/M \\ \dot{x}_2 &= \mu(\lambda)F_N r/J + \mu(\lambda)k_w x_4 r/J - B\omega/J \\ &\quad -u_b/J \qquad (7) \\ \mathbf{y} &= [\lambda] \qquad (8)\end{aligned}$$

where the states are $\mathbf{x} = [x_1 \;\; x_2]^T$, $x_1 = v$, $x_2 = \omega$.
y is the output slip (4).
u_b is the braking torque τ_b as the system braking control input.

The system model can thus be defined in the state-space form as the non-linear single-input single-output system:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}\mathbf{u}(t) \qquad (9) \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) = [\lambda] \qquad (10)\end{aligned}$$

where

$$\begin{aligned}\mathbf{g} &= [0 \;\;\; 1/J]^T \qquad (11) \\ \mathbf{u} &= [u_b] = \tau_b \qquad (12)\end{aligned}$$

Highly non linear **f(x)** consists of all terms of (7) except for the terms in u_b. For stabilisation, and tracking control this ABS model thus requires linearisation.

B. Input-State Feedback Linearisation

Input-output feedback linearisation, $IOFBL$, is one approach to linearise a system and has been applied in various of our previous works [9], [18], [6]. Yet as highlighted in [6] $IOFBL$ often requires elaborate internal stability analysis. This is mainly because $IOFBL$ transforms the states of ABS by transforming the original ABS states and often resulting in hidden states. The stability of the hidden states then consequently demands of necessity the internal stability analysis.

Input-state feedback linearisation $ISFBL$ referred to FBL in this work avoids the above hidden states of $IOFBL$ by performing a state transformation that maintains the number of system states. Without hidden states the internal stability challenge is totally circumvented.

Yet the one key challenge of $ISFBL$ referred to as FBL is this work is that $ISFBL$ formulates a linear relationship between the states and the input. Thus $ISFBL$ only performs an input to state linearisation transformation as the name $ISFBL$ suggests. Linear reference tracking of slip is still not possible due to the key challenge of the nonlinear output slip equation. To avoid this problem this work proposes an initial state transformation such that the output is one of the ABS states. Though this transformation is not new to ABS analysis yet we propose the use of this transformation as a novel linearisation approach.

Thus this paper seeks to generate a linear transformed-input to slip-output relation for the non-linear ABS model. Additionally, FBL allows us to check on the stability of the linearising transformations on the ABS model and in so doing this paper identifies an output to state transformation for the slip controlled ABS model. Furthermore, we then design stable slip reference tracking feedback controllers for the linearised model.

Taking the total derivative of the state output equation (8) we get

$$\dot{y} = \dot{\lambda} = \frac{\partial\lambda}{\partial v}\dot{v} + \frac{\partial\lambda}{\partial\omega}\dot{\omega} \qquad (13)$$

Substituting (4) into (13) allows for the slip state equation formulation

$$\dot{y} = \dot{\lambda} \;=\; -\frac{\dot{v}}{v}\lambda + \frac{\dot{v} - r\dot{\omega}}{v}$$

and substituting (1) and (2) in the above slip-state equation gives a slip to braking input u_b equation of relative degree 1 when considered as an $IOFBL$ linearisation transformation:

$$\begin{aligned}\dot{y} &= \dot{\lambda}(t) = f_{ab}(\lambda) + g_{ab}(t)u_b(t) = u_{bnew} \qquad (14) \\ f_{ab} &= -\frac{\dot{v}}{v}\lambda + \frac{\dot{v}}{v} - \frac{\mu F_z r^2}{vJ} + \frac{Br\omega}{vJ} \\ g_{ab} &= \frac{-r}{vJ}\end{aligned}$$

From (14) $\dot{y}$ is a function of the braking input $u_b(t)$ and by letting u_{bnew} be the transformed braking input such that $\dot{y}_1 = u_{bnew}$ we thus have a linear relation between the transformed braking input u_{bnew} and the output slip, λ.

FBL theory states that $IOFBL$ is equivalent to $ISFBL$ if the relative degree of the $IOFBL$ linearisation is equal to the degree of the ABS system. Given that ABS model is of degree 2 from equation (7). Hence output to state transformation for $ISFBL$ must transform the ABS model from a system of degree 2 to and ABS model of degree 1 so as to be equal to the relative degree of $IOFBL$ as shown in (14) above.

C. Slip Control Criteria

The primary objective of the slip controller is to bring a car traveling with an initial speed v_0 down to stop in a shortest possible distance or time while using admissible control ($0 \leq \tau_b \leq \tau_{b_{max}}$). In doing so the slip value should rise to its optimum value λ_0 as fast as possible and track this value through the deceleration process with minimal deviation from the set reference value λ_0 until the car stops. The braking system should also use admissible braking torques, i.e. have limited control input through the braking process including initial transient response, steady state control and the final stage of the braking process as the car comes to rest.

Correspondingly the suspension system should have admissible active suspension forces i.e have limited transient and steady state suspension control input through the braking process. The suspension must also have a limited suspension travel as this is a physical restriction due to a finite space between the vehicle's wheel and body.

Fig. 3 shows the comparison results for locked wheel braking and perfect tracking of ideal slip control with no suspension effects. The ideal slip control is obtained by assuming an ideal braking actuator with zero transient time delays in its response. A similar braking actuator is also assumed for the locked wheel braking case, locking the brakes at $t = t_0 = 0$ and throughout the braking process until the car comes to rest. As can be observed from these results slip controlled braking gives far better results of about half the braking time and half the braking distance.

The goal of the FBL analysis for ABS with suspension effects is to at least achieve improved performance than the ideal slip controlled ABS results.

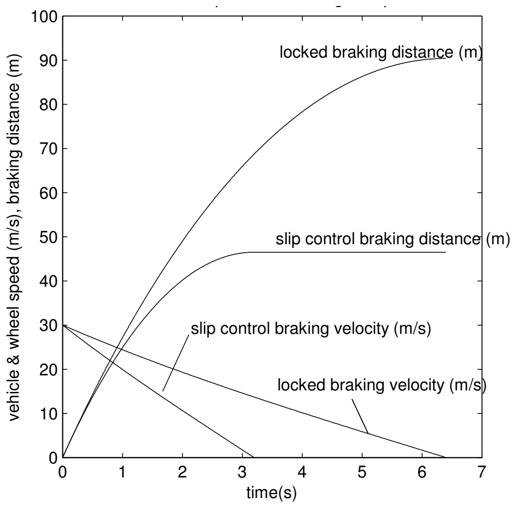

Fig. 3. locked and slip control braking for ABS.

For this quarter-car model the maximum braking torque is $2250Nm$ while the optimal slip value as obtained from the dry-asphalt graph on Fig.2 is $\lambda_0 = 0.2$ with $\mu_0 = 0.9$. The ideal braking time of $3.15s$ and the ideal braking distance of $46.5m$ from Fig.3 are to be used as a basis for evaluating the various modeling parameter variations. The suspension system travel trajectory is hand-tuned to provide an approximate 25% increase in nominal weight of the car body and wheel as exerted on the road surface. During the braking process the suspension travel is limited in its peak maximum travel so the individual wheel travel and body travel.

IV. Control Formulation

A. Linearising Input-Output Transformation

Control systems theory has various ways of linearising nonlinear systems the most applicable of which is $Feedback\ Linearisation$, see [16], [19], [17], [18], [6]. The Input-Output Feedback Linearisation, $IOFBL$, performed to obtain (14) is a linearising state transformation from $\mathbf{x}$ to $\mathbf{z}_o = [\mu_i]^T$.

$$\begin{aligned} \dot{\mathbf{y}} &= \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \dot{\mathbf{x}} \\ &= \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) + \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{g} u_b(t) \\ &= \mathbf{f}_{ab}(\mathbf{x}) + \mathbf{g}_{ab} u_b(t) \end{aligned} \tag{15}$$

Using differential geometry notation (i.e. Lie derivatives and diffeomorphism notation) see [17], [6]

$$\begin{aligned} \dot{\mathbf{y}} &= \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \dot{\mathbf{x}} = \nabla \mathbf{h} (\mathbf{f} + \mathbf{g} u_b) \\ &= L_{\mathbf{f}} \mathbf{h}(\mathbf{x}) + L_{\mathbf{g}} \mathbf{h}(\mathbf{x}) u_b \end{aligned} \tag{16}$$

$$u = \frac{1}{L_{\mathbf{g}} \mathbf{h}} \left(-L_{\mathbf{f}} \mathbf{h} + u_{b_{new}} \right) \tag{17}$$

The input transformation on the control law (14) gives the linear relation of the form $\dot{y} = u_{new}$. The transformed state is $\mathbf{z}_o = [\mu_1]$. $IOFBL$ theory allows us to choose $\mu_1 = \lambda$ see [6], [16].

Thus we have a linearised system utilising practically feasible new states additionally λ is our output and enables the direct design of the controller for λ and γ reference tracking purposes. The ABS control input torque can be generated from (14).

B. Linearising Input-State Transformation

As noted in FBL theory see [16], [20] $ISFBL$ is equivalent to $IOFBL$ if the relative degree (1 for the ABS model) from $IOFBL$ is equal to the system degree. Since the ABS model system degree from (7) is of degree 2 we choose the output variable as the only system state such as to have a system degree of 1. Hence the ABS model equation becomes (14).

FBL requires the following procedure to be performed to effect linearisation by $ISFBL$ see [16], [20].

Construct the vector field
$\mathbf{M} = [\mathbf{g}_{ab}, \quad ad_{\mathbf{f}_{ab}} \mathbf{g}_{ab}, \ldots, \quad ad_{\mathbf{f}_{ab}}^{n-1} \mathbf{g}_{ab}]$

with $n = 1$
so

$$\mathbf{M} = \mathbf{g}_{ab} = \frac{-r}{Jv} \tag{18}$$

Since $\mathbf{M}$ is both controllable and involutive for both $v \neq 0$ and for finite v, FBL theory allows the formulation of the new linearised state z such that:

$$\nabla z \mathbf{g}_{ab} \neq 0 \tag{19}$$

so

$$\frac{\partial z}{\partial \lambda} \mathbf{g}_{ab} \neq 0 \tag{20}$$

from which is chosen the new linear state z as $z = \lambda$ simultaneously being the linearising state transform.

Finally FBL allows us to formulate the input transformation as

$$u_b = \tau_b = \alpha(\lambda) + \beta(\lambda) u_{b_{new}} \tag{21}$$

$with$

$$\alpha = \frac{L_{\mathbf{f}_{ab}}^{n} z}{L_{\mathbf{g}_{ab}} L_{\mathbf{f}_{ab}}^{n-1} z} \tag{22}$$

$$\beta = \frac{1}{L_{\mathbf{g}_{ab}} L_{\mathbf{f}_{ab}}^{n-1} z} \tag{23}$$

with n=1 we get

$$L_{\mathbf{f}_{ab}} z = \nabla z f_{ab} = f_{ab} \tag{24}$$

similarly

$$L_{\mathbf{g}_{ab}} L_{\mathbf{f}_{ab}}^{0} z = L_{\mathbf{g}_{ab}} z = \nabla z g_{ab} = g_{ab} \tag{25}$$

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so the input transformation is thus

$$u_b = \tau_b = \alpha(\lambda) + \beta(\lambda) u_{b_{new}} \quad (26)$$
$$= \frac{-f_{ab}}{g_{ab}} + \frac{u_{new}}{g_{ab}} \quad (27)$$
$$= \frac{-Jvf_{ab}}{r} + \frac{Jvu_{new}}{r} \quad (28)$$

and the new state equation is

$$\dot{\lambda} = u_{b_{new}} = \mathbf{f}_{ab}(\mathbf{x}) + \mathbf{g}_{ab} u_b(t) \quad (29)$$

Remarks

- the transformation from ABS states v and ω to the new ABS single state λ is a linearising state transform
- the transformation from ABS states v and ω to the new ABS single state λ is also an output to state transformation
- (16) and (17) are obtained by $IOFBL$ while the same equations are obtained for $ISFBL$ in (29) and (26)
- thus equivalence of $IOFBL$ and $ISFBL$ are thus shown in the results of pairs (16),(17) and (29), (26) as is required by FBL theory
- the fact that an ABS model initially with two system states is reduced to a system with only one state to which $ISFBL$ is successfully applied warrants further analysis for internal stability purposes.

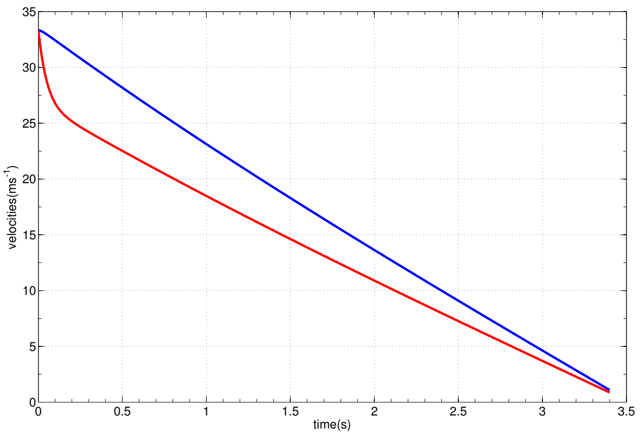

Fig. 4. Braking velocities with with IS-FBL

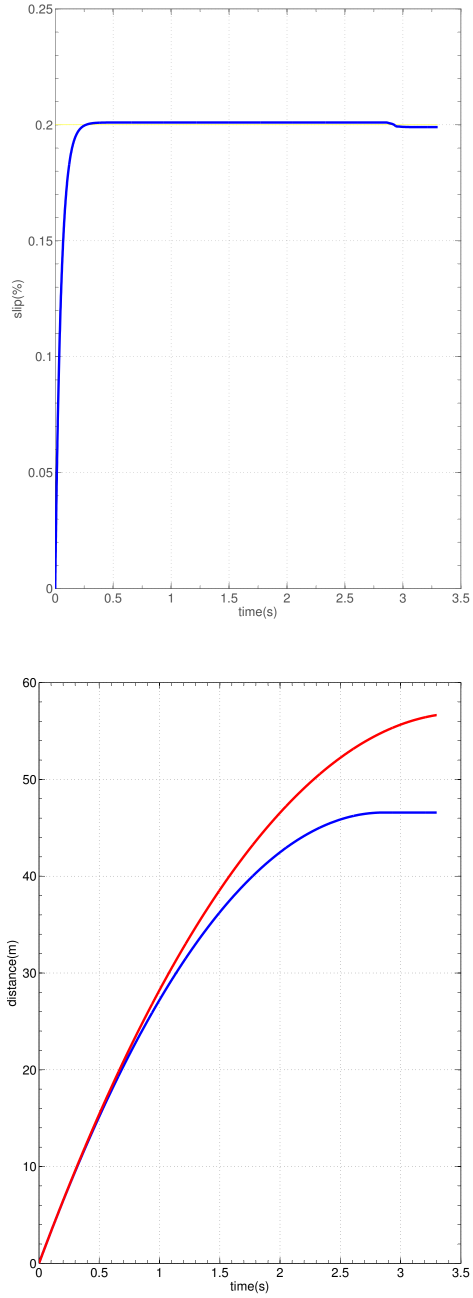

Fig. 5. Braking slip (top) and distances (bottom) with IS-FBL

C. Stability Analysis

The transformation from an ABS model with two states namely v and ω to only one state λ means that one of the states has been internalised, which further demands that the stability of this transform be analysed. In fact the transform is made valid since the linearised system tracks the slip instead of tracking both v and ω in the non-linear ABS.

Simultaneously a control law (26) is formulated only for the linear tracking of λ while the internal state v needs no control law for its trajectory. Without an internal state v the linearised slip λ controller would only be a system that tracks the slip but would have unobservable velocity. So internalising the state v makes v still observable and only λ is both observable and controllable. The v state is known to have stable dynamics as the car slows to rest due to the aerodynamic friction force $C_x v^2$ if no braking torque is applied. Hence the stable zero dynamics of v mean that the internalised v dynamics are also stable as long as $v \neq 0$ and indeed the linearising FBL transform is also stable.

D. Linear Tracking Slip Controller Design

Linear pole placement is finally applied to (29)

$$\lambda = u_{new} = \dot{z}_d - a_0 \tilde{z} \quad (30)$$
$$where \quad \dot{z}_d = 0$$
$$and \quad (31)$$
$$\tilde{z} = z - z_d = \lambda - \lambda_0 \quad (32)$$

hence for asymptotically stable slip tracking error dynamics

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TABLE I
PARAMETERS OF THE QUARTER-CAR MODEL USED FOR SIMULATION.

Parameters	Parameters	Parameters
B = 0.08 kgm^2/s	C_x = 0.856 kg/m	I = 1.6 kgm^2
m_s = 400 kg	M = 440 kg	m_u = 40 kg
r = 0.3 m	$\mu_0 = 0.9$	$\lambda_0 = 0.2$
$\alpha = 0.1$	$\tau_{b_{max}} = 10000$ Nm	$t_{settling}$ = 0.3s

$$\dot{\tilde{z}} + a_0 \tilde{z} = 0 \tag{33}$$

choose a negative a_0 that gives fast tracking dynamics without saturating the control torque in this case $a_0 = -200$ was chosen.

V. RESULTS AND DISCUSSION

Results are compared with perfect slip reference tracking rsults of Fig.3. The settling time for slip control is 3.4s from Fig.5. There is a near perfect time to come to rest from a speed of $33.\dot{3}ms^{-1}(120km/hr)$ as noted in Fig.4 in 3.4s instead of 3.2s. The near perfect braking time is due to the presence of the formulated $ISFBL$ based linear slip controller. Similarly the braking distance is near perfect as seen in Fig.5 for the ABS slip control. The braking distances for ABS with perfect slip tracking, Fig.3, and with $ISFBL$ formulated linear pole placement controller are 48.5m instead of 46.5m. Yet again a near perfect braking performance as measured by the braking distance

A point to note is that all the above tests are run until "rest" yet stopped when the vehicle's speed is about $1ms^{-1}$. The tests are run to as near $0ms^{-1}$ as possible because firstly the stability of the linearising transform is not guaranteed when the vehicle velocity is $0ms^{-1}$ and secondly that the slip itself becomes undefined when the vehicle velocity reaches $0ms^{-1}$. Hence it is traditional to apply slip controlled ABS until just before the car comes to rest. For our tests $1ms^{-1}$ is regarded as near $0ms^{-1}$.

VI. CONCLUSION

This paper has demonstrated a linearising technique for a highly non-linear ABS model. The feedback linearisation approach presented can be applied to a wide range of ABS non linear model parameters that introduce non-linearities. Simulation results demonstrate a consequent simple but effective linear system analyses where a linear controller as basic as pole placement is applied to significantly improve the braking performance of ABS. A key feature is the identification of a transformation that not only linearises the input output relationship but also simultaneously identifies the output as a new system state that facilitates a linearisation transform equivalent to input-output linearisation.

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