Optimal Control of Water Supply in Pipe Networks by Retention Tanks: A Linear Case

Nagib G. N. Mohammed

Abstract. The methodology for optimal control of supply in water networks is presented. It is based on the assumption that the water network exhibits a steady state of flow and the water heads in the network's nodes can be measured for an inspected part of the network. Making use of the graph-based model of a water network and employing the so-called Virtual Distortion Method, the problem of optimal control can be formulated as a constrained optimisation problem. The water supply is minimised subject to constraints on the water head in the network's nodes. The optimal solution is expected for active constraints. Analogies between the presented water network modelling and the truss modelling in structural mechanics are described. The paper deals with the optimal control of water supply in pipe networks based on the graph-based model of a water network, the problem of optimal control has been formulated as a constrained optimisation problem in the paper. Linear constitutive relation (water flow vs. pressure head) has been assumed in order to develop the first step of the methodology. However, generalisation of the proposed approach for a piece-wise-linear case is planned in the near future.

Key words: *Water supply, optimal supply control, VDM based design.*

1 INTRODUCTION

Global demand for water is continuously increasing due to population growth, industrial development, and improvements of economic conditions, while accessible sources keep decreasing in number and capacity, moreover, the applications involving manipulation and transport of water and fluids in general demand high power consumption. The optimal use of such water supply networks seems to be the best solution for the present and thus it is necessary to carefully manage water transfer [6, 7]. A software tool for signal processing in control of supply in water networks is presented. It is assumed that the water pressure in the network's nodes in a distance from a controlled inlet can be measured and also that the inlet pressure can be modified in real time in a controlled way. Then, making use of the analytical network model (cf.Refs.3,1) of this installation and using presented below, the so-called Virtual Distortion Method (VDM), the control of water supply can be performed. The problem of management of water sources is more and more important in the world scale. Therefore, there is a requirement for an automatic water supply control. The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed below numerical procedure, the correction of water supply can be determined.

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The proposed methodology is based on the so-called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation (Ref. 2). This technique (called *Piezodiagnostics*) is focused on efficient numerical performance of inverse, non-linear, dynamic analysis. The crucial point of the concept is pre-computing of structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in the so-called *influence matrix* allow composition of all possible linear combinations of the influence of local non-linearities (due to defect) on final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modelling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

2 DEFINITIONS AND LINEAR ANALYSIS

Let me describe the *network analysis* (cf. Ref.4) based approach to modelling of water systems using oriented *graf* of small example shown in Fig .1, with topology defined by the following incidence matrix:

$$L=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \tag{1}$$

where rows correspond to the network's nodes while columns correspond to the branches.

Defining the following quantities describing the state of the water network:

H – the vector of water head in network's nodes
ε - the vector of pressure head in network's branches
Q – the vector of water flow in network's branches
R – the vector of hydraulic compliance in network's branches (depends on pipes' cross-sections, length, material, etc.)

$$R = \frac{K^2}{l},$$

K- the characteristic of the element, l - the element's length,
the following equations governing the water distribution can be formulated:

- equilibrium of inlets and outlets for nodes:

$$\mathbf{L}\ \mathbf{Q} = -\mathbf{q} \qquad (2)$$

- continuity equation for the network's branches:

$$\mathbf{L}^{\mathbf{T}}\ \mathbf{H} = \varepsilon \qquad (3)$$

- constitutive relation governing local flow in branches
-

$$\mathbf{Q} = \mathbf{R}\ \varepsilon \qquad (4)$$

where **q** denotes external inlet to the system.

The constitutive relation (4) is non-linear. Nevertheless, let us assume temporarily linearity of this relation. Substituting Eqs. (4) and (3) to (2), the following formula can be obtained:

$$\mathbf{L}\ \mathbf{R}\ \mathbf{L}^{\mathbf{T}}\ \mathbf{H} = \mathbf{q} \qquad (5)$$

Describing the water network shown in Fig.1, the above set of equations takes the following form

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (6)$$

where:

$$q_4 = R_4'(H_0 - H_4),$$

$$R = \frac{K^2}{l},$$

K- the characteristic of the element,
l - the element's length,
H – denotes the water pressure in the node (*height* of water)
q – denotes the flow in the branch,
and it was assumed that the network is supplied only through the node No.1 (inlet with intensity q_1) and the only outlet is through the node No.4 (the cofiecient R_4' =1).

$R_2' = R_3' = 0$, what means, that the outlets in nodes No.2 and 3 vanish.

3 PROBLEM FORMULATION

$$\min f = \min \sum_i \gamma_i q_i^{RT} = \min \sum_i \left(\gamma_i \sum_m B_{jm} H_m^{RT(i)} \right) \qquad (3)$$

3.1 Linear programming:

Assuming temporarily linearity of constitutive relation, the control problem for a network with constant inlet, equipped with a few retention tanks, can be formulated as follows:

where:
γ – the coefficients (control variables) modifying the unit inlet q^{RT} at Retention Tanks (RT),
q^{RT} – unit inlet at RTs,
$H^{RT(i)}$ – water head at all nodes, of the network due to unit supply by the i-th RTs.
And:

$$B = L\ R\ L^T \qquad (8)$$

The used indices refer to the following ranges:
i - over retention tanks
m - over all nodes
j – fixed index corresponding to the node in which the i-th RT is jointed to the network. The index j determines the appropriate row of the matrix B

The objective function is subject to constraints on a minimum value of the required water head in a set of outlet nodes:
plus a set of engineering constraints:

$$\gamma_i \geq 0 \quad \text{if} \quad H^* \geq H_{min}^{MS}$$
$$\gamma_i < 0 \quad \text{if} \quad H^* < H_{min}^{MS} \qquad (10)$$

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saying that there are two operational modes of the system. Namely, water is either supplied from all retention tanks to the network or it is taken by all retention tanks from the network. So, a situation in which water is supplied from tank A, and simultaneously taken by tank B is not permitted. The notation in the formula (8) is the following:
H – total water head,
H^* – arbitrarily determined minimum required level of water head,
H^{MS} – water head due to the Major Source (MS) supply,
H^{MS}_{min} – minimum value in the vector of water head due to the MS supply

Let me now discuss the problem of control by retention tanks for the network, shown in Fig. 2. The network is supplied through the node No. 1 (constant inlet with intensity q_1) and equipped with 6 controllable retention tanks, incorporated to the network at the nodes Nos. 2, 4, 5, 9, 12, 14. The outlets are located in the nodes Nos. 8, 13, 15.

The optimisation problem posed by (7), (8), (9) is a linear programming problem, which may be solved by standard optimisation procedures i.e. SIMPLEX [4]. The network was tested with 9 different levels of supply, ranging from 0 l/s (level 1) to 2 l/s (level 9), in several non-uniform intervals. Fig. 3 justifies the application of constraints (8) at the highest supply level 9.

For the unconstrained case (without constraints (9) as many as six tanks have to be employed for optimal control, supplying water to or taking it from the network. For the constrained case, only three tanks are operational, all of them supplying water to the network. It will be shown later on that the solution with 3 tanks is very close to the solution with 6 tanks, however much more practical form the engineering point of view.

3.2 One fixed inlet + controllable retention tanks (linear)

Let me now find the water head distribution for one fixed inlet as well as 6 tanks built into the network for linear case. Considering the constraints (9), Fig. 4 presents contribution of each tank to the optimal value of the objective function at certain level. So, it is apparent that for low water heads (below H^{MS}_{min}), the dominant role of taking water from the network is played by the tank No. 2 (highest contribution at levels 1-4). For high water heads (above H^{MS}_{min}), the dominant role of supplying water to the network is played by the tank No. 6 (highest contribution at levels 8-10).

Taking the facts into account and having in mind a practical engineering solution, it is proposed to consider a network equipped with only 2 retention tanks, as shown in Fig. 5, instead of 6. In order to prove that the configuration of retention tanks, proposed in Fig. 5, is reasonable, SIMPLEX optimisation has been carried out for the case and compared with the case of 6 tanks. Fig. 6 shows a difference in the objective function (6) value for the two cases. It is clear that the qualitative difference of performance for 2 and 6 tanks is very little.

However, the quantitative difference is quite substantial, which is demonstrated in Fig. 7. For the case of 2 tanks working in the network, only one tank is active within the water head level 3-7. At the remaining levels, two (all) tanks are active. For the case of 6 tanks working in the network, only one tank is active for levels 5, 6, two tanks are active for levels 3, 4, 7, 8 and more tanks are employed for other levels. So, for providing a reasonable engineering control, the configuration with 2 tanks is much more convenient, as the number of tanks is drastically reduced, while their performance is almost the same (cf. Fig. 6).

4 CONCLUSIONS AND FURTHER STEPS (non-linear case)

It has been demonstrated (on basis of analysis of particular cases) that optimal control of water supply for the system with linear constitutive equations require solving of two sets of linear equations before each modification of the inlet pressure. However, taking into account real, non-linear constitutive characteristics the optimal control problems become more difficult and cannot be converted just to solving linear equation systems. Nevertheless, further exploitation of analogy between elasto-plastic structures and water networks can be examined. Assuming piece-wise-linear constitutive relations describing water flow in the considered networks and introducing concept of *influence matrix,* an analogy to the problem of load minimisation for elsato-plastic structures (with deflections of chosen nodes not lower than some given values) can be explored. In the consequence, an algorithm similar to that for VDM based load maximisation for elasto-plastic truss structures (Ref.5) can be expected. This path will be explored in the separate paper.

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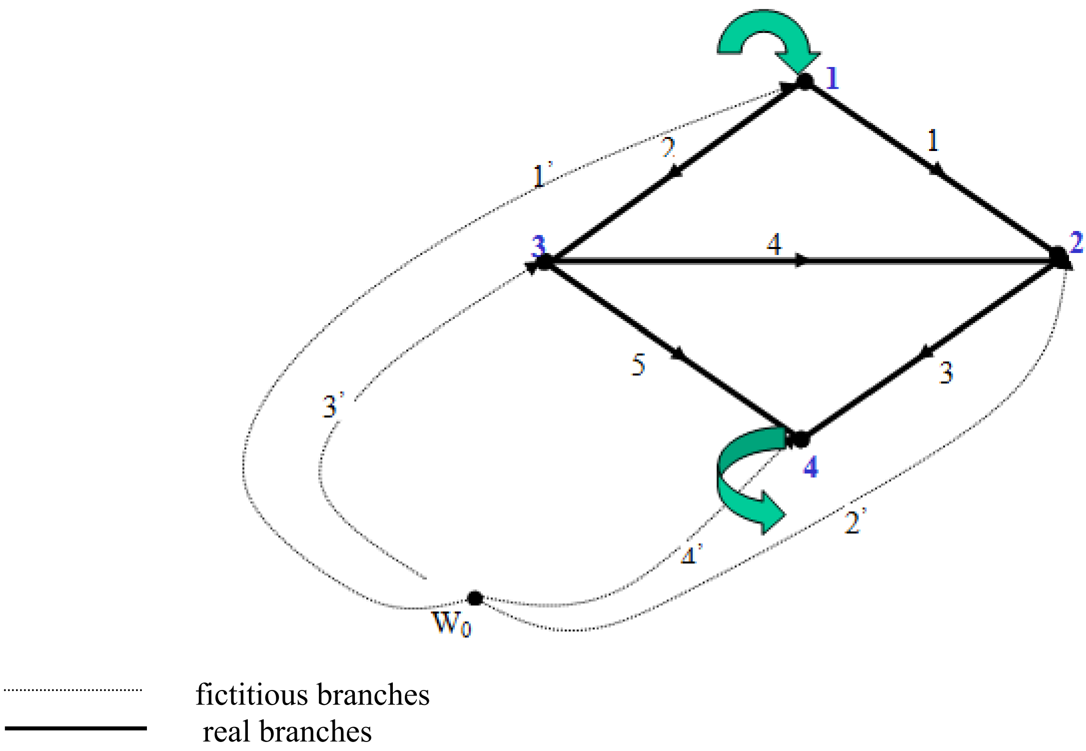

Figure 1. Network analysis (cf. Ref.4) based approach to modelling of water systems using oriented graf of small example

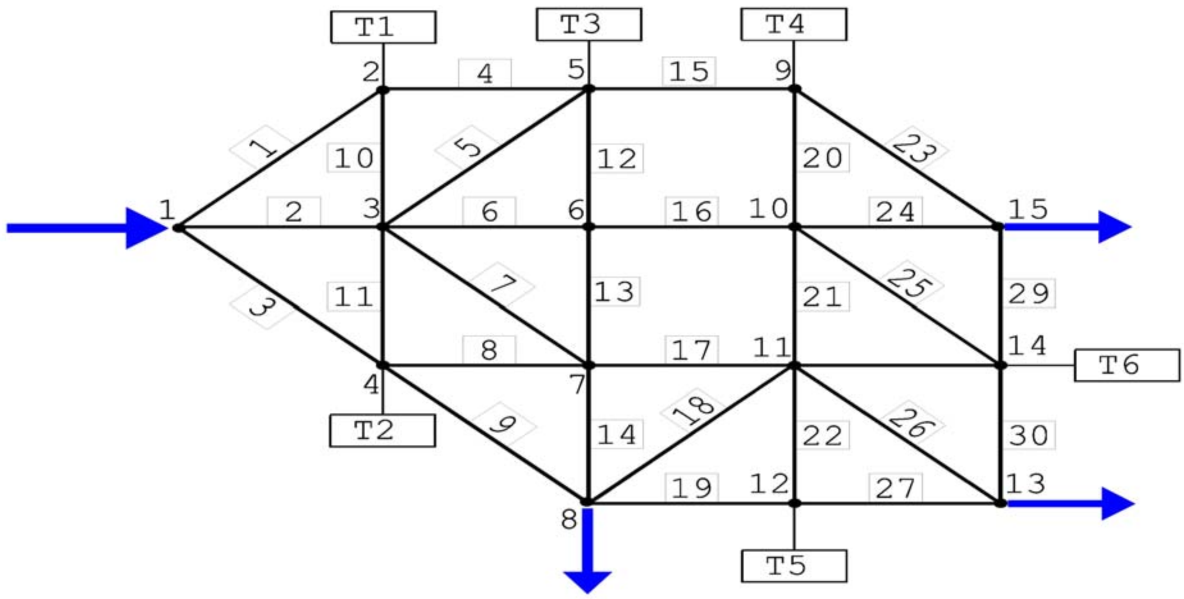

Fig. 2 Water network with one fixed inlet and six retention tanks

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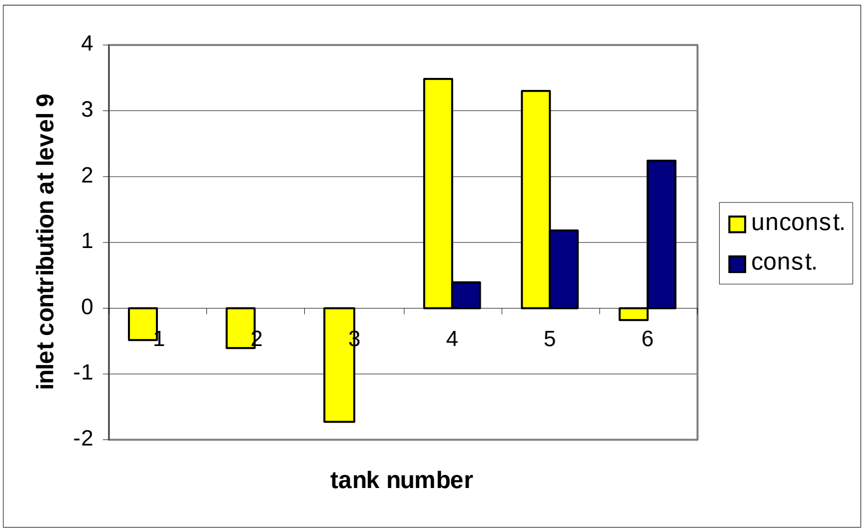

Fig. 3 Influence of constraints (9) on the SIMPLEX solution

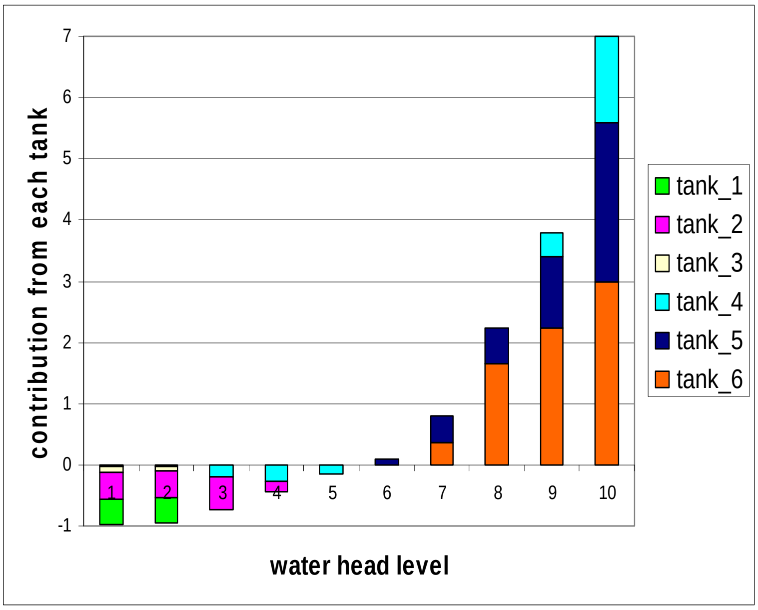

Fig. 4 Contribution of each tank to the objective function value at the optimum

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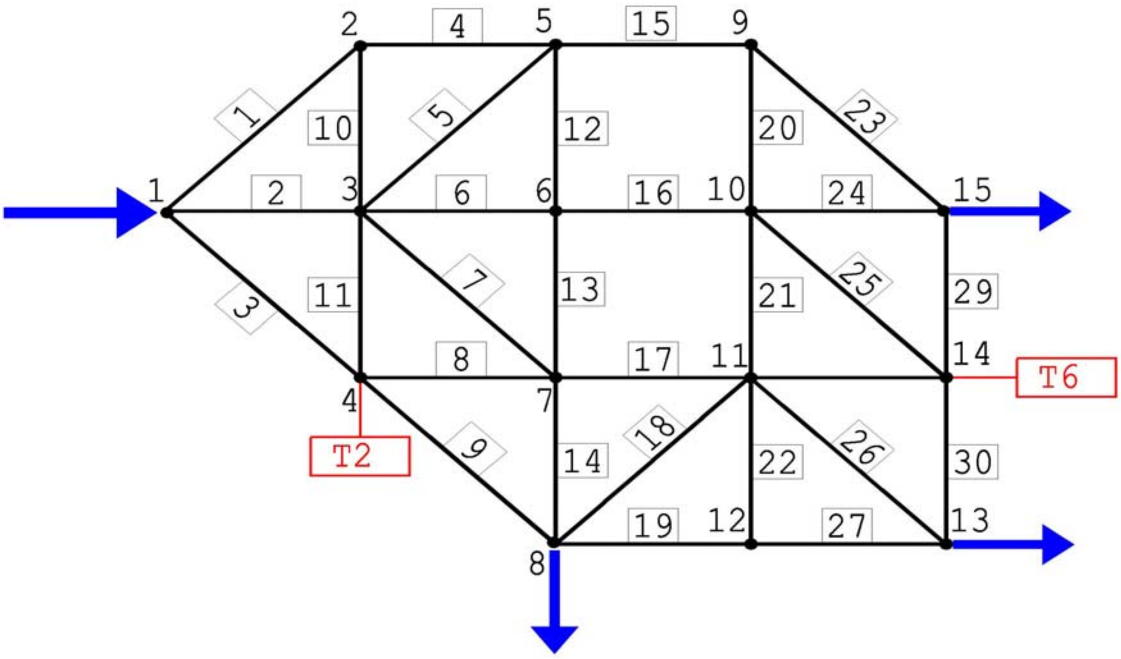

Fig. 5 Practical configuration of retention tanks in the network

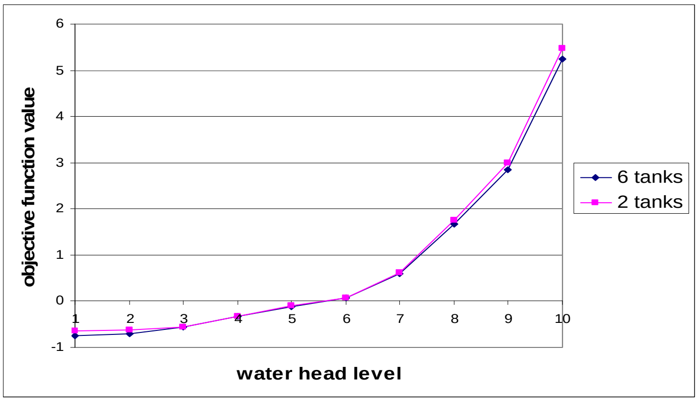

Fig. 6 Objective function values for the cases of 2 and 6 tanks in the network

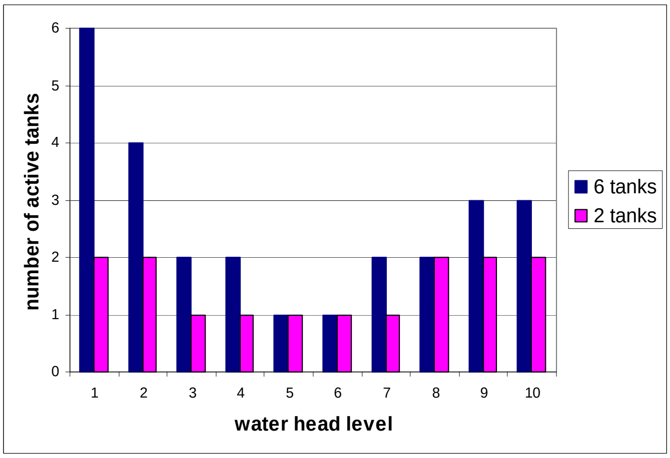

Fig. 7 Number of operational tanks for the cases of 2 and 6 tanks in the network