On Possible and Necessary Optimality of Schedules in the Single Machine Scheduling Problem with Imprecise Processing Times and Due Dates

Siamak Haji Yakhchali, Ali Husseinzadeh Kashan

***Abstract*—The possible and necessary optimality of a given schedule for the problem of minimizing maximum tardiness on single machine with imprecisely defined processing times and due dates is analyzed. Imprecise processing times and due dates are represented by means of interval or fuzzy numbers. In the interval case, it is shown that both the problems of ascertaining whether a given schedule is possibly and necessarily optimal are polynomially solvable.**

The results are extended to the problem with fuzzy processing times and due dates. Novel mathematical programming formulations are proposed for calculating the degree of possibility and necessity that a given schedule for the problem is optimal.

***Index Terms*— Scheduling; Possibility and necessity; Fuzzy scheduling; Maximum tardiness**

I. Introduction

The fuzzy scheduling problem which seems to be an interesting alternative to the deterministic and stochastic approaches constitutes an important and challenging problem. This problem has received intensive attention in the recent decade e.g. [15] and [17]. Dubois et al. [6] discussed two very distinct approaches of fuzzy scheduling in the literature. In the first approach, fuzzy sets are used to model local or global requirements in the form of flexible constraints. Flexible requirements include due-dates, release times, and durations, e.g. [7]. In the second approach, the aim is to analyze the main characteristics of a scheduling problem when data is ill-known and modeled by fuzzy numbers, in the setting of possibility theory [9]. Based on second approach in this paper, fuzzy numbers express uncertainty connected with the ill-known parameters.

One of the best known problems in scheduling theory is the problem of minimizing maximum tardiness on single machine. This problem can be solved optimally in the crisp case by EDD rule (Earliest Due Date) [1]. Several fuzzy approaches to the single machine scheduling problems were appeared. Ishii and Tada investigated a two objective problem with fuzzy precedence relation [13]. Han et al. [12] provided a generalized problem with fuzzy due dates and controllable machine speeds. Chanas and Kasperski [3] have considered a single machine scheduling problem with fuzzy processing times and fuzzy due dates and showed that the Lawler's algorithm [14] can be generalized into the fuzzy case. They [4] defined the fuzzy tardiness of a job in a given sequence as a fuzzy maximum of zero and the difference between the fuzzy completion time and the fuzzy due date of this job. They [5] introduced the notions of possible and necessary optimality of a given schedule and determined the possibility and necessity of optimality of a schedule in the single machine scheduling problem with the objective of minimizing the weighted completion time when the processing times are fuzzy numbers. In this paper, the problems of determining the possible and necessary optimality of a given schedule in the single machine scheduling problem with the objective of minimizing the maximum tardiness are considered. The literature on this realistic generalized problem is completely void.

After positioning this paper in the scope of the single machine scheduling problem under interval processing times and due dates, a complete solution to the problem of ascertaining whether a given schedule is possibly and necessarily optimal schedule are provided. Based on some propositions, it is shown that proposed algorithms solve the problem in polynomial time. Then, the obtained results are generalized to case of fuzzy processing times and due dates and the degrees of possible and necessary optimality of a schedule are computed.

II. Terminology and Representation

The scheduling problems to be dealt with throughout this paper can be stated as follows. A set $J=\{1,2,\ldots,n\}$ of jobs has to be processed on a single machine which can process only one job at a time. With each job j, $j\in J$, we shall assume a processing time p_j and a due date d_j. A given schedule $\theta=(\theta_1,\theta_2,\ldots,\theta_n)$ denotes a permutation of jobs where θ_j address the index of the job sitting in position j. It is assumed that there is no precedence relation between jobs, no idle time is allowed and all jobs are available at time zero (there are no release times in the problem). The objective function is minimizing the maximum tardiness defined as follows:

$$\min T_{max} = \min \max_{j=1,\ldots,n}\{T_j\} = \min \max_{j=1,\ldots,n}\{\max(0, C_j - d_j)\} \quad (1)$$

where C_j is the completion of job j, $j\in J$, leaving the system. Using the three notation $\alpha|\beta|\gamma$, the problem is represented

Siamak Haji Yakhchali, Department of Industrial Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran; e-mail: yakhchali@ut.ac.ir

Ali Husseinzadeh Kashan, Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran, e-mail: a.kashani@aut.ac.ir.

as $1||T_{max}$. Proposition 1 characterizes the optimality of a given schedule in the problem $1||T_{max}$.

Proposition 1: The necessary and sufficient condition for schedule θ to be optimal is that:

$$d_{\theta_j} \le d_{\theta_{j+1}} \quad j = 1, \dots, n-1 \qquad (2)$$

Proof: The proof is based on jobs interchange.■

A. The notion of Configurations

Assume that each processing time is assigned to a set of possible time P_j under the form of a closed interval. $P_j = [\underline{p}_j, \overline{p}_j]$ means that the real processing time p_j of the job j is not precisely known, but lies between $\underline{p}_j$ and $\overline{p}_j$ ($p_j \in P_j$). Each due date is assigned to a set of possible time d_j under the form of a closed interval, $D_j = [\underline{d}_j, \overline{d}_j]$. Intuitively, if the processing times and due dates are interval, then the value of the objective function, T_{max}, will be also interval.

The notation of *configuration* denoted by Ω has been defined by Buckley [2] to relate the interval case to the deterministic case of classical problems. We redefine a configuration as a tuple of processing times and due datas, $(p_1, p_2, \dots, p_n, d_1, d_2, \dots, d_n)$, such that $\forall j \in J,\ p_j \in P_j$ and $d_j \in D_j$. For a configuration Ω, $p_i(\Omega)$ and $d_i(\Omega)$ will denote the processing time and due date of job j, respectively. H denotes the set of all possible configurations. A configuration defines an instance of the deterministic scheduling problem (classical single machine scheduling), to which the classical methods can be applied.

B. Notions of Optimality Under Interval Processing Times and Due Dates

Using configurations, the possible and necessary optimality of solutions can be defined as follows.

Definition 1: A schedule θ is possibly optimal if and only if there exists a configuration Ω, $\Omega \in H$, such that θ is optimal in the usual sense in the configuration Ω.

Definition 2: A schedule θ is necessarily optimal if and only if for each configuration Ω, $\Omega \in H$, θ is optimal in the usual sense in the configuration Ω.

These definitions differ slightly from the ones proposed by Chanas and Kasperski [5]. The notions of the possible and necessary non-optimality are introduced. The following two definitions are complementary to those given in Definition 1 and Definition 2.

Definition 3: A schedule θ is possibly non-optimal if and only if there exists a configuration Ω, $\Omega \in H$, such that θ is not optimal in the usual sense in the configuration Ω.

Definition 4: A schedule θ is necessarily non-optimal if and only if for each configuration Ω, $\Omega \in H$, θ is not optimal in the usual sense in the configuration Ω.

The following statements are obvious. They result from the previously given definitions.

Statement 1: A schedule θ is necessarily non-optimal if and only if it is not possibly optimal.

Statement 2: A schedule θ is possibly non-optimal if and only if it is not necessarily optimal.

Statement 3: A schedule θ is necessarily optimal then it is possibly optimal.

Based on above statements, it is enough to determine the possible and necessary optimality of schedule.

III. Optimality with Interval Processing Times and Due dates

In this section, the possible and necessary optimality of a given schedule for the problem $1||T_{max}$ is analyzed.

A. . Possibly Optimal Solutions

Preposition 2 gives necessary and sufficient conditions for establishing the possible optimality of a given schedule θ.

Proposition 2: A given schedule θ is possibly optimal in the problem $1||T_{max}$ if and only if there exists a configuration Ω^*, $\Omega^* \in H$, such that the following system of inequalities is feasible for Ω^*:

$$\underline{p}_j \le p_j(\Omega^*) \le \overline{p}_j \qquad j = 1, \dots, n$$
$$\underline{d}_j \le d_j(\Omega^*) \le \overline{d}_j \qquad j = 1, \dots, n$$
$$d_{\theta_j}(\Omega^*) \le d_{\theta_{j+1}}(\Omega^*) \quad j = 1, \dots, n-1 \qquad (3)$$

Proof: The *only if* direction: Based on Definition 1, there exists a configuration Ω', $\Omega' \in H$, such that θ is optimal in the usual sense in the configuration Ω'. It yields that $\underline{p}_j \le p_j(\Omega') \le \overline{p}_j$ and $\underline{d}_j \le d_j(\Omega') \le \overline{d}_j$, $j = 1, \dots, n$. According to Proposition 1 and θ is optimal in Ω', the inequalities $d_{\theta_j}(\Omega') \le d_{\theta_{j+1}}(\Omega')$ for $j = 1, \dots, n-1$ hold. It deduces that Ω' fulfills the system of inequalities (3), hence the system is feasible.

The *if* direction: Assume there exists a configuration Ω^*, $\Omega^* \in H$, that fulfills the systems of inequalities (3). Based on Proposition 1, θ is optimal in Ω^*. So there exists a configuration Ω^* such that θ is optimal in the usual sense in Ω^*. We can conclude that the schedule θ is possibly optimal.■

The system of linear inequalities is obtained for a given schedule θ based on Preposition 2. The feasibility of this system can be checked in polynomial time. Algorithm1 checks the feasibility of the system of inequalities (3), thus this algorithm determines whether a given schedule, θ, is possibly optimal or not.

Algorithm 1: Checking the feasibility of the system of inequalities in Proposition 2

Input: A given schedule θ, Intervals of due dates $D_i = [\underline{d}_i, \overline{d}_i]$, $i = 1, \dots, n$
Output: *PossOptimal* = *true* if θ is possibly optimal, *false* otherwise

$PossOptimal = true$
$d_{\theta_1}(\Omega^*) = \underline{d}_{\theta_1}$
for $i=2$ to n **do**
... $d_{\theta_i}(\Omega^*) = \max\{\underline{d}_{\theta_i}, \min(d_{\theta_{i-1}}(\Omega^*), \overline{d}_{\theta_i})\}$
...**if** $d_{\theta_{i-1}}(\Omega^*) > d_{\theta_i}(\Omega^*)$ **then**
...... $PossOptimal = false$; exit
...**end if**
end for

It is clear that the computational complexity of Algorithm 1 is $O(n)$. It is worth noticing that the configuration after the termination of Algorithm 1 is Ω^* in Proposition 2.

B. Necessarily Optimal Solutions

The possible optimality is a week measure of optimality of a given schedule, θ, since it is enough that there exists only one configuration Ω, $\Omega \in H$, that θ is optimal. On the other hand, θ is not optimal for any configuration, $\forall \Omega \in H$, if θ is not possibly optimal. Thus the possible optimality is deal with a strong measure of non-optimality of θ, the necessary optimality. Proposition 3 gives necessary and sufficient conditions for establishing the necessary optimality of a given schedule θ.

Proposition 3: A given schedule θ is necessarily optimal in the problem $1||T_{max}$ if and only if there exists a configuration Ω^*, $\Omega^* \in H$, such that the following system of inequalities is feasible for Ω^*

$$\underline{p}_j \leq p_j(\Omega^*) \leq \overline{p}_j \qquad j = 1, \dots, n$$

$$\overline{d}_{\theta_j} \leq \underline{d}_{\theta_{j+1}} \qquad j = 1, \dots, n-1 \qquad (4)$$

Proof: The *only if* direction: Assume that the schedule θ is necessarily optimal. Based on Definition 2, θ is optimal in the usual sense for each configuration Ω, $\Omega \in H$. Assume on the contrary that the system of inequalities (4) is not feasible. Thus, there exist $j \in \{1, \dots, n-1\}$ such that:

$$\overline{d}_{\theta_j} > \underline{d}_{\theta_{j+1}}$$

A configuration Ω', $\Omega' \in H$, with $d_{\theta_j}(\Omega') = \overline{d}_{\theta_j}$ and $d_{\theta_{j+1}}(\Omega') = \underline{d}_{\theta_{j+1}}$ is considered. According to Proposition 1 and (5), it is deduced that θ is not optimal in Ω'. This fact contradicts the assumption that θ is necessarily optimal.

The *if* direction: Assume that the system of inequalities (4) is feasible. Hence, the following inequalities for each configuration Ω', $\Omega' \in H$ are fulfilled

$$d_{\theta_j}(\Omega') \leq \overline{d}_{\theta_j} \leq \underline{d}_{\theta_{j+1}} \leq d_{\theta_{j+1}}(\Omega') \quad (6)$$

According to Proposition 1, θ is optimal in the usual sense in for each configuration Ω', $\Omega' \in H$. The necessary optimality of θ is deduced from Definition 2.■

The system of linear inequalities is obtained for a given schedule θ based on Proposition 3 to ascertain whether θ is necessarily optimal or not. The feasibility of this system can be checked in polynomial time by Algorithm 2. This algorithm checks the feasibility of the system of inequalities (4).

Algorithm 2: Checking the feasibility of the system of inequalities in Proposition 3

Input: A given schedule θ, Intervals of due dates $D_i = [\underline{d}_i, \overline{d}_i]$, $i = 1, \dots, n$
Output: $NecOptimal = true$ if θ is necessarily optimal, *false* otherwise

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NecOptimal = true
for i=1 to n-1 do
...if d̄_θi > d_θi+1 then
...... NecOptimal = false; exit
...end if
end for
```

It is obvious that the complexity of Algorithm 1 is $O(n)$.

IV. Optimality with Fuzzy Processing Times and Due Dates

Let us focus on the fuzzy case. All the elements of the problem are the same as in the interval case except for the processing times and due dates, determined by means of fuzzy numbers. A fuzzy number is a normal convex fuzzy set in the space of real numbers with an upper semi-continuous membership function. $\tilde{p}_j$ imprecisely determine the processing time of job j, $j \in J$, and $\tilde{d}_j$ determine the due date of j. Intuitively, if any processing time or due date is fuzzy, the objective function becomes fuzzy as well (its value can be calculated by means of the Zadeh's extension principle [20]). In such a situation This is not possible to calculate the optimal schedule unless an order on the set of fuzzy numbers is defined. It is not an easy task since there is no such a natural order and a lot of orders are considered in the literature devoted to fuzzy sets [3]. The obtained optimal schedule depends on the choice of this order, which is a disadvantage of this approach.

Fuzzy numbers $\tilde{p}_j$ and $\tilde{d}_j$ express uncertainty connected with the ill-known processing time and due date modeled by these numbers, respectively. These fuzzy numbers generate possibility distributions for the sets of values containing the unknown processing time and due date. More formally, the assertion of the form "δ_j is $\tilde{d}_j$", where δ_j is a variable and $\tilde{d}_j$ is a fuzzy number, generates the possibility distribution of δ_j with respect to the following formula:

$$Poss(\delta_j = x) = \mu_{\tilde{d}_j}(x), \quad x \in \mathcal{R}_+ \qquad (7)$$

Let Ω be a configuration of processing times and due dates, p_i *and* $d_i \in \mathcal{R}_+, j \in J$ and H denote the set (5) of all configurations. Hence, the joint possibility distribution over configurations, induced by the p_i and d_i, is obtain by the following formula:

$$\pi(\Omega) = \min\{\min_{j \in J} \mu_{\tilde{p}_j}(x), \min_{j \in J} \mu_{\tilde{d}_j}(x)\} \quad \Omega \in \mathcal{R}_+^{n+n}$$

(8)

Thus, the following formula determines the possibility that a schedule θ is optimal:

$$Poss(\theta \text{ is optimal}) = \sup_{\Omega:\, \theta \text{ is optimal in } \Omega} \pi(\Omega) \qquad (9)$$

Due to possibility-necessity relations (see [8]) a necessity measure of optimality can be obtain by the following formula:

$$Nec(\theta \text{ is optimal}) = 1 - Poss(\theta \text{ is not optimal}) = \inf_{\Omega:\, \theta \text{ is not optimal in } \Omega}(1 - \pi(\Omega)) \qquad (10)$$

Before we pass on to the basic consideration let us define some sets of notations, which will be helpful in formulating and proving propositions.

A fuzzy number $\tilde{A}$ is called a *trapezoidal* fuzzy number if its membership function $\mu_{\tilde{A}}$ has the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & for\ x \in [\underline{a}, \overline{a}] \\ 1 - \frac{\underline{a} - x}{\alpha} & for\ x \in [\underline{a} - \alpha, \underline{a}) \\ 1 - \frac{x - \overline{a}}{\beta} & for\ x \in (\overline{a}, \overline{a} + \beta] \\ 0 & for\ x \in (-\infty, \underline{a} - \alpha) \cup (\overline{a} + \beta, \infty) \end{cases}$$

Where $\underline{a} \leq \overline{a}$ and $\alpha, \beta > 0$.

A trapezoidal fuzzy number is denoted by $\tilde{A} = (\underline{a}, \overline{a}, \alpha, \beta)$. A trapezoidal fuzzy number is positive if an only if $\underline{a} - \alpha \geq 0$. Let $\tilde{A}$ be a fuzzy number. The following interval is called $\lambda - cut$ of the fuzzy number $\tilde{A}$.

$$\tilde{A}^\lambda = [\underline{a}^\lambda, \overline{a}^\lambda] = \{x \in \mathcal{R} | \mu_{\tilde{A}}(x) \geq \lambda\} \qquad \lambda \in (0,1]$$

The $\lambda - cut$ of a trapezoidal fuzzy number has the following form:

$$\tilde{A}^\lambda = [\underline{a} - \alpha(1-\lambda), \overline{a} + \beta(1-\lambda)] \qquad \lambda \in (0,1]$$

Let us denote by H^λ the $\lambda - cut$ of the H, i.e. the set of all configurations with interval processing times, $\tilde{p}_j^\lambda = [\underline{p}_j^\lambda, \overline{p}_j^\lambda]$, and interval due dates, $\tilde{d}_j^\lambda = [\underline{d}_j^\lambda, \overline{d}_j^\lambda]$. The following proposition determines the relationship between indices $Poss(\theta \text{ is optimal})$ and $Nec(\theta \text{ is optimal})$ and the notions of possible and necessary optimality.

Proposition 4: For a given schedule, θ, the following equivalences hold for a fixed $\lambda \in (0,1]$

$Poss(\theta \text{ is optimal}) \geq \lambda \Leftrightarrow$ θ is possibly optimal in H^λ

$Nec(\theta \text{ is optimal}) > \lambda \Leftrightarrow$

θ is necessarily optimal in $H^{1-\lambda}$

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Proof: It follows directly from definitions of degree of possibility and necessity that a schedule is optimal and the fact that if $\alpha < \beta$ then $\tilde{p}_j^{\alpha} \subseteq \tilde{p}_j^{\beta}$ *and* $\tilde{d}_j^{\alpha} \subseteq \tilde{d}_j^{\beta}$.■

Proposition 5 provides a tool for calculating the $Poss(\theta\ is\ optimal)$ and $Nec(\theta\ is\ optimal)$ indices.

Proposition 5: For a given schedule, θ, the following equalities hold:

$Poss(\theta\ is\ optimal) = \{\lambda | \theta \text{ is possibly optimal in } H^{\lambda}\}$

$Nec(\theta\ is\ optimal) = \{\lambda | \theta \text{ is necessarily optimal in } H^{\lambda}\}$

Proof: Straightforward. The proposition is a direct consequence of the definitions and Proposition 4. ■

There are two effective methods of calculating the values of indices $Poss(\theta\ is\ optimal)$ and $Nec(\theta\ is\ optimal)$. The first one is adapted to fuzzy numbers given in a general form and the second one, based on mathematical programming, is valid only for fuzzy processing times and due dates determined by trapezoidal fuzzy numbers. The first method is based on the idea of bisection of the unit interval of values of λ to compute the values of indices. This method is used by several authors, e.g. [5] and [19]. In this paper for the sake of brevity, the second method is only applied to calculate the values of indices.

Additionally, Fortin and Dubois [10] have shown that the algorithms in the interval-valued case can be adapted to fuzzy intervals considering them as crisp intervals of gradual numbers. The notions of gradual numbers are introduced by Fortin et al. [11]. This is a topic for future research.

A. 4. 1. Calculating the Degree of Possibility That a Schedule Is Optimal

The problem of calculating the degree of possibility that a schedule is optimal can be reduced under certain assumptions about membership functions of fuzzy processing times and due dates, to that of determining the optimal solution of a classical linear programming problem. It is assumed that $\tilde{p}_j = (\underline{p}_j, \overline{p}_j, \tau_j, \upsilon_j)$ and $\tilde{d}_j = (\underline{d}_j, \overline{d}_j, \alpha_j, \beta_j)$, $j \in J$. Based on Proposition 2, $Poss(\theta\ is\ optimal)$ is equal to the maximal value of the objective function in the following linear programming problem:

$$\lambda \rightarrow max$$
$$0 \leq \lambda \leq 1$$
$$\underline{p}_j - \tau_j(1-\lambda) \leq p_j \leq \overline{p}_j + \upsilon_j(1-\lambda) \qquad j = 1, \dots, n$$
$$\underline{d}_j - \alpha_j(1-\lambda) \leq d_j \leq \overline{d}_j + \beta_j(1-\lambda) \qquad j = 1, \dots, n$$
$$d_{\theta_j} \leq d_{\theta_{j+1}} \qquad j = 1, \dots, n-1$$

If λ_{max} is the optimal objective value of (18) then the $Poss(\theta\ is\ optimal) = \lambda_{max}$. If problem (18) is infeasible then $Poss(\theta\ is\ optimal) = 0$. The standard simplex algorithm cab be used to solve the problem (18) [16].

B. Calculating the Degree of Necessity That a Schedule Is Optimal

Mathematical programming has been used in the literature for calculating the degree of necessity, e.g. [18]. In this section, a novel mathematical programming is proposed for calculating the degree of necessity that of a given schedule in the problem $1||T_{max}$. is optimal. Assumed that all the fuzzy processing times and fuzzy due dates are positive trapezoidal fuzzy numbers, i.e. $\tilde{p}_j = (\underline{p}_j, \overline{p}_j, \tau_j, \upsilon_j)$ and $\tilde{d}_j = (\underline{d}_j, \overline{d}_j, \alpha_j, \beta_j)$, $j \in J$. The following mathematical programming problem is based on Proposition 3.

$$\lambda \rightarrow min$$
$$0 \leq \lambda \leq 1$$
$$\underline{p}_j - \tau_j(1-\lambda) \leq p_j \leq \overline{p}_j + \upsilon_j(1-\lambda) \qquad j = 1, \dots, n$$
$$\overline{d}_{\theta_j} - \alpha_{\theta_j}(1-\lambda) \leq \underline{d}_{\theta_{j+1}} + \beta_{\theta_{j+1}}(1-\lambda) \quad j = 1, \dots, n-1$$

(16)

(17)

According to Proposition 5, $Nec(\theta\ is\ optimal) = 1 - \lambda_{min}$, where λ_{min} is the minimal value of the objective function of problem (19). System (19) is linear for a given schedule θ and it can be calculated by means of the standard simplex algorithm [16]. If problem (19) is infeasible, then $Nec(\theta\ is\ optimal) = 0$.

V. Conclusions

In this paper the possible and necessary optimality of a given schedule for the problem of minimizing maximum tardiness on single machine with imprecise processing times and due dates (by means of interval and fuzzy interval numbers) is discussed. It is assumed that the optimal schedule in such a problem cannot be determined precisely, thus the processing times and due dates of the single machine scheduling problem are imprecise a priori.

In the interval case, the problems of stating whether a given schedule is possibly and necessarily optimal are easy to solve. Based on the propositions, it is shown that proposed algorithms solve the problems in polynomial time. Instead of being optimal or not,, schedules are divided into thee groups, those that are for sure optimal despite uncertainty (necessarily optimal schedules), those that are for sure not optimal (necessarily non-optimal schedules), and schedules whose optimality is unknown, called possibly optimal schedules.

Then, the obtained results are generalized to case of fuzzy processing times. In this paper, it is shown how to calculate the degrees of possible and necessary optimality of a given schedule in one of the special cases of the single machine scheduling problems. Linear programming formulations are proposed for calculating the degree of possibility and necessity that of a given schedule is optimal in the problem.

It seems possible to further develop the problems of ascertaining whether a given schedule is possibly and necessarily optimal schedule for more than one machine. Another research line is the generalization of the calculation of degrees of possible and necessary optimality where processing times and due dates are represented (18) gradual numbers.

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