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Discrete Hamiltonians for Minimum of Available Energy in Bubble Fluidized Drying Operations

Artur Poświata, and Zbigniew Szwast

***Abstract*—In this paper we study optimization of multistage drying process in fluidized bed. The drying of fine particles in fluidized bed is very effective operation because of good contact of phases and a large interfacial surface. Using a relatively high gas velocity improves conditions of mass and heat transfers but a portion of gas begins to flow as bubbles or canals, so an energy carried by this gas is not used efficiently. This is a reason to search for best operating parameters to minimize cost or to maximize efficiency of the process.**

A method of determining optimal parameters of drying gas (flowrate, temperature and humidity) is described in this paper. As a performance index, i.e. function being optimized, we apply a function describing the exergy input in the drying process. This function consists of three parts, the first two describing the thermal and chemical exergy of gas whereas the third one referring to investment costs expressed in exergy units. The chemical exergy of gas is connected with the difference between gas humidity and ambient humidity. In the optimization calculations we search for a minimum of exergy consumption

Any effective method of optimization involves an algorithm of maximum principle for multistage problems. The earliest algorithms of this sort was the Katz' and Fan's algorithm. As it is less suitable for our purposes due to its undefined symplectic structure, we apply a modification of Sieniutycz's and Szwast's algorithm with a constant Hamiltonian (formally similar to the well-know continuous algorithm of Pontryagin). In this paper a modification of the basis algorithm is applied. This modification admits constrained intervals of time.

In our calculations we take into consideration the bed hydrodynamics and kinetics of transport processes To describe behavior of fluidized bed we use the well-known two-phase model of Kunii and Levenspiel In this model, the gas excess flows in form of bubbles. The heat and mass transfer between the solid and gas as well between the dense phase (with gas and solid) and bubble phase determines values of optimal gas parameters. We discuss optimal parameters of gas and solid associated with minimum exergy consumption in drying proces.

***Index Terms*— exergy, fluidized drying, optimization.**

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I. Introduction

The optimization problem considered here deals with minimization of exergy consumption in the drying process in which thermodynamic state of solid is transformed from an initial state to a final one. The initial values of solid parameters (moisture content and temperature or enthalpy) are fixed, whereas final solid moisture content is fixed and solid final temperature is usually free. In this study decision variables, for which optimal values are searched, are inlet gas temperature, humidity and its flow rate. Optimal decisions have to minimize process performance index describing total cost of the process. The process cost is transformed to the form which describes exergy consumption in the multistage fluidized drying process.

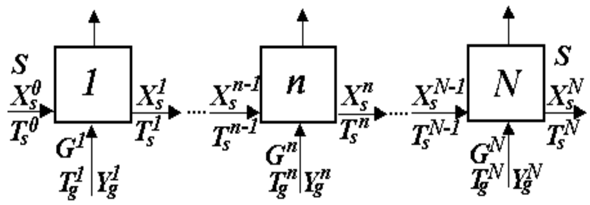

Fig. 1. Scheme of the multistage fluidized drying process.

II. Perfomance Index

For N stage cascade (n=1,..N) of fluidized dryers and for unit stream of dried material the total cost is given by equation (1) [1]:

$$K_e = \left(\frac{\tilde{z}}{\tau_m} + \beta_r \right) \frac{J}{\tau_u S} + \sum_{n=1}^{N} \frac{e_e \Delta P G^n}{\eta \rho_g S} + \sum_{n=1}^{N} c_g \left(T_g^n, Y_g^n \right) \frac{G^n}{S} \qquad (1)$$

where symbols are employed: $\tilde{z}$ is a factor describing the freezing of capital cost; τ_m – the maximum acceptable payout time [yr]; β_r – a coefficient describing renovations [yr^{-1}]; J – the total cost [\$] of the new cascade of fluidized dryer; τ_u – the utilization time [s/yr]; e_e – the unit price of electrical energy [\$/kJ] for gas pumping; ΔP – the pressure drop [Pa] in the fluidized layer; η – the pumping efficiency; $c(T_g^n,Y_g^n)$ – the specific price of drying gas [\$/kg] as function of its temperature T_g^n and humidity Y_g^n; S is a mass flow rate of solid [kg/s] and G^n is a gas flow rate on the stage n [kg/s].

The cost of new cascade of fluidized dryer can be expressed as linear function of total fluidized apparatuses bottom area, A_a, and then also as a linear function of total gas flow rate i.e. sum of gas flow rate for all stages:

$$J = j_0 + p\sum_{n=1}^{N} A_a^n = j_0 + \frac{p}{\rho_g u_g}\sum_{n=1}^{N} G^n \tag{2}$$

where j_0 is a fixed term of apparatus price independent on its size, and p is the fluidized apparatus price per unit area of bottom.

The thermo-economic studies indicate that the economic value of the stream of matter is proportional to the exergy of this stream. Therefore the specific price of drying gas can be substituted by the specific exergy of this gas

$$c_g\left(T_g^n, Y_g^n\right) = eb_g\left(T_g^n, Y_g^n\right) \tag{3}$$

where e is a price of exergy unit according to the so-called exergy tariff of prices.

The exact analytic formula for exergy of drying gas has the following form

$$\begin{aligned} b_g\left(T_g, Y_g\right) = &\left(c_g + Y_g c_w\right)\left(T_g - T_a - T_a \ln\frac{T_g}{T_a}\right) + \\ &+ \frac{RT_a}{M_g}\ln\frac{\frac{M_w}{M_g} + Y_a}{\frac{M_w}{M_g} + Y_g} + \frac{RT_a Y_g}{M_w}\ln\frac{Y_g\left(\frac{M_w}{M_g} + Y_a\right)}{Y_a\left(\frac{M_w}{M_g} + Y_g\right)} \end{aligned} \tag{4}$$

After Taylor expansion of this function around the point (T_a, Y_a) and omitting terms higher than second order one can obtain

$$b_g\left(T_g, Y_g\right) = \tfrac{1}{2}A\left(T_g - T_a\right)^2 + \tfrac{1}{2}B\left(Y_g - Y_a\right)^2 \tag{5}$$

where

$$A = \frac{c_g + Y_a c_w}{T_a}, \quad B = \frac{RT_a}{Y_a\left(M_w + Y_a M_g\right)} \tag{6}$$

Combining equations (1), (2), (3) and (5) yields the total cost of the process cost expressed in exergy unit in the following form

$$\begin{aligned} I' = &N\left(\frac{\tilde{z}}{\tau_m} + \beta_r\right)\frac{j_0}{\tau_u S} + \\ &+ \sum_{n=1}^{N}\left[\frac{1}{2}A\left(T_g^n - T_a\right)^2 + \frac{1}{2}B\left(Y_g^n - Y_a\right)^2 + \kappa\right]\theta_g^n \end{aligned} \tag{7}$$

where θ_g^n is dimensionless gas flow rate defined as a ratio of gas flow rate for stage n to the solid flow rate, and κ defined by expression

$$\kappa = \left(\frac{\tilde{z}}{\tau_m} + \beta_r\right)\frac{p}{e\tau_u \rho_g u_g} + \frac{e_e \Delta P}{e\eta\rho_g} \tag{8}$$

is so-called exergy coefficient of investment and gas pumping cost [1].

The first term on the right hand side of the equation (7) is independent on any decision or state variables so for fixed number of stages is constant, and can be omitted in performance index accepted to calculation. Furthermore it should be noticed that the price of thermal exergy related to the gas temperature, and price of chemical exergy related to the humidity of the gas can be different. Taking this into account, we introduced parameter ζ defined as the ratio of the chemical exergy price to the thermal one. Hence the final form of performance index is given by the following equation:

$$I = \sum_{n=1}^{N}\left[\frac{1}{2}A\left(T_g^n - T_a\right)^2 + \frac{1}{2}B\zeta\left(Y_g^n - Y_a\right)^2 + \kappa\right]\theta_g^n \tag{9}$$

III. Model of The Drying Process

In our studies two-phase Kunii-Levenspiel's model is applied [2]. In this model the following assumptions are made: fluidized bed consists of two phases: a dense phase and a bubble phase; dense phase is ideally mixed and gas flowing through this phase has velocity of minimal fluidization; the plug flow of gas is accepted for the bubble phase; diameter of bubbles is constant along bed height and is a bed parameter; in the bubble phase any solid particles are not present.

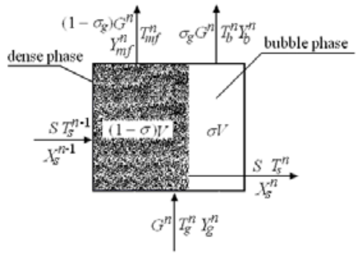

Fig. 2. Scheme of two-phase fluidized bed.

Moreover, only first period of drying is considered in this study, therefore the moisture evaporation proceeds like from the liquid surface, so process rate depends only on the heat and mass transfer in gaseous phase and is independent on the processes in solid particles.

Taking into account that the gas forms two distinct phases, moisture balance for stage n takes the following form

$$X_s^n - X_s^{n-1} = \theta_g^n\left[\left(1-\sigma_g\right)\left(Y_g^n - Y_{mf}^n\right) + \sigma_g\left(Y_g^n - Y_b^n\right)\right] \tag{10}$$

and the enthalpy balance takes the analogous form:

$$h_s^n - h_s^{n-1} = \theta_g^n\left[\left(1-\sigma_g\right)\left(i_g^n - i_{mf}^n\right) + \sigma_g\left(i_g^n - i_b^n\right)\right] \tag{11}$$

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For the first period of drying the kinetic equation describing moisture transfer between solid and gas can be written as

$$X_s^n - X_s^{n-1} = \frac{Y_{mf}^n - Y_e^n}{W_{mf}}\theta_g^n \tag{12}$$

where Y_e is an equilibrium humidity on the particles surface and is equal to saturation humidity in temperature of solid particles, whereas W_{mf} is dimensionless coefficient describing resistances of mass transfer defined by following expression:

$$W_{mf} = \frac{u_g\,\rho_g}{k_{mf}h_{mf}} \tag{13}$$

The moisture transfer between gas in dense phase and bubble phase is described by equation (14). The driving force for the transfer is expressed as logarithmic average of differences of humidity between gas in dense phase and gas in bubble phase, whereas the coefficient W_b describes resistance of moisture transfer and is defined by equation (15).

$$Y_g^n - Y_b^n = \frac{\Delta Y_b^n}{W_b} \tag{14}$$

$$W_b = \frac{u_g\,\rho\sigma_g(1-\sigma)}{k_b h_{mf}\sigma} \tag{15}$$

The kinetic equations for heat transfer contain except the terms describing convective transfer of heat also the terms which describe enthalpy transfer together with the stream of moisture. Hence, the equation describing the heat transfer from gas to solid is of the form

$$h_s^n - h_s^{n-1} = -\frac{c_g\left(T_{mf}^n - T_s^n\right)}{Z_{mf}}\theta_g^n - (X_s^n - X_s^{n-1})i_{vmf} \tag{16}$$

The first term on the right hand side of the equation (16) – convection term – is described by the ratio of driving force described by temperatures difference to the coefficient Z_{mf}, analogous to W_{mf}, describing resistance of heat transfer. The second term describes the total heat flux carried by the stream of moisture. The heat transfer between gas in dense phase and bubble phase is expressed by the following equation

$$i_g^n - i_b^n = \frac{c_g\Delta T_b^n}{Z_b} - \left(Y_b^n - Y_g^n\right)i_{vb} \tag{17}$$

where, analogous to the equation (16), first term on the right hand side of above equation describes the convection heat transfer whereas the second term - the heat flux connected with moisture flux. The driving force for convective transfer is logarithmic average of temperature differences while resistance of the convective transfer is described by dimensionless coefficient Z_b, like W_b. Both dimensionless coefficients describing resistances of heat transfer (Z_{mf}, Z_b) are defined by the following expressions, respectively:

$$Z_{mf} = \frac{u_g\rho_g\,c_g}{\alpha_{mf}h_{mf}} \tag{18}$$

$$Z_b = \frac{u_g\,\rho_g\,\sigma_g\,(1-\sigma)c_g}{\alpha_b h_{mf}\sigma} \tag{19}$$

For the air-water system commonly found in drying processes the ratio of the corresponding coefficients of Z and W can be accepted as equal to 1, so the proper coefficients Z and W are equal, $Z_{mf} = W_{mf}$ and $Z_b = W_b$.

The set of the equations (10) – (19) being the model of fluidized drying process has not the analytical solution, so there are not analytical forms of state transformations i.e. functions describing changes of state variables on the each stage of cascade. Therefore, the values of state transformations must be determined by numerical solution of the model. General forms of state transformations are shown below:

$$X_s^n - X_s^{n-1} = f_X^n\left(X_s^n, T_s^n, T_g^n, Y_g^n, \theta_g^n\right)\theta_g^n \tag{20}$$

$$T_s^n - T_s^{n-1} = f_T^n\left(X_s^n, T_s^n, T_g^n, Y_g^n, \theta_g^n\right)\theta_g^n \tag{21}$$

$$t^n - t^{n-1} = \theta_g^n \tag{22}$$

In the above equations one can distinguish three state variables: X_s and T_s describing solid moisture content and solid temperature, respectively, and state variable t called a "time" in optimization theory, which describes the gas flow rates from beginning of cascade to the current stage n, as well three decision variables T_g, Y_g, and θ_g being temperature and humidity of inlet gas as well gas flow rate for a stage. Note that in contrast to the problems optimized by the classic version of a discrete algorithm with constant Hamiltonian [1, 3, 4] in the problem considered in this paper the decision variable θ_g can be one of the arguments in the functions f_X and f_T, and also appears outside these functions. Therefore the generalized version of a discrete algorithm with constant Hamiltonian presented below is applied in this paper.

IV. Optimization Algorithm

Using performance index (9) and general forms of state transformations (20 – 22), generalized Hamiltonian takes the following form (the list of arguments for function f_X and f_T are omitted) [5]

$$\begin{aligned}&\widetilde{H}^{n-1}\left(X_s^n, T_s^n, T_g^n, Y_g^n, \theta_g^n, z_X^{n-1}, z_T^{n-1}, z_t^{n-1}\right) = \\ &= \tfrac{1}{2}A\left(T_g^n - T_a\right)^2 + \tfrac{1}{2}B\zeta\left(Y_g^n - Y_a\right)^2 + \kappa + z_X^{n-1}f_X^n + z_T^{n-1}f_T^n + z_t^{n-1}\end{aligned} \tag{23}$$

The variables z_X, z_T, and z_t are called adjoint variables and are analogous to Lagrange multipliers. If the equations (20), (21) and (22) (state equations) describing changes in state variables can be obtained again from the Hamiltonian by

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differentiating it with respect to the adjoint variables, the equations describing the change of adjoint variables (adjoint equations) are obtained by differentiating of Hamiltonian with respect to the state variables.

$$z_X^n - z_X^{n-1} = -\frac{\partial \widetilde{H}^{n-1}}{\partial X_s^n}\theta_g^n = -\left[z_X^{n-1}\frac{\partial f_X^n}{\partial X_s^n} + z_T^{n-1}\frac{\partial f_T^n}{\partial X_s^n}\right]\theta_g^n \quad (24)$$

$$z_T^n - z_T^{n-1} = -\frac{\partial \widetilde{H}^{n-1}}{\partial T_s^n}\theta_g^n = -\left[z_X^{n-1}\frac{\partial f_X^n}{\partial T_s^n} + z_T^{n-1}\frac{\partial f_T^n}{\partial T_s^n}\right]\theta_g^n \quad (25)$$

$$z_t^n - z_t^{n-1} = -\frac{\partial \widetilde{H}^{n-1}}{\partial t^n}\theta_g^n = 0 \quad (26)$$

From equation (26) it results that the adjoint variable z_t is constant along cascade for optimal processes.

Boundary values for adjoint variables result from boundary values for right state variables. Thus, for fixed value of state variable appropriate adjoint variable is undetermined, and for free value of state variable appropriate adjoint variable is equal to 0. Hence in the considered case it is assumed that for the beginning of the cascade, adjoint variables are undefined because state variables are fixed. At the end of the cascade, variable z_X^N is unconstrained as solid final moisture content must be determined, whereas the variable z_T^N is 0 because the final temperature of the solid is free. Moreover, for unspecified total gas flow rate, t^N, z_t^N variable is equal to 0, whereas from the equation (26) it follows that it is also constant along the cascade, therefore z_t is always equal to 0.

Conditions for optimality of the decision variables T_g^n and Y_g^n are defined by equations

$$\frac{\partial \widetilde{H}^{n-1}}{\partial T_g^n} = A\left(T_g^n - T_a\right) + + z_X^{n-1}\frac{\partial f_X^n}{\partial X T_g^n} z_T^{n-1}\frac{\partial f_T^n}{\partial T_g^n} = 0 \quad (27)$$

$$\frac{\partial \widetilde{H}^{n-1}}{\partial Y_g^n} = B\zeta\left(Y_g^n - Y_a\right) + z_X^{n-1}\frac{\partial f_X^n}{\partial Y_g^n} + z_T^{n-1}\frac{\partial f_T^n}{\partial Y_g^n} = 0 \quad (28)$$

whereas for optimality of gas flow rate θ_g^n appropriate condition has a following form:

$$\frac{\partial \widetilde{H}^{n-1}}{\partial \theta_g^n}\theta_g^n + \widetilde{H}^{n-1} = 0 \quad (29)$$

The above equation (29) is suitable for generalized version of discrete algorithm with constant Hamiltonian. As it was mentioned above, for classical version of this algorithm the functions in state transformation cannot include an interval of time (θ_g^n) as open argument, so the derivative of Hamiltonians for variable θ_g is zero, then the equation (29) reduces to expression $\widetilde{H}^{n-1} = 0$, which is valid for classical version of the algorithm.

Solution set of state equations (20 – 22), adjoint equations (24 – 26), and equations for optimality conditions (27 – 29) for each stage of cascade is optimal solution giving optimal values of decision variables for each stage, and optimal interstage values of state variables.

V. Optimization Results

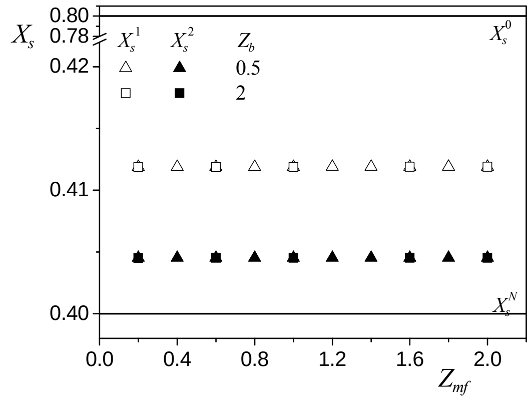

Fig. 3. Solid moisture content after first X_s^1 and second stage X_s^2 as a function of coefficient Z_{mf} and for two values of coefficient Z_b.

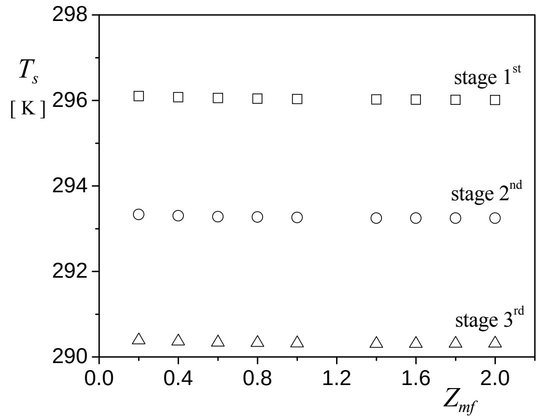

Fig. 4. Optimal solid temperature for 1st, 2nd and 3rd stage.

The optimization calculation was performed for the three-stage cascade ($N = 3$) as well for initial value of solid moisture content and temperature equal to $X_s^0 = 0.8$ (kilogram of moisture per kilogram of dry matter) and $T_s^0 = T_a = 293$ [K] and final solid moisture content equal to $X_s^N = 0.4$. In Fig. 3. the interstage solid moisture content as a function of coefficient Z_{mf} is presented for two values of coefficient Z_b. As it is shown in this figure the all dependences are constant so interstage moisture contents in the solid are independent on the values of the coefficients Z_{mf} and Z_b, and thus are independent on the kinetics of the drying process. Moreover analyzing the values of interstage moisture contents it can be concluded that the drying process proceeds mainly on the first stage whereas on the second and third stages changes of moisture contents are rather small. The next graph in Fig. 4. shows solid temperatures on each stage of the cascade as a function of coefficient Z_{mf}. Also solid temperatures are independent on value of coefficient Z_{mf} so they are independent on process kinetics. Note that the highest temperature of solid occurs at the first stage and falls on the next steps.

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In the next two figures, Fig. 5. And Fig. 6., the optimal values of the decision variables are shown. These values are presented in the form of so-called substitute driving forces for convective heat and mass transfer. And so, for the moisture transfer in the first period of drying substitute driving force is defined as the difference between the saturation humidity at the surface of solid particles and the drying gas inlet humidity. For heat transfer the substitute driving force is defined as the difference between gas inlet temperature and solid temperature on the stage. The driving forces are constant for various values of coefficient Z_{mf}, as it is shown in Fig. 5. and Fig. 6. Taking into account that the temperature of the solid is independent on the values of the coefficients Z_{mf} and Z_b, also the absolute values of decision variables (T_g, Y_g) are independent on these coefficients.

Note that substitute driving force for heat transfer (the differences of temperatures) is the highest on the first stage and decreases on next stages but the gas temperature is always higher than solid temperature, so heat is always transferred by convection from gas to solid, nevertheless the solid temperature falls. The drop in solid temperature on the second and third stages results from the transfer of enthalpy together with moisture stream. The opposite situation occurs for the driving forces of moisture transfer. The driving force increases along the cascade. This growth occurs despite the drop in solid temperature and thus despite the drop in saturation humidity. Therefore the optimal humidity of drying gas is the highest on the first stage and rapidly drops on the next stages.

The total optimal gas flow rate is presented in Fig. 7. As it is shown in this figure the total gas flow rate strongly depends on process kinetics and bed hydrodynamics because increases if the value of coefficient Z_{mf} increases or if the value of coefficient Z_b increases. Moreover it was easily found that the dependences between total flow rate of drying gas and coefficient Z_{mf} is practically linear.

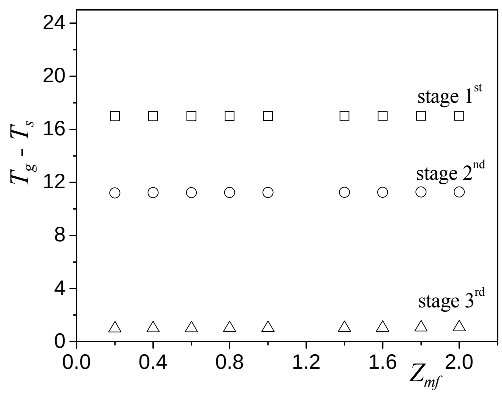

Fig. 5. Optimal substituted driving forces for heat convective transfer on stages 1st, 2nd, and 3rd.

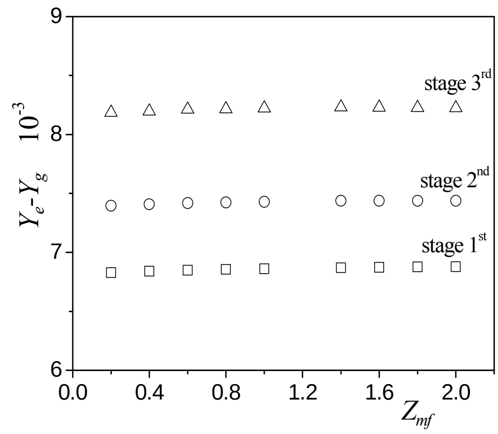

Fig. 6. Optimal driving forces for mass transfer on stages 1st, 2nd, and 3rd.

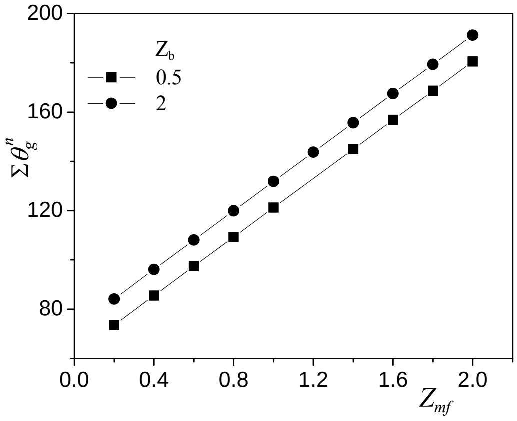

Fig. 7. Optimal total gas flow rate.

VI. Conclusions

The optimization problem of fluidized drying processes proceeding in the first period of drying is considered in the paper. The hydrodynamics of two phase fluidized bed is included. The mathematical model of this process cannot be derived in the form required by the well-known classical algorithm with the constant Hamiltonian i.e. the all state transformation and the performance index are linear with respect to one of decision variables (an interval of a time). The paper presents generalized version of discrete algorithm with the constant Hamiltonian which does not require linearity of the optimization model. Generalized version of the algorithm preserves the structure of the equations and boundary conditions such as for the classic version. The main difference relates to a single equation, which is the optimality condition for the decision variable describing interval of the time.

Some results of optimization calculations for drying process proceeding in the cascade of fluidized dryer are presented and discussed. The results presented show that the bed hydrodynamics and heat transfer kinetics practically no effect on the optimal values of state variables and on the optimal values of drying gas temperature and moisture content, whereas bed hydrodynamics and heat transfer kinetics reveal in the optimal value of the gas flow rate.

Thus, regardless of the drying kinetics, the optimal values of drying gas property remain the same, but changes the duration of the process – residence time of the solid particles. Moreover, it can be concluded that the relationship between optimum gas flow rate and the coefficients that describe the kinetics of drying is linear. Concluding one can observe that intensive parameters (gas temperature and humidity) are independent on process kinetics, whereas extensive parameter (gas flow rate) strongly depends on process kinetics.

Finally, it should be noted that the presented generalization for the optimization algorithm allows finding its application in a wider range of processes in which nonlinearity of the optimization models excludes the use of the classical version of the discrete algorithm with constant Hamiltonian.

VII. Symbols

b_g – specific exergy of gas, kJ/kg
c – specific price of drying gas, $/kg
c_g – specific heat capacity of gas, kJ/(kg·K)
c_w – specific heat capacity of moisture, kJ/(kg·K)
e – unit price of exergy, $/kJ
e_e – unit price of electrical energy, $/kJ
G – gas flow rate, kg/s
h_s – specific solid enthalpy, kJ/kg
h_{mf} – bed height for minimum fluidization, m
i – specific gas enthalpy, kJ/kg
i_{vb} – enthalpy of moisture in bubble phase, kJ/kg
i_{vmf} – enthalpy of moisture in dense phase, kJ/kg
J – total cost of the new cascade of fluidized dryer, $
j_0 – fixed term of apparatus price, $
k – mass transfer coefficient, kg/(s·m^3)
M_g – molar mass of gas, kg/kmol
M_w – molar mass of moisture, kg/kmol
N – number of stages,
p – apparatus price per unit area of bottom, $/m^2
ΔP – pressure drop in fluidized layer, Pa
R – gas constant, kJ/(kmol·K)
S – solid flow rate, kg/s
T – temperature, K
u_g – superficial gas velocity, m/s
X – moisture content,
Y – gas humidity,
$\tilde{z}$ – adjoint variables,
z – factor describing the freezing of capital cost

Greek Letters

α – heat transfer coefficient, kW/(m^3·K)
β_r – coefficient describing renovations, 1/yr
η – pumping efficiency,
ρg – gas density, kg/m^3
σ – volume fraction of bubble phase
σg – fraction of gas flowing through bubble phase
θ_g – dimensionless gas flow rate, $\theta_g=G/S$
τ_m – maximum acceptable payout time, yr
τ_u – utilization time, s/yr

Subscripts

a – ambient
b – outlet gas from bubble phase
g – inlet gas
mf – dense phase
s – solid particles

Superscripts

n – stage number

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