

Percentile Analysis of Probability Density Function of the Concomitants of Lower Record Values from Bivariate Inverse Rayleigh Distribution

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Abstract— Percentile analysis of probability density function (pdf) of the r th concomitant of lower record values has been developed for Bivariate Inverse Rayleigh distribution.

Index Terms— Lower record values, concomitants, percentiles analysis.

1. INTRODUCTION

Let $\{X_i, Y_i, i = 1, 2, \dots\}$ be a sequence of i.i.d bivariate random variables with cdf $F_{12}(x, y)$ and pdf $f_{12}(x, y)$. Let $X_{L(1)}, X_{L(2)}, \dots$ be the lower record values of the X 's and $Y_{(1)}, Y_{(2)}, \dots$ be the corresponding values of Y 's in the sequence $\{X_i, Y_i, i = 1, 2, \dots\}$. Then we will call $Y_{[r]}$ as the r th concomitant of the r th lower record value. The conditional distribution of $Y_{[r]}$ given $X_{L(r)}$ is the same as the conditional pdf of Y_i given X_i . hence the joint pdf g_{12} of $x_{L(r)}$ and $y_{[r]}$ is given by:

$$g_{12}(x, y) = f_{12}(x, y) [-\ln F_1(x)]^{r-1} / \Gamma_r$$

Where $F_1(x)$ is the marginal cdf of x . The marginal pdf $g_{2[r]}(y)$ of $Y_{[r]}$ is:

$$g_{2[r]}(y) = c_r \int_{-\infty}^{\infty} f_{12}(x, y) [-\ln F_1(x)]^{r-1} dx \dots (1.1)$$

$$\text{where } c_r = \frac{1}{\Gamma_r}$$

The joint pdf of the $Y_{[r]}$ and $Y_{[s]}$, which are the r th and s th concomitants of the r th and s th lower record values is.

$$g_{[r],[s]}(y_1, y_2) = c_{r,s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, y_1) \cdot f(x_2, y_2) [H(x_1)]^{r-1} [H(x_2) - H(x_1)]^{s-r-1} h(x_1) f(x_2) dx_1 dx_2 \dots (1.2)$$

$$\text{where } c_{i,s} = \frac{1}{\Gamma_r \Gamma(s-r)}$$

For properties of concomitants of record value refer to [1]-[7]. Here we consider Bivariate Inverse Rayleigh Distribution [8] with distribution function (df):

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$F = F_1 F_2 (1 + \alpha \bar{F}_1 \bar{F}_2)$ where $\bar{F}_1(x) = 1 - F_1(x)$ and the pdf:

$$f(x, y) = f_1 f_2 [1 + \alpha(1 - 2f_1)(1 - 2f_2)] \dots (1.3)$$

where $f_1 = F_1'$, $-1 \leq \alpha \leq 1$

$$\text{and } f(x, \theta) = \frac{2\theta}{x^3} \exp\left(-\frac{\theta}{x^2}\right) x, \theta > 0 \dots (1.4)$$

$$F(x) = \exp\left[-\frac{\theta}{x^2}\right] \dots (1.5)$$

Expression for single i.e r th and joint pdf of r th and s th concomitants of lower record values from Bivariate Rayleigh distribution are obtained.

2. PROBABILITY DENSITY FUNCTION OF $Y_{[r]}$

For Bivariate Inverse Rayleigh distribution with pdf (1.3), the pdf of the r th concomitant of the lower record values using (1.3), (1.4) in (1.1), we get

$$g_{2[r]}(y) = 4c_r \theta_2 \theta_1^r y^{-3} \cdot \exp\left[-\frac{\theta_2}{y^2}\right] \int_0^{\infty} x^{-2r-1} \exp\left(-\frac{\theta_1}{x^2}\right) \left[1 + \alpha \left\{1 - 2 \exp\left(-\frac{\theta_1}{x^2}\right)\right\} \left\{1 - 2 \exp\left(-\frac{\theta_2}{y^2}\right)\right\}\right] dx$$

where $y, \theta_1, \theta_2 \geq 0$.

After integrating and simplifying, we get

$$g_{2[r]}(y) = \frac{\theta_2}{2^{r-1}} y^{-3} \exp\left(-\frac{\theta_2}{y^2}\right)$$

$$\left[\left\{2^r + \alpha(2^r + 2)\right\} - \left\{2\alpha(2^r - 2) \exp\left(-\frac{\theta_2}{y^2}\right)\right\} \right]$$

where $y, \theta_2 > 0$.

...(2.1)

We have checked that $\int g_{2[r]}(y) dy = 1$.

3. PERCENTILE ANALYSIS OF PROBABILITY DENSITY FUNCTION OF $Y_{[r]}$

The distribution function of the concomitant $Y_{[r]}$ is given as:

$$F[Y_{[r]} = y] = \frac{\theta_2}{2^{r-1}} \int_0^y x^{-3} \exp\left(\frac{-\theta_2}{x^2}\right) \left[\left\{ 2^r + \alpha(2^r + 2) \right\} - \left\{ 2\alpha(2^r - 2) \exp\left(\frac{-\theta_2}{x^2}\right) \right\} \right] dx$$

after integrating and simplifying, we get

$$F(y) = \left[1 + \alpha(1 - 2^{1-r}) \right] \exp\left(\frac{-\theta_2}{y^2}\right) - \alpha(1 - 2^{1-r}) \exp\left(\frac{-\theta_2}{y^2}\right) \quad \dots (3.1)$$

the pth percentile of the concomitant of rth lower record values of Bivariate Inverse Rayleigh distribution by using (3.1) is

$$\xi_p = \left[\frac{+2\theta_2 B}{L_n \left\{ A \pm \sqrt{A^2 - 4BP} \right\}} \right]^{\frac{1}{2}} \quad \dots (3.2)$$

where $A = 1 + \alpha(1 - 2^{1-r})$, $B = \alpha(1 - 2^{1-r})$, when $P = 0.5$, we get the median of the distribution. The probability density function of the Concomitants of Lower Record values from Bivariate Inverse Rayleigh Distribution (3.1) and pth percentile (3.2) are given by Aleem and Ahmed (2003), here we give its percentile analysis.

Fig. 1 of table 1 shows the multiple patterns of concomitant distribution for B-life. Here the B-1 life shows the maximum values of percentiles life. The minimum value of B-1 life is 4.762134 for $\theta = 0.1$ and the maximum value for B-1 life is 33.67337 for $\theta = 5$. The minimum value of B-2 life is 3.341009 for $\theta = 0.1$ and the maximum value for B-2 life is 23.6245 for $\theta = 5$. The minimum value of B-3 life is 2.705908 for $\theta = 0.1$ and the maximum value for B-3 life is 19.13366 for $\theta = 5$. Similarly all of these B-lives from B1 to B-10-lives lies between 1.38768 to 4.762134. All of these B-lives are of increasing patterns. From the above we observe that as the parameter θ increasing then these percentile lives are also increasing this shows that both are proportional increasing. The concomitant distribution for percentile life is applying in those situations where the B-lives are of increasing order. This distribution is appropriate for particularly perfect positive correlation ($\alpha = 1$) case. This distribution provides the maximum B-1 life 33.67337 for $\theta = 5$.

Fig. 2 of table 2 shows the multiple patterns of concomitant distribution for B-life. Here the B-1 life shows the minimum values of percentiles life. The minimum value of B-1 life is 0.27553 for $\theta = 0.1$ and the maximum value for B-1 life is 1.948295 for $\theta = 5$. The minimum value of B-2 life is 0.276235 for $\theta = 0.1$ and the maximum value for B-2 life is 1.953277 for $\theta = 5$. The minimum value of

B-3 life is 0.276948 for $\theta = 0.1$ and the maximum value for B-3 life is 1.958321 for $\theta = 5$. Similarly all of these B-lives from B1 to B-10-lives lies between 0.27553 to 0.282201 for $\theta = 0.1$. All of these B-lives are of increasing patterns. But as the B-lives increasing then the pattern of these values become also increasing. On the other side as the parameter θ increasing then these percentile lives are also increasing this shows that both are proportional increasing. This distribution is appropriate for particularly perfect positive correlation case. This distribution provides the maximum B-10 life 1.995463 for $\theta = 5$.

Fig. 3 of table 3 shows the multiple patterns of concomitant distribution for B-life. Here the B-1 life shows the minimum values of percentiles life. The minimum value of B-1 life is 0.228602 for $\theta = 0.1$ and the maximum value for B-1 life is 1.616457 for $\theta = 5$. The minimum value of B-2 life is 0.229186 for $\theta = 0.1$ and the maximum value for B-2 life is 1.620591 for $\theta = 5$. The minimum value of B-3 life is 0.229778 for $\theta = 0.1$ and the maximum value for B-3 life is 1.624779 for $\theta = 5$. Similarly all of these B-lives from B1 to B-10-lives lies between 0.228602 to 0.23415 for $\theta = 0.1$. All of these B-lives are of increasing patterns. But as the B-lives increasing then the pattern of these values become also increasing. On the other side as the parameter θ increasing then these percentile lives are also increasing, this shows that both are proportional increasing. This distribution is appropriate for particularly perfect positive correlation case. This distribution provides the maximum B-10 life 1.655693 for $\theta = 5$. Therefore from the above comparison we conclude that as the value of " r " increases then the pattern of the percentile life become decreasing. We also conclude that as the parameter θ increases then these patterns also increases.

4. CONCLUSION

From the all multiple patterns of the concomitants distribution for B-life concluded that these parameters and the concomitant variate are helpful for accessing the life of components.

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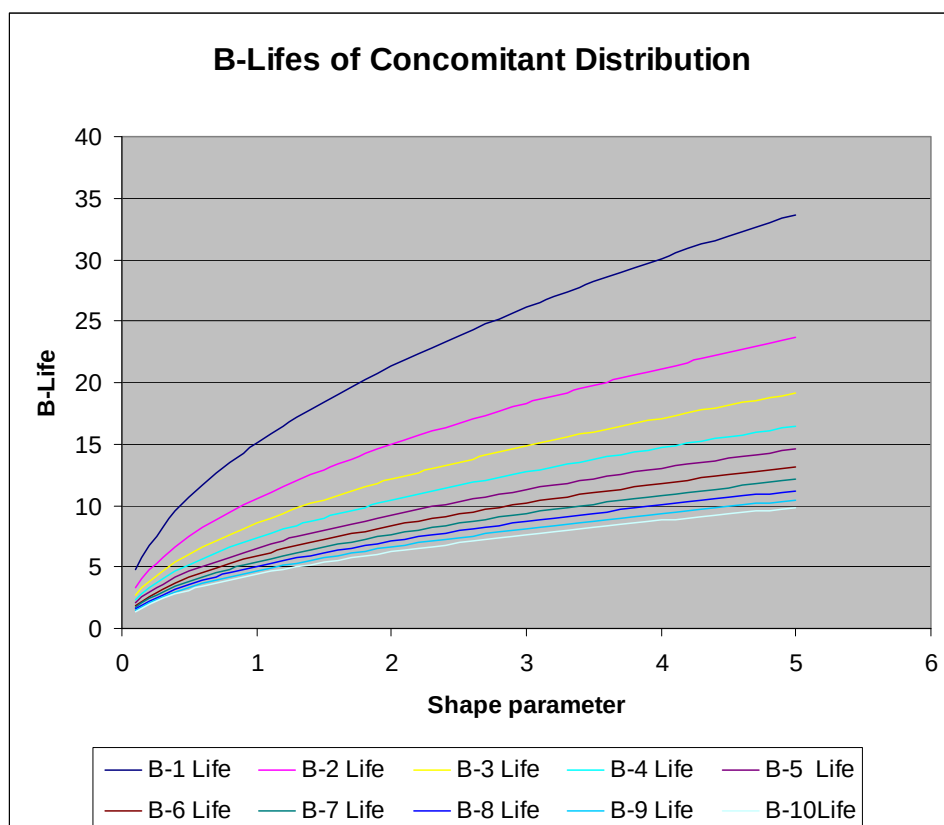


Fig.1. B-Lives of Concomitant Distribution (for data in table 1)

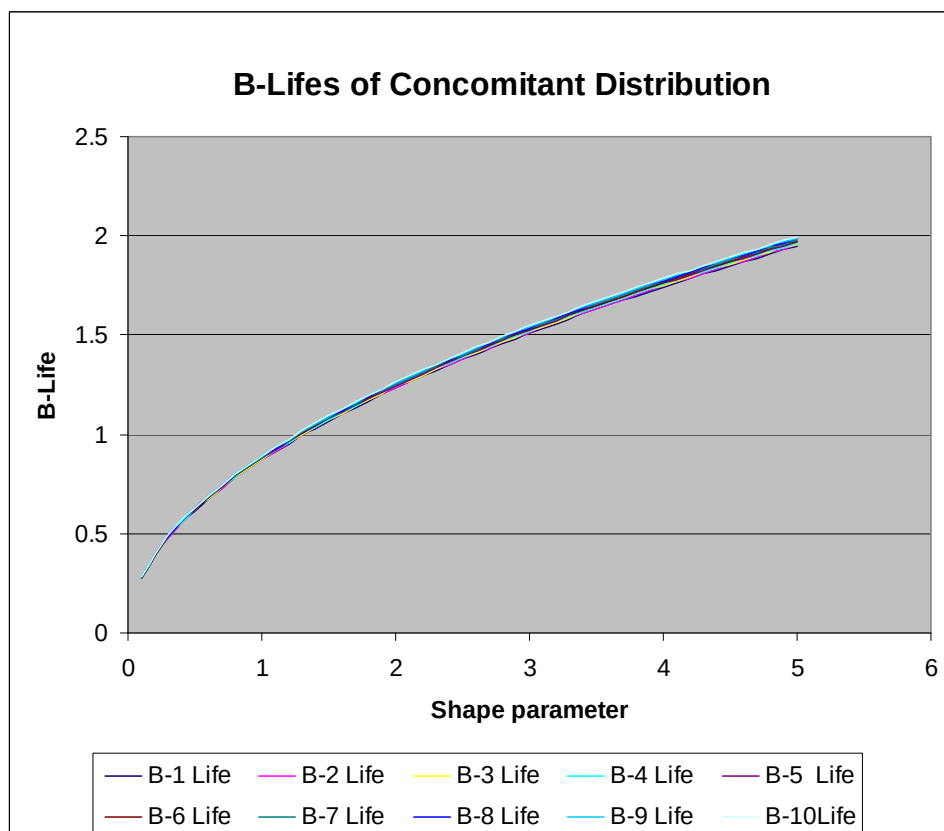


Fig.2. B-Lives of Concomitant Distribution (for data in table 2)

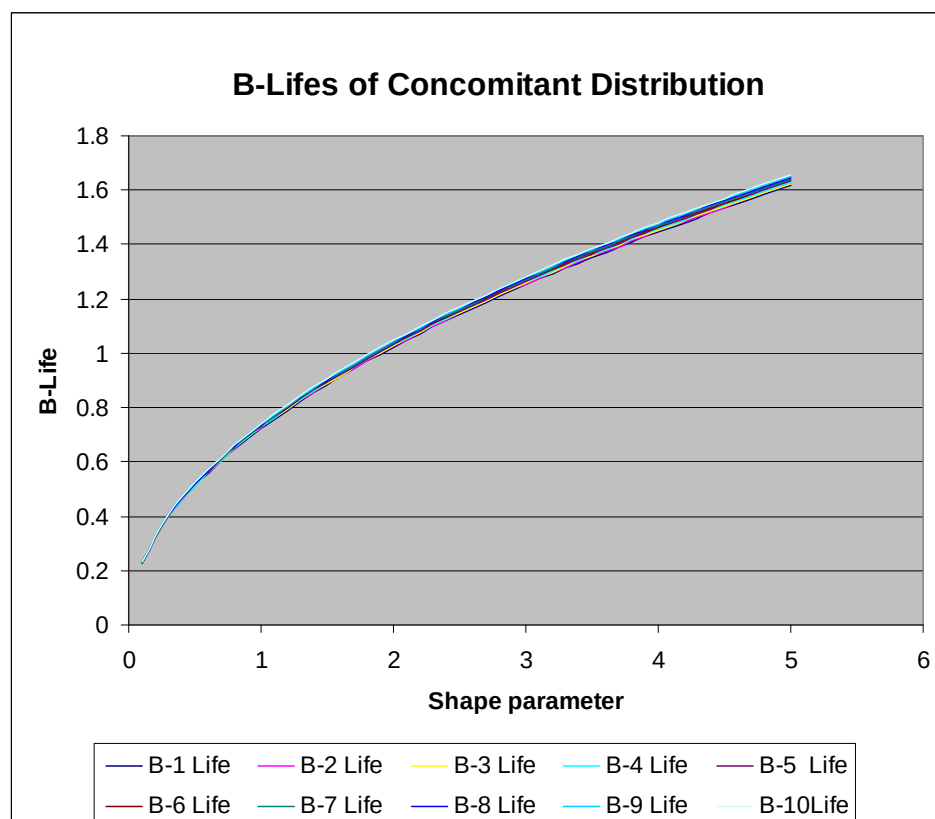


Fig.3. B-Lives of Concomitant Distribution (for data in table 3)

TABLE 1

B-lives For the Concomitant Distribution of lower record Values from Bivariate Inverse Rayleigh ($r=1$ and $\alpha=1$) by using (3.2):

θ	B-1 Life	B-2 Life	B-3 Life	B-4 Life	B-5 Life
0.5	10.64845	7.470724	6.050593	5.196261	4.607554
1	15.05919	10.5652	8.556831	7.348623	6.516065
1.5	18.44366	12.93967	10.47994	9.000189	7.980517
2	21.29691	14.94145	12.10119	10.39252	9.215107
2.5	23.81067	16.70505	13.52954	11.61919	10.3028
3	26.08328	18.29946	14.82087	12.72819	11.28616
3.5	28.17316	19.76568	16.00837	13.74802	12.19044
4	30.11838	21.1304	17.11366	14.69725	13.03213
4.5	31.94536	22.41217	18.15178	15.58878	13.82266
5	33.67337	23.6245	19.13366	16.43202	14.57036

θ	B-6 Life	B-7 Life	B-8 Life	B-9 Life	B-10 Life
0.5	4.168459	3.823376	3.541857	3.305627	3.102948
1	5.895091	5.40707	5.008943	4.674863	4.388231
1.5	7.219982	6.622281	6.134677	5.725515	5.374463
2	8.336918	7.646752	7.083715	6.611255	6.205895
2.5	9.320957	8.549328	7.919834	7.391608	6.938402
3	10.2106	9.36532	8.675744	8.0971	7.600638
3.5	11.02871	10.1157	9.370874	8.745868	8.209628
4	11.79018	10.81414	10.01789	9.349726	8.776461
4.5	12.50538	11.47013	10.62557	9.916882	9.308843
5	13.18182	12.09058	11.20034	10.45331	9.812382

TABLE 2

B-lives For the Concomitant Distribution of lower record Values from Bivariate Inverse Rayleigh ($r=2$ and $\alpha=1$) using (3.2):

θ	B-1 Life	B-2 Life	B-3 Life	B-4 Life	B-5 Life
0.5	0.616105	0.617681	0.619276	0.62089	0.622525
1	0.871304	0.873532	0.875788	0.878072	0.880384
1.5	1.067125	1.069854	1.072617	1.075414	1.078246
2	1.23221	1.235361	1.238551	1.241781	1.245051
2.1	1.262639	1.265868	1.269137	1.272447	1.275797
2.2	1.292353	1.295658	1.299003	1.302391	1.30582
2.3	1.321398	1.324777	1.328198	1.331661	1.335168
2.4	1.349818	1.35327	1.356765	1.360303	1.363885
2.5	1.377652	1.381176	1.384742	1.388353	1.392009
2.6	1.404935	1.408528	1.412165	1.415848	1.419576
2.7	1.431698	1.43536	1.439066	1.442819	1.446618
2.8	1.45797	1.461699	1.465473	1.469295	1.473164
2.9	1.483777	1.487572	1.491413	1.495302	1.49924
3	1.509143	1.513002	1.516909	1.520864	1.52487
3.1	1.534089	1.538012	1.541984	1.546004	1.550076
3.2	1.558636	1.562622	1.566657	1.570742	1.574878
3.3	1.582802	1.58685	1.590948	1.595096	1.599297
3.4	1.606605	1.610714	1.614873	1.619084	1.623348
3.5	1.63006	1.634229	1.638449	1.642721	1.647047
3.6	1.653183	1.657411	1.661691	1.666024	1.670411
3.7	1.675987	1.680273	1.684612	1.689004	1.693452
3.8	1.698484	1.702828	1.707225	1.711677	1.716184
3.9	1.720687	1.725088	1.729542	1.734052	1.738619
4	1.742608	1.747064	1.751576	1.756143	1.760768
4.1	1.764256	1.768768	1.773335	1.777959	1.782641
4.2	1.785642	1.790208	1.794831	1.799511	1.80425
4.3	1.806774	1.811395	1.816072	1.820808	1.825603
4.4	1.827662	1.832337	1.837068	1.841858	1.846709
4.5	1.848315	1.853042	1.857827	1.862671	1.867576
4.6	1.868739	1.873518	1.878356	1.883254	1.888213
4.7	1.888942	1.893773	1.898663	1.903614	1.908627
4.8	1.908931	1.913813	1.918755	1.923758	1.928824

θ	B-1 Life	B-2 Life	B-3 Life	B-4 Life	B-5 Life
4.9	1.928713	1.933646	1.938639	1.943694	1.948813
5	1.948295	1.953277	1.958321	1.963428	1.968598

θ	B-6 Life	B-7 Life	B-8 Life	B-9 Life	B-10 Life
0.5	0.624181	0.625858	0.627557	0.629277	0.631021
1	0.882725	0.885097	0.887499	0.889933	0.892398
1.5	1.081113	1.084018	1.08696	1.08994	1.09296
2	1.248362	1.251716	1.255113	1.258555	1.262041
2.1	1.279191	1.282627	1.286108	1.289635	1.293208
2.2	1.309293	1.312811	1.316374	1.319983	1.32364
2.3	1.338719	1.342316	1.345959	1.34965	1.353389
2.4	1.367512	1.371186	1.374908	1.378678	1.382497
2.5	1.395711	1.399461	1.403259	1.407107	1.411005
2.6	1.423352	1.427176	1.431049	1.434973	1.438949
2.7	1.450466	1.454363	1.45831	1.462308	1.46636
2.8	1.477082	1.48105	1.48507	1.489142	1.493268
2.9	1.503227	1.507266	1.511356	1.515501	1.519699
3	1.528925	1.533033	1.537194	1.541408	1.545679
3.1	1.554198	1.558374	1.562603	1.566888	1.571229
3.2	1.579067	1.58331	1.587607	1.59196	1.59637
3.3	1.60355	1.607858	1.612222	1.616643	1.621122
3.4	1.627665	1.632038	1.636467	1.640955	1.645501
3.5	1.651428	1.655865	1.660359	1.664911	1.669524
3.6	1.674854	1.679353	1.683911	1.688528	1.693206
3.7	1.697956	1.702518	1.707139	1.711819	1.716562
3.8	1.720749	1.725372	1.730054	1.734798	1.739604
3.9	1.743243	1.747926	1.75267	1.757476	1.762345
4	1.765451	1.770194	1.774998	1.779865	1.784796
4.1	1.787383	1.792185	1.797049	1.801976	1.806968
4.2	1.809049	1.813909	1.818832	1.823819	1.828872
4.3	1.830458	1.835376	1.840357	1.845403	1.850516
4.4	1.85162	1.856595	1.861634	1.866738	1.87191
4.5	1.872543	1.877574	1.88267	1.887832	1.893062
4.6	1.893235	1.898321	1.903473	1.908693	1.913981
4.7	1.913703	1.918844	1.924052	1.929328	1.934673
4.8	1.933954	1.93915	1.944413	1.949745	1.955146
4.9	1.953996	1.959246	1.964563	1.96995	1.975407
5	1.973834	1.979137	1.984508	1.98995	1.995463

TABLE 3

B-lives For the Concomitant Distribution of lower record Values from Bivariate Inverse Rayleigh ($r=5$ and $\alpha=1$) using (3.2):

θ	B-1 Life	B-2 Life	B-3 Life	B-4 Life	B-5 Life
0.1	0.228602	0.229186	0.229778	0.230378	0.230986
0.2	0.323291	0.324118	0.324956	0.325804	0.326664
0.3	0.39595	0.396962	0.397988	0.399027	0.40008
0.4	0.457203	0.458372	0.459557	0.460757	0.461972
0.5	0.511169	0.512476	0.5138	0.515142	0.516501
0.6	0.559957	0.561389	0.56284	0.564309	0.565798
0.7	0.604823	0.60637	0.607937	0.609524	0.611132
0.8	0.646583	0.648237	0.649912	0.651608	0.653327
0.9	0.685805	0.687559	0.689335	0.691135	0.692958
1	0.722902	0.724751	0.726623	0.72852	0.730442
1.1	0.758186	0.760125	0.762089	0.764079	0.766094
1.2	0.791899	0.793924	0.795976	0.798054	0.800159
1.3	0.824235	0.826343	0.828478	0.830641	0.832832
1.4	0.855349	0.857536	0.859752	0.861997	0.864271
1.5	0.88537	0.887634	0.889928	0.892251	0.894605
1.6	0.914406	0.916745	0.919114	0.921513	0.923945
1.7	0.942548	0.944959	0.947401	0.949874	0.95238
1.8	0.969874	0.972355	0.974867	0.977413	0.979991
1.9	0.996451	0.999	1.001581	1.004196	1.006845
2	1.022337	1.024952	1.0276	1.030283	1.033001
2.1	1.047584	1.050263	1.052977	1.055726	1.058511
2.2	1.072236	1.074979	1.077756	1.08057	1.083421
2.3	1.096335	1.099139	1.101979	1.104856	1.107771
2.4	1.119914	1.122779	1.12568	1.128619	1.131596
2.5	1.143008	1.145931	1.148892	1.151892	1.154931
2.6	1.165644	1.168625	1.171645	1.174704	1.177803
2.7	1.187849	1.190887	1.193964	1.197081	1.200239
2.8	1.209646	1.21274	1.215873	1.219048	1.222264

2.9	1.231057	1.234206	1.237395	1.240625	1.243898
3	1.252102	1.255305	1.258548	1.261834	1.265163
3.1	1.2728	1.276055	1.279352	1.282692	1.286076
3.2	1.293166	1.296473	1.299823	1.303217	1.306655
3.3	1.313216	1.316575	1.319977	1.323423	1.326914
3.4	1.332965	1.336374	1.339827	1.343325	1.346869
3.5	1.352425	1.355884	1.359388	1.362937	1.366532
3.6	1.371609	1.375117	1.378671	1.38227	1.385917
3.7	1.390529	1.394085	1.397688	1.401337	1.405034
3.8	1.409195	1.412799	1.416449	1.420148	1.423894
3.9	1.427616	1.431268	1.434966	1.438712	1.442508
4	1.445803	1.449501	1.453246	1.457041	1.460885
4.1	1.463764	1.467508	1.4713	1.475141	1.479033
4.2	1.481507	1.485297	1.489134	1.493022	1.496961
4.3	1.499041	1.502875	1.506758	1.510692	1.514677
4.4	1.516371	1.52025	1.524178	1.528157	1.532189
4.5	1.533506	1.537428	1.541401	1.545425	1.549502
4.6	1.550451	1.554417	1.558433	1.562502	1.566624
4.7	1.567213	1.571222	1.575282	1.579394	1.583561
4.8	1.583798	1.587849	1.591952	1.596108	1.600319
4.9	1.600211	1.604304	1.608449	1.612648	1.616903
5	1.616457	1.620591	1.624779	1.629021	1.633319

θ	B-6 Life	B-7 Life	B-8 Life	B-9 Life	B-10 Life
0.1	0.231602	0.232226	0.232859	0.2335	0.23415
0.2	0.327535	0.328417	0.329312	0.330219	0.331139
0.3	0.401146	0.402228	0.403323	0.404434	0.40556
0.4	0.463204	0.464452	0.465718	0.467	0.468301
0.5	0.517878	0.519274	0.520688	0.522122	0.523576
0.6	0.567307	0.568836	0.570385	0.571956	0.573549
0.7	0.612761	0.614413	0.616086	0.617783	0.619504
0.8	0.655069	0.656835	0.658624	0.660438	0.662277
0.9	0.694806	0.696679	0.698576	0.7005	0.702451
1	0.73239	0.734364	0.736364	0.738392	0.740448
1.1	0.768137	0.770207	0.772305	0.774432	0.776589
1.2	0.802293	0.804455	0.806647	0.808868	0.811121
1.3	0.835053	0.837303	0.839584	0.841897	0.844241
1.4	0.866575	0.868911	0.871278	0.873677	0.87611
1.5	0.896991	0.899408	0.901858	0.904342	0.90686
1.6	0.926408	0.928905	0.931435	0.934	0.936601
1.7	0.95492	0.957493	0.960101	0.962746	0.965427
1.8	0.982604	0.985252	0.987936	0.990657	0.993416
1.9	1.00953	1.01225	1.015008	1.017803	1.020638
2	1.035756	1.038547	1.041376	1.044244	1.047152
2.1	1.061334	1.064194	1.067093	1.070032	1.073012
2.2	1.08631	1.089237	1.092205	1.095213	1.098262
2.3	1.110724	1.113718	1.116752	1.119827	1.122945
2.4	1.134614	1.137671	1.14077	1.143912	1.147098
2.5	1.15801	1.161131	1.164294	1.167501	1.170752
2.6	1.180943	1.184126	1.187352	1.190622	1.193937
2.7	1.203439	1.206683	1.20997	1.213302	1.216681
2.8	1.225523	1.228825	1.232173	1.235566	1.239007
2.9	1.247215	1.250576	1.253983	1.257437	1.260938
3	1.268537	1.271955	1.27542	1.278933	1.282494
3.1	1.289505	1.292981	1.296503	1.300074	1.303694
3.2	1.310139	1.31367	1.317248	1.320876	1.324554
3.3	1.330452	1.334038	1.337672	1.341356	1.345091
3.4	1.35046	1.3541	1.357788	1.361528	1.365319
3.5	1.370176	1.373869	1.377611	1.381405	1.385252
3.6	1.389612	1.393357	1.397153	1.401001	1.404902
3.7	1.40878	1.412577	1.416425	1.420326	1.424281
3.8	1.427691	1.431538	1.435438	1.439391	1.4434
3.9	1.446354	1.450252	1.454203	1.458208	1.462268
4	1.46478	1.468727	1.472728	1.476784	1.480897
4.1	1.482977	1.486973	1.491024	1.49513	1.499294
4.2	1.500953	1.504998	1.509097	1.513254	1.517468
4.3	1.518716	1.522809	1.526957	1.531163	1.535426
4.4	1.536274	1.540414	1.544611	1.548865	1.553178
4.5	1.553634	1.557821	1.562064	1.566366	1.570728
4.6	1.570801	1.575035	1.579325	1.583675	1.588085
4.7	1.587783	1.592062	1.596399	1.600796	1.605254
4.8	1.604586	1.60891	1.613293	1.617736	1.622241
4.9	1.621214	1.625583	1.630012	1.634501	1.639052
5	1.637674	1.642087	1.64656	1.651095	1.655693