Comparison of Stabilizing and Tracking Designs of Back-stepping Controller for Wind **Tunnel Application**

Rini Jones S.B., P.Poongodi, and Binu L.S.

Abstract— Wind tunnel is a facility used to investigate the aerodynamic properties of objects by passing a stream of velocity-controlled air over them. Hypersonic wind tunnels operate at hypersonic speeds ie, with a mach number greater than 6. For doing experiments, it is necessary to maintain a constant pressure in the settling chamber of the tunnel so that we get the desired mach number and mass flow rate through the nozzle. The Back stepping control is a systematic and recursive design methodology for nonlinear feedback control. Here, a back stepping controller is designed using the stabilizing and tracking design approach and the mass flow rate of hypersonic intermittent blow down type wind tunnel is controlled. The servo and regulator operations of the system are studied and it is found that the tracking design is good for the servo operation and the stabilizing design for regulator operation.

Index Terms- Hypersonic wind tunnel, back stepping controller, tracking design, stabilizing design, Lyapunov function.

I. INTRODUCTION

IND tunnels are being used to study the aerodynamic W properties of space crafts, fighter planes etc. Although the form of a wind tunnel can vary, all wind tunnels have a drive system and a test section where a model that is supported in airstream whose characteristics are measured by test instrumentation. Wind tunnels are an excellent example of a technological innovation that supports aircraft design. According to Speed of operation they can be classified into: Subsonic (Low Speed)Mach No: range: 0.5m/s – 55m/s, Supersonic (High Speed) Mach No: range: 1.5 - 4 and Hypersonic (High Speed)Mach No: range: 6 – 12.

Hypersonic intermittent blow down-type wind tunnel is a ground based facility to simulate flight conditions of space vehicles in hypersonic flow regime [7].

The common control method for today's industries is based on a classical linear system design approach using PI controllers. However, when the system parameter changes

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become significant, it is difficult to achieve optimum response. Also, the cross-interaction between the subsystems could not be conveniently incorporated by the traditional PI-control loop approach, thus limiting the performance of the system.

In recent years, significant efforts of developing nonlinear control approaches have been made for industrial applications. In this paper, a back stepping controller using Lyapunov function is derived after formulating the control problem. The objectives are to improve the system's settling time against the set point variations and achieve stabilization for the disturbances. The paper is organized as follows. Section 2 states the model of the hypersonic wind tunnel. Section 3 details the stabilizing and tracking designs of back stepping controller. The main results are stated in section 4 whilst concluding remarks are contained in section 5.

II. WIND TUNNEL MODEL

Obtaining the mathematical models of the wind tunnel process is very complicated since they involve viscous effects and distributed characteristics. Models of different components are developed to obtain the process model of the wind tunnel system for designing a controller considering the total system as three pressure vessels. The continuity equations [6] of the pressure vessels are used to develop the non linear model considering the effects of temperature.

Flow rate of compressible fluid F1 is given by

$$F_1 = mC_v N_8 F_p P_1 Y \sqrt{\frac{XM}{T_1 Z}}$$

Where, 'm' is the position of the valve,

 C_v is the valve coefficient,

 N_8 is the constant for engineering units,

Fp is the constant for pipeline geometry,

M is molecular weight of air,

Z is the compressibility factor,

Expansion factor
$$Y = 1 - \frac{X}{3F \cdot X}$$

Where, X_t is critical pressure drop ratio factor

 F_k is the ratio of specific heats factor and

$$X = \frac{P_1 - P_2}{P_1},$$

Where, P_2 is the downstream pressure of PRV. The outflow from heater F_2 is given by

$$F_2 = C_v N_8 F_p P_2 Y \sqrt{\frac{XM}{T_2 Z}}$$

 $F_2 = C_v N_8 F_p P_2 Y \sqrt{\frac{XH}{T_2 Z}}$ Where, the Expansion factor $Y = 1 - \frac{X}{3F_k X_t}$

and $X = \frac{P_2 - P_3}{P_2}$. Where, P3 is the settling chamber pressure.

The mass flow rate through nozzle, F₃ is given by

$$F_3 = \frac{K_n P_3}{\sqrt{T_2}}$$

Where, k_n is the nozzle constant and P_3 is the settling chamber pressure and T_3 is the settling chamber temperature.

The continuity equations for three pressure vessels may be written as,

$$C_1 \frac{dP_1}{dt} = -F_1; \ C_2 \frac{dP_2}{dt} = F_1 - F_2; \ C_3 \frac{dP_3}{dt} = F_2 - F_3$$

Where,

$$C_1 = \frac{V_1}{nRT_1}$$
; $C_2 = \frac{V_2}{nRT_2}$; $C_3 = \frac{V_3}{nRT_3}$

III. BACK STEPPING CONTROLLER

Back stepping is a nonlinear control design technique, developed by V. Kokotovic and others in 1990's. They are built from subsystems that radiate out from irreducible subsystem, which can be stabilized using Lyapunov synthesis. Furthermore, back stepping can accommodate large nonlinearities and uncertainties in the system's model. Here the complex non linear system is broken down into smaller subsystems and we apply Lyapunov function for each. In this paper, implementation of the back stepping controller using the tracking technique [11-13] and stabilizing technique [1-4] for a hypersonic wind tunnel is discussed, so that the mass flow rate is controlled.

A. Controller Design

From the modeling equations, the state equations of the system can be written as follows

$$\dot{x}_1 = \frac{F_2 - F_3}{C_3}; \ \dot{x}_2 = \frac{F_1 - F_2}{C_2}; \ \dot{x}_3 = \frac{-F_1}{C_1}$$

Then, the three flow equations as below

$$F_{1} = 2.39 * 10^{-5} m \sqrt{P_{1}^{2} - P_{1}P_{2}}$$

$$F_{2} = 1.57 * 10^{-5} \sqrt{P_{2}^{2} - P_{2}P_{3}}$$

$$F_{3} = 2.24 * 10^{-4}P_{3}$$

$$x_{1} = P_{3}; x_{2} = P_{2}; x_{3} = P_{1}$$

Now using binomial expansion, the state equations can be approximated as

1. Stabilizing Design Control Lyapunov function is selected as

$$V_1(x) = \frac{1}{2} \cdot \chi_1^2$$

Derivative of $V_1(x)$ becomes,

$$V_1(x) = x_1 x_1$$

$$V_1(x) = x_1(1.557x_2 - 22.97x_1)$$

Now the desired value of x_2 to make $\dot{V}_1(x)$ negative definite is given by,

$$x_2(des) = -C_1 x_1$$
,
Where, C_1 is a positive constant.
Let, $Z = x_2 - x_2(des) = x_2 + C_1 x_1$.
Now, $x_1 = 1.557z - 1.557C_1 x_1 - 22.97x_1$

$$z = m(0.455x_3 - 0.227x_2) - 0.029x_2 + 0.014x_1 + C_1(1.557z - 1.557C_1x_1 - 22.97x_1)$$



Figure 1. The system with back stepping controller using stabilizing design.

The second Lyapunov function is selected as

$$V_2(x) = \frac{1}{2} \cdot \chi_1^2 + \frac{1}{2} \cdot Z^2$$

 $\cdot V_2(x) = x_1 x_1 + z_1 z_1$
 $\cdot V_2(x) = x_1 (1.557z - 1.557C_1x_1 - 22.97x_1)$
 $+ z[m(0.455x_3 - 0.227x_2) - 0.029x_2 + 0.014x_1$
 $+ C_1 (1.557z - 1.557C_1x_1 - 22.97x_1)]$

Now the desired value of m, m(des) to make $\dot{V}_2(x)$ negative definite is given by

$$m(des) = -C_2 z - (\frac{1.557x_1 + 1.557C_1 z}{0.455x_3 - 0.227x_2})$$

2. Tracking Design $z = x_{ref} - x_1$ $\cdot \qquad \cdot$ $z = x_{ref} -1.557x_2 + 22.97x_1$

Control Lyapunov function is selected as

$$V_1(z) = \frac{1}{2} \cdot z^2$$

Derivative of $V_1(x)$ becomes,

$$\dot{V}_1(z) = z \dot{z}$$

 $\dot{V}_1(x) = z \dot{x}_{ref} - 1.557 z x_2 + 22.97 z x_1$

Now the desired value of x_2 to make $\dot{V}_1(z)$ negative definite is given by,

$$x_{2}(des) = -C_{1}x_{1} + \frac{22.97}{1.557}x_{1} + \frac{x_{ref}}{1.557},$$

Where, C₁ is a positive constant.
$$y = x_{2} - x_{2des}$$

Let, $x_{2} = y + C_{1}z + \frac{22.97}{1.557}x_{1} + \frac{x_{ref}}{1.557}.$
Now, $\dot{z} = -1.557y - 1.557C_{1}z$
 $\dot{y} = x_{2} + 1.557C_{1}y + 1.557C_{1}^{2}z$
The second Lyapunov function is selected
 $V_{2}(z) = \frac{1}{2} \cdot z^{2} + \frac{1}{2} \cdot y^{2}$
 $\dot{V}_{2}(z) = zz + yy$

•

$$V_2(x) = -1.557 zy - 1.557 C_1 z^2 + my(0.455 x_3 - 0.227 x_2) - 0.029 yx_2 + 0.014 yx_1 + 1.557 C_1 y^2 + 1.557 C_1^2 yz$$

as



Now the desired value of m, m(des) to make $V_2(z)$ negative definite is given by

IV.

$$m(des) = -C_2 y - \left(\frac{0.029x_2 - 0.014x_1 - 1.557C_1 y - 1.557C_1^2 z}{0.455x_3 - 0.227x_2}\right)$$

IV. SIMULATION AND RESULTS

Extensive simulations have been performed using Matlab-Simulink Software to examine stabilizing and tracking control action of the nonlinear back stepping controller applied for Hypersonic wind tunnel.

A. Servo operation

In servo operation, the ability of a controller to track changes in set point (requirement) is evaluated. Simulations were done at the two set points of 70 bar and 100 bar, with the values of the positive constants, C1 and C2 of the backstepping controller as 750, 1000, 1500, 2000 and 3000.

1. Controller using Stabilizing design

Figure 3 to figure 6 show the servo operation of the backstepping controller using stabilizing design and the performance comparison results are tabulated in table 1.



Figure 3. Output response of the system with backstepping stabilizing controller for a set point of 70 bar with $C_1=C_2=1000$.



Figure 4. Output response of the system with backstepping stabilizing controller for a set point of 70 bar with $C_1=C_2=2000$.



Figure 5. Output response of the system with backstepping stabilizing controller for a set point of 100 bar with $C_1=C_2=1000$.



Figure 6. Output response of the system with backstepping stabilizing controller for a set point of 70 bar with $C_1=C_2=2000$.

Set Point	C ₁ ,C ₂	Settling time in sec	Offset (bars)	%Over shoot	IAE *10 ⁸	ISE *10 ¹⁴
70	1000	4	1.5	50	1.11	6.7
bar	1500	4.4	0.7	57	1.17	7.03
	2000	3.3	0.5	50	1.5	8.41
100	1000	4.2	2	45	1.63	14
bar	1500	4.6	1	36	2.19	17.6
	2000	3.5	0.9	29	3.57	26.8

Table 1: Performance comparison of servo operation- Stabilizing design

2. Controller using Tracking design

Figure 7 to figure 10 show the servo operation of the backstepping controller using tracking design and the performance comparison results are tabulated in table2.



Figure 7. Output response of the system with backstepping tracking controller for a set point of 70 bar with $C_1=C_2=1000$.







Figure 9. Output response of the system with backstepping tracking controller for a set point of 100 bar with $C_1{=}C_2{=}1000$



Figure 10. Output response of the system with backstepping tracking controller for a set point of 100 bar with $C_1=C_2=2000$

Table 2: Performance comparison of servo operation- Tracking design

Set	C_1, C_2	Settling	Offset	%	IAE	ISE
Point	-1,-2	time in sec	(bars)	Over		
		(2%)	· /	shoot	X10 ⁸	X10 ¹⁴
		tolerance)				
70	1000	3.8	0.5	51	1.08	6.72
bar	1500	4.3	003	57	1.13	7.03
	2000	3.9	0.01	50	1.47	8.35
100	1000	3.9	0.6	46	1.57	13.9
bar	1500	3.5	0.01	35	2.11	17.5
	2000	3.4	0.01	30	3.57	26.9

In the servo operation, comparing to the results obtained in table 1 and table 2, the performance of the controller using tracking design has less offset compared to the controller with stabilizing design. The settling time is also less for the tracking design controller. So we can infer that, when a sudden change in set point is made to the system (servo operation), the tracking design of controller has better performance than the stabilizing design of the controller.

B. Regulator Operation

In regulator operation, the ability of a controller to resist the disturbances affecting the system (load changes) is evaluated.

An external disturbance in valve position in the form of an impulse with amplitude, period and pulse width of 10, is given to the system at 10 sec, when the system is in the set point. Results are obtained by giving this disturbance when the system is at the set points of 70 bar and 100 bar, and with different values of the positive constant C1 and C2.

1. Controller using Stabilizing design

Figure 11 to figure 14 show the regulator operation of the backstepping controller using stabilizing design and the performance comparison results are tabulated in table3.







Figure 12. Regulator response of the system with backstepping controller when a disturbance is applied, at the set point is 70 bar with $C_1=C_2=2000$.







Figure 14. Regulator response of the system with backstepping controller when a disturbance is applied, at the set point is 100 bar with $C_1=C_2=2000$.

Table 3: Performance comparison of regulator operation- Stabilizing

SetPoint	C ₁ ,C ₂	%Overshoot	IAE X10 ⁷	ISE X10 ¹⁴
70 bar	1000	157	5.46	2.78
	1500	92	4.26	1.78
	2000	57	2.30	0.525
100 bar	1000	70	5.61	2.6
	1500	60	3.37	1.2
	2000	40	2.11	0.439

1. Controller using Tracking design

Figure 15 to figure 18 show the regulator operation of the backstepping controller using tracking design and the performance comparison results are tabulated in table4.



Figure 15. Regulator response of the system with backstepping controller when a disturbance is applied, at the set point is 70 bar with $C_1=C_2=1000$.

In the regulator operation, comparing to the results obtained in table 3 and table 4, the performance of the controller using tracking design and stabilizing design are almost same. Even then, we can say that for a disturbance given, when the system is at the set point (regulator operation), there is a slight improvement in performance for the stabilizing design in terms of IAE and ISE.



Figure 16. Regulator response of the system with backstepping controller when a disturbance is applied, at the set point is 70 bar with $C_1=C_2=2000$.



when a disturbance is applied, at the set point is 100 bar with $C_1=C_2=1000$.



Table 4: Performance comparison of regulator operation- Tracking

Set Point	C ₁ ,C ₂	% Overshoot	IAE X 10 ⁷	ISE X10 ¹⁴
70 bar	1000	128	4.87	2.64
	1500	94	4.46	1.98
	2000	57	1.91	0.421
100 bar	1000	75	4.94	2.4
	1500	60	3.25	1.26
	2000	40	5.93	1.43

V. CONCLUSION

A stabilizing and tracking design controller using the back stepping technique is designed and implemented for a hypersonic wind tunnel. Compared with the PI and the fuzzy assisted PI controller [7,8], the back stepping controller has improved settling time at different set points and also, when a disturbance is given, the controller takes corrective action to stabilize the system at the desired set point. The back stepping controller utilizes feedback of internal states so that the controller is more predictive and feed forward in nature. This may be the reason for fast settling and better disturbance rejection.

From the studies made by the servo and regulator operation in the tracking and stabilizing design, we can infer that the tracking design is best for the servo operation and the stabilizing design has better performance in the regulator operation. The results show slight offset, which is inherent with a back stepping controller [10]. It can be eliminated by incorporating an integrator in the controller.

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