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Multiple Winding Transformer Model for Power Supply Applications in Circuit Simulations

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Abstract—**The new technologies in the electronics industry are fast replacing current designs. With electronic circuits becoming obsolete in a very short time, the expected turnaround times for new designs are becoming shorter and shorter. This is why circuit simulation is becoming a crucial part of the design cycle.**

This research develops a simulation model for multiple winding transformers. The current transformer models are very specific to certain applications and have a limited number of windings. The objective of this research is to develop a general multiple winding transformer model. This will prove to be very useful in simulating power electronic circuits with multiple winding transformers.

Index Terms— **circuit simulations, multiple winding transformer, switch mode power supply**

I. INTRODUCTION

Circuit simulation programs are very common tools in designing electronic circuits. Currently, these programs do not have available models for multiple winding transformers. Simulations of circuits with these kinds of transformers are limited to having all devices referred to the primary side. Existing transformer models only have two windings. There are some multiple winding models but are specific to certain applications only. The development of more models can lead to more possibilities in the design and development of electronic circuits with transformers.

In many circuit simulation programs, only sinusoidal waveforms are allowed as input signals for transformer models. These are very useful in line frequency circuit simulations. Using these models in high frequency switching converter simulations will not be appropriate. Signals will be distorted and simulation results will be very far from expected outputs.

There are specialized simulation programs that offer models appropriate for switching converter applications. These models may be modified to model a physical transformer. The parameters that may be varied include magnetizing inductance, leakage inductance and core permeability, etc. This level of detail is very good for some simulations when the construction of the physical transformer is already considered. In most simulations however, the design of the physical component is not yet considered because the initial intention is to verify the behavior of the circuit. When the requirement of the model is to get the parameters of the physical device, the model becomes too cumbersome to work with.

An additional requirement in switching converters is the use of multiple winding transformers. Transformer models that are available for use in high frequency circuit simulations only have two windings.

This research proposes a model that may be used for simulating multiple winding transformers. The model proposed is a two winding transformer model and may be expanded easily to increase the number of windings in the simulation. The basic equations used in the derivation of the model, as well as the proposed transformer model, are presented in Section II. The expanded model is shown in Section III and simulation results using the model are discussed in Section IV. A summary of the results is shown in the conclusion.

II. TRANSFORMER MODEL

A. Transformer Equation

The proposed transformer model is derived from the basic equations of a transformer. Two inductors that are coupled to each other may represent a transformer. In order to derive the equations of a transformer, we can look at the voltage and current characteristics of a single inductor.

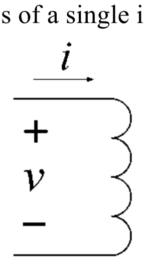

Figure 1. An isolated Inductor

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In Fig. 1, the current is entering the positive terminal of the inductor. The equation that relates the two parameters is,

$$v = L\left(\frac{di}{dt}\right) \qquad (1)$$

In a single and isolated inductor, the voltage across the winding is equal to the product of the inductance and the time derivative of the current flowing through the inductor. When an inductor is placed in close proximity to another inductor, some of the magnetic flux produced by one winding may pass through the other winding. This occurs because of magnetic coupling. The two windings are coupled to each other because they share some of the flux generated by either winding. This is the basis of the operation of transformers.

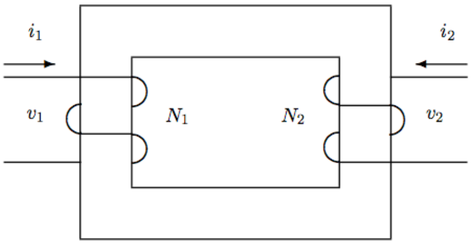

Figure 2. Two-Winding Transformer

Figure 2 shows a two-winding transformer with both currents i_1 and i_2 entering the positive terminals of the two windings. The two windings are essentially two inductors placed in close proximity of each other. They are normally wound on the same core. The core material and the method of construction dictate the amount of flux that will reach one winding.

We can derive the equations of the two winding transformer shown in Figure 2. The voltages of the two windings may be defined as:

$$V_1 = L_{11}\left(\frac{di_1}{dt}\right) + L_{12}\left(\frac{di_2}{dt}\right) \qquad (2)$$

$$V_2 = L_{21}\left(\frac{di_1}{dt}\right) + L_{22}\left(\frac{di_2}{dt}\right) \qquad (3)$$

where

L_{11} = self inductance of winding 1
L_{22} = self inductance of winding 2
i_1 = current in winding 1
i_2 = current in winding 2
L_{12} = mutual inductance in winding 1 due to winding 2
L_{21} = mutual inductance in winding 2 due to winding 1

The second term in the equation is the voltage due to the mutual inductance of the two windings. This voltage is caused by the flux created by the current in the other winding. The inductances L_{12} and L_{21} are equal as seen in [6]. This may be replaced by the symbol M. The value of M is dependent on the coupling of the two windings, which is primarily determined by the core material. We can define the coupling coefficient *k* as

$$k = \frac{M}{\sqrt{L_{11}L_{22}}} \qquad (4)$$

where $-1 \leq k \leq 1$

In iron core transformers, *k* can reach as high as 0.998 while it is very difficult to make *k* above 0.5 in air core transformers. For switching converters, this is generally desired to be as high as possible. The higher the coupling coefficient, the better the coupling of the windings and this translates to a smaller leakage inductance. In many topologies, the leakage inductance is the source of voltage spikes that can damage the power switch.

For transformers with multiple windings, (2) and (3) may be expanded easily to include the effects of the additional windings. The multiple-winding transformer may be represented by the following equations:

$$V_1 = L_{11}\left(\frac{di_1}{dt}\right) + L_{12}\left(\frac{di_2}{dt}\right) + \cdots \quad L_{1n}(\frac{di_n}{dt}) \qquad (5)$$

$$V_2 = L_{21}\left(\frac{di_1}{dt}\right) + L_{22}\left(\frac{di_2}{dt}\right) + \cdots \quad L_{2n}(\frac{di_n}{dt}) \qquad (6)$$

…

$$V_n = L_{n1}\left(\frac{di_1}{dt}\right) + L_{n2}\left(\frac{di_2}{dt}\right) + \cdots \quad L_{nn}(\frac{di_n}{dt}) \qquad (7)$$

Each winding will have an effect on all the other windings. The voltage equation of each winding will have n terms for an n-winding transformer.

B. *Two-Winding Model*

The proposed model was derived from (2) and (3).

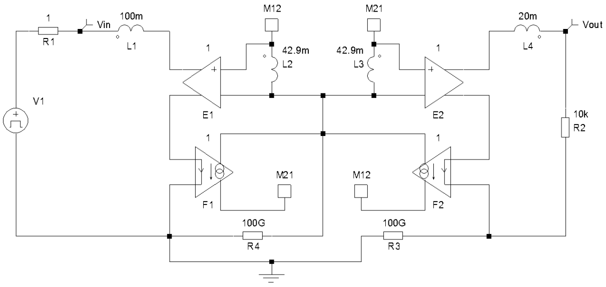

Figure 3. Proposed Two-Winding Transformer Model

A circuit model that satisfies the system of equations may be seen in Figure 3. Standard inductors were used for the self-inductances and mutual inductances. L1 and L4 are the self-inductances of the two windings. L2 and L3 are the mutual inductances. E1 and E2 are voltage controlled voltage sources to include the voltages across the mutual inductances without inserting an inductor into the loop. F1 and F2 are current controlled current sources to sample the current in the windings without breaking the loop. The sampled currents are fed to the mutual inductances and the voltages developed across these inductances will be sampled by the dependent sources to be inserted in the voltage loop of each winding. Each winding will only have one inductance in the loop but will also have the effects of the mutual inductances through the use of the controlled sources.

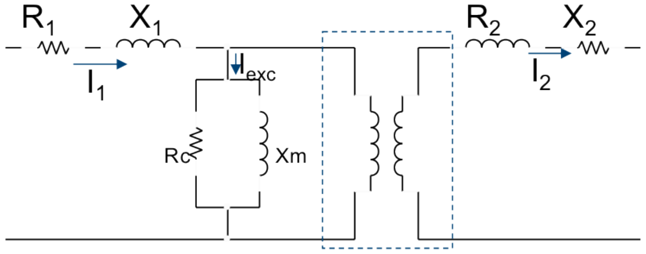

Figure 4. Exact Equivalent Circuit of a Transformer

The model proposed will have self-inductances and mutual inductances. When looking at the exact equivalent circuit of a transformer as seen in figure 4, certain resistances may be seen that are not included in the proposed model. These resistances may be added in the circuit simulation. The leakage inductances are already covered in the model via the self-inductances.

In the cantilever model seen in [5], the leakage inductance is already incorporated in the self-inductance. The only parameter needed is the coefficient of coupling which may be assumed or computed depending on the design procedure followed.

The first test performed to check the validity of the model was to inject a sinusoidal and square waveform in the first winding and look at the output across the second winding.

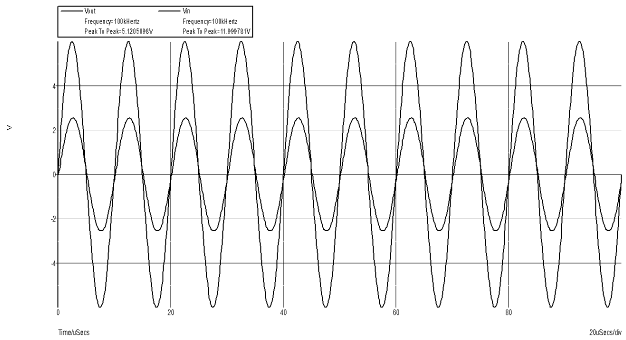

Figure 5. Input and Output Waveforms Using a Sine Wave

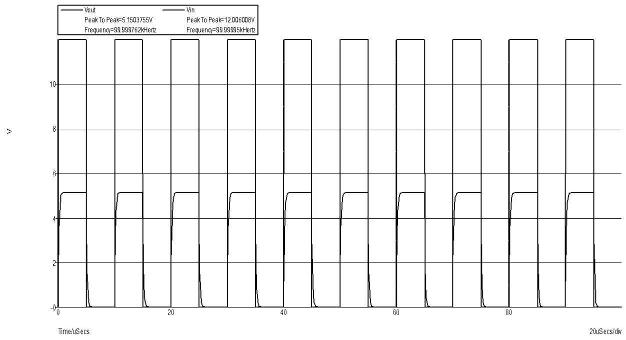

Figure 6. Input and Output Waveforms Using a Square Wave

The waveforms in Figures 5 and 6 show that a sinusoidal input waveform yields a sinusoidal output. A square waveform at the input yields a square waveform at the output. The values used in this simulation may be seen in Figure 1. The waveforms also show that it has a step down behavior that is consistent with the values used.

C. Three Winding Model

In order to observe the validity of the model with more than two windings, the model was tested with three windings based on (5), (6) and (7) with $n = 3$.

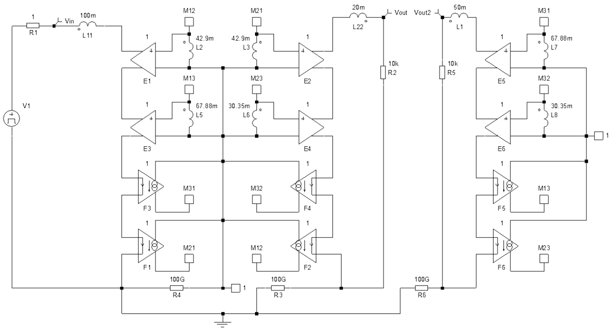

Figure 7. Three Winding Transformer Model

The model in Figure 7 shows the circuit implementation with three windings. This model was also tested with the sinusoidal and square waveforms at the input.

Figure 8. Input and Output Waveforms Using a Sine Wave

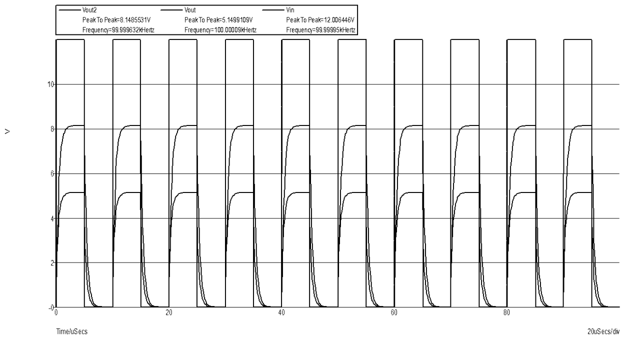

Figure 9. Input and Output Waveforms Using a Square Wave

The output waveforms, for the two types of input waveforms, show that the model is behaving properly as a transformer as seen in Figures 8 and 9. The values used as seen in Figure 7 show that the two output windings should have lower voltages than the input voltage. This is also evident in the shown waveforms in Figures 8 and 9.

D. Flyback Simulation Test

The model was tested in an actual switching converter simulation. The chosen topology was the flyback converter with a regenerative clamp.

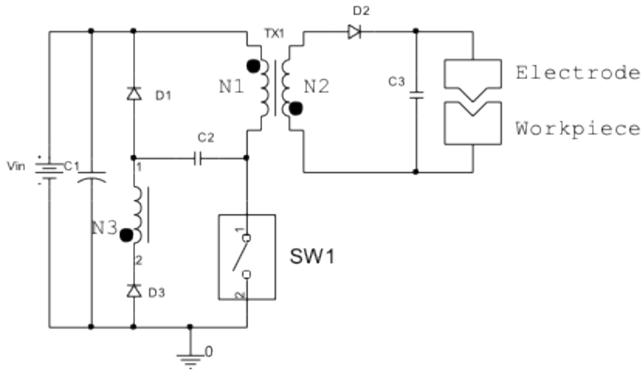

Figure 10. Energy-Saving Flyback Converter

In the flyback converter of Figure 10, the transformer TX1, with the three windings N1, N2 and N3, was replaced with the three winding model in Figure 7.

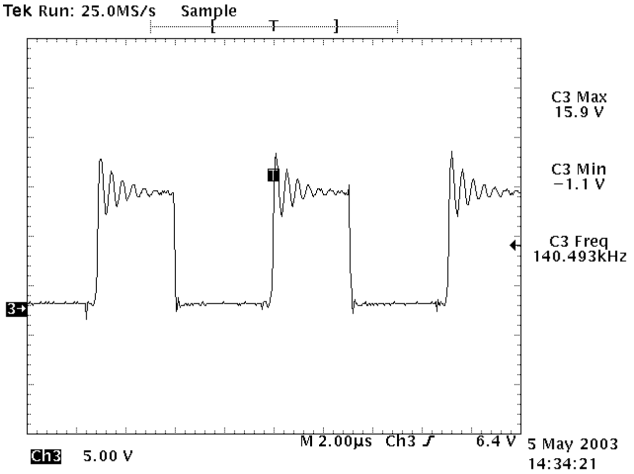

Figure 11. Actual Drain Voltage of Flyback Converter with Regenerative Clamp

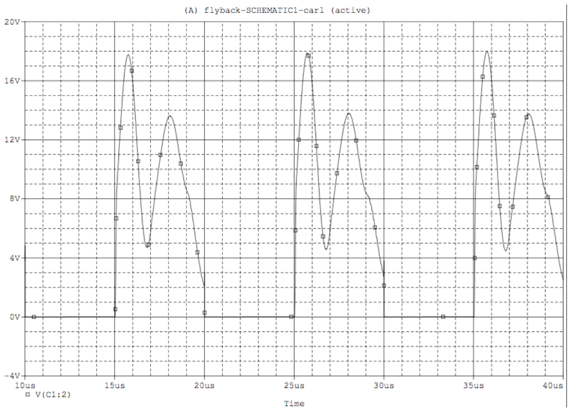

Figure 12. Simulated Drain Voltage Waveform of the Flyback Converter with Regenerative Clamp

The waveform seen in Figure 11 is the actual drain voltage of a MOSFET in a hardware implementation. The circuit used in this setup may be seen in [10].

The drain voltage seen in Figure 12 is the result of a simulation using the three winding model in the same topology.

The two waveforms are very similar except for the minor difference in the frequency of oscillation. This may be attributed to the actual value of the leakage inductance in the actual transformer.

The objective of the research on the flyback converter with a regenerative clamp was to find an optimal configuration to dump the energy in a particular state of the converter. The transformer model would have been very appropriate in trying the different configurations of the transformer without the need to design the physical parameters of the device.

III. CONCLUSION

The proposed transformer model has shown that it is a good representation of the basic transformer equations. The model may be used for simulating switch mode power supply circuits in spice environments. The capability of the model to be expanded to n-windings is an advantage in using it for different types of topologies of switching converters. The model is appropriate for observing the behavior of switching converters in simulations without the need to go through the detailed transformer construction and design.

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REFERENCES

[1] L. Mandache and K. Al-Haddad, “High Precision Modeling of Saturable Transformers used as Voltage Regulators,” IEEE ISIE 2006, July 9-12, 2006

[2] L. Mandache, D. Topan, M. Iordache, I.G. Sirbu, “SPICE Model for Effective and Accurate Time Domanin Simulation of Power Transformers,” ISTET 2011

[3] D. Muthumuni, P.G McLaren, W. Chandrasena, A. Parker, “Simulation Model of an Air Gapped Current Transformer,” IEEE Power Engineering Society Winter Meeting 2011

[4] G.L. Johnson, Solid State Tesla Coil, Twenty First Century Books, 2004

[5] B. Hesterman, Analysis and Modeling of Magnetic Coupling, Denver PELS, 2007

[6] L. O. Chua, *Linear and Nonlinear Circuits*. New York: McGraw-Hill, 1987, pp. 771-780

[7] R.W. Erickson and D. Maksimovic, “A multiple-winding magnetics model having directly measurable parameters,” in *Proc. IEEE Power Electronics Specialists Conf.*, May 1998, pp. 1472-1478

[8] K. D. T. Ngo, S. Srinivas, and P. Nakmahachalasint, “Broadband extended cantilever model for multi-winding magnetics,” *IEEE Trans. Power Electron.*, vol. 16, pp. 551-557, July 2001

[9] K.D.T. Ngo and A. Gangupomu, "Improved method to extract the short-circuit parameters of the BECM," *Power Electronics Letters, IEEE* , vol.1, no.1, pp. 17- 18, March 2003

[10] C.M.F. Odulio, M.T. Escoto, Jr., L.G. Sison, “Energy-Saving Flyback Converter for EDM Applications,” Proceedings of IEEE Region 10 Conference 2005, TENCON 2005, November 2005