Proceedings of the World Congress on Engineering 2013 Vol II,
WCE 2013, July 3 - 5, 2013, London, U.K.

Output-Feedback Control for Networked Systems with Probabilistic Delays

Magdi S. Mahmoud, *Member, IAENG,*

***Abstract*—The networked control system (NCS) design for continuous- time systems with probabilistic delays is discussed in this paper. The delay is assumed to follow a given probability density function. A design scheme for the observer-based output feedback controller is proposed to render the closed-loop networked system exponentially mean-square stable with H_∞ performance requirement. The design method is fulfilled through solving linear matrix inequalities. A numerical example is provided to show the advantages of the proposed technique.**

***Index Terms*—Networked control systems, Delays, Varying sample interval, Packet dropout.**

I. Introduction

In many modern complex and distributed control systems, remotely located sensors, actuators, controllers and filters are often connected over a sharing communication network. Systems with such architectures are often called the network-based systems, which bring a lot of advantages such as low cost, simple installation and maintenance, increased system agility and so on [1]. In spite of these advantages, the sharing network makes the analysis and synthesis of such network-based systems challenging. Recently, the network-based control system, which is called the networked control system (NCS), has attracted much research interest [2]. So far, there has been considerable research work appeared to address modelling, stability analysis, control and filtering problems for NCSs, [3]. In [4], an LMI-based robust $\mathcal{H}_\infty$ dynamic output feedback control design was provided using discrete time-delay system approach. Most of the studies on NCSs have concentrated on state feedbacks [14], and the commonly investigated systems have been discrete-time models, sampled-data models, continuous-time models through sampled-data feedback controls. Upon unavailable state information, observer-based feedbacks have to be performed to achieve control purposes [5]-[13] and [15].

As has been mentioned above that it is difficult to deal with the NCS with long time-varying or random delays, and one aspect of the difficulties lies in providing an appropriate modeling method for such NCSs. Since the delay may be larger than one sampling period, more than one control signals may arrive at the actuator during one sampling interval. Moreover, the numbers of the arriving control signals vary over different sampling intervals, thus the dynamic model of the overall closed-loop NCS varies from sampling period to sampling period [16]. So, the closed-loop NCS is naturally a switched system with the subsystems describing various system dynamics on the different sampling intervals. The switched system model has been used to describe the NCS with delays [17], [18]. However, it is assumed in most of the existing results that the delay is smaller than one sampling period. In [19], the switched system model was used to describe the NCS with long time-varying delays. However, the arbitrary switching scheme was used, which may be conservative and infeasible when some subsystems of the NCS are unstable. Recently, the observer- based feedback controls have been further studied for discrete-time NCSs with random measurements and time delays. In [20], the closed-loop system was transformed into a delay-free model, and an observer-based $\mathcal{H}_\infty$ control design scheme was given in terms of a linear matrix inequality (LMI) to render the closed-loop systems exponentially mean-square stable.

Manuscript received October 8, 2012; revised November 8, 2012. M. S. Mahmoud is with the Systems Engineering Department, KFUPM, P. O. Box 5067, Dhahran 31261, Saudi Arabia, e-mail: magdim@yahoo.com, msmahmoud@kfupm.edu.sa.

Motivated by the above observations, in this paper, we provide a generalized approach to treating NCSs with probabilistic delays. Specifically, we study the problem of the exponential stability of NCSs with probabilistic time-varying delay. By adopting a LyapunovKrasovskii functional (LKF) approach and linear matrix inequalities (LMIs), new criteria for the exponential stability of such NCSs are derived in the form of feasibility testing of LMIs, which can be readily solved by using standard numerical software based on inner-minimization methods [22]. We also adopt an appropriate free-weighting matrix method [23] suitable for the derivation of the main results for our considered problem. Numerical example is provided to illustrate that when the variation probability of the time delay is given, the upper bound of the time delay could be much larger than that when only the variation range of the time delay is known.

$Notation$: We use I and 0 to denote, respectively, the identity matrix and the zero matrix with compatible dimensions; the superscripts T and $'-1'$ stand for the matrix transpose and inverse, respectively; $W > 0$ means that W is a real symmetric positive definite matrix; $\|\cdot\|$ is the spectral norm; $\mathbb{E}\{\cdot\}$ denotes the expectation and $\mathbf{Pr}\{\cdot\}$ means the probability; $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ denote, respectively, the maximum eigenvalue and the minimum eigenvalue of a matrix. In symmetric block matrices or complex matrix expressions, we use the symbol $\bullet$ to represent a term that is induced by symmetry.

II. Problem formulation

Consider a continuous-time system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_{xw}w(t), \\ z(t) &= A_z x(t) + B_z u(t) + B_{zw}w(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $z(t) \in R^p$ and $w(t) \in R^q$ are the state, the control input, the controlled output and the disturbance input belonging to $\mathcal{L}_2[0,\infty)$, respectively. $A, B, B_{xw}, B_z, B_{zw}$ and C_z are known constant real matrices with appropriate dimensions. The pair (A, B) is assumed stabilizable. The measured output $y(t) \in R^r$ frequently experiences sensor delay, and it can be described by two random events:

$$\begin{cases} Event1 : y(t) & dose\ not\ experience\ sensor\ delay, \\ Event2 : y(t) & experience\ sensor\ delay, \end{cases}$$

Recall from the theory of functional differential equations that a continuous and piecewise differentiable initial condition guarantees the existence of the solutions. Assume that the measurement delay $\tau(t)$ from sensor to controller is a random variable whose density function is given by $p(\tau;\pi(t))$, where $\pi(t)$ is a vector of parameters of p. In this paper, we assume that the experience sensor delay distribution is stationary, that is,$\pi(t) = \pi$,where π is a given vector. For example, if p is the normal density function, then $\pi(t) = \{\mu(t), \sigma(t)\}$, where $\mu(t)$ and $\sigma(t)$ are the mean and variance of $\tau(t)$. If the support of p contains values that the experience sensor delay cannot attain such as negative values, one could truncate the density function p to have a specified range $[0, \vartheta]$. In this case, the truncated distribution, p_T is given by

$$f_T(\tau;\pi(t)) \quad = \quad \frac{f(\tau;\pi(t))}{\int_\alpha^\beta f(r;\pi(t))dr}, \ \varrho_1 \le \tau(t) \le \varrho_2 \quad (2)$$

Next, consider partitioning the range $[\alpha, \beta]$ into n mutually exclusive partitions whose end points are: $[\tau_0, \tau_1][\tau_1, \tau_2] \dots [\tau_{n-2}, \tau_{n-1}][\tau_{n-1}, \tau_n]$ where $\tau_0 = \varrho_1, \tau_n = \varrho_2$. Let $\rho_j = \mathbf{Pr}(\tau_{j-1} \le \tau(t) \le \tau_j)$. Define the indicator functions $\varphi_j(t)$ as follows

$$\varphi_j(t) = \begin{cases} 1 : & \tau_{j-1} \le \tau(t) \le \tau_j, \\ 0 : & otherwise, \end{cases} \quad (3)$$

Further we introduce the time-varying sensor delay $\tau_j(t), j = 1, \dots, n$ where $\tau_{j-1} \le \tau_j(t) \le \tau_j$. In this paper, we will consider the application where the sensor delay $\tau(t)$ is stationary, that is, $\mu(t) = \mu$ and $\sigma(t) = \sigma$, for all t. Observe that

$$\begin{aligned} \mathbf{Pr}(\varphi_j = 1) &= \mathbf{Pr}(\tau_{j-1} \le \tau(t) \le \tau_j) = \rho_j, \\ \mathbf{Pr}(\varphi_j = 0) &= 1 - \rho_j & (4) \\ \mathbb{E}(\varphi_j) &= \rho_j, \ \mathbf{Var}(\varphi_j) = \rho_j(1 - \rho_j) & (5) \end{aligned}$$

In this paper, we consider two cases for the time delay $\tau(t)$:

- **Case 1**: There exist scalars ϱ_1, ϱ_2 and h with $0 \le \varrho_1 < \varrho_2$ such that
$$\varrho_1 \le \tau(t) \le \varrho_2, \quad \dot{\tau(t)} \le h. \quad (6)$$
- **Case 2**: There exist scalars ϱ_1 and ϱ_2 with $0 \le \varrho_1 < \varrho_2$ such that
$$\varrho_1 \le \tau(t) \le \varrho_2.$$

Case 1 means that the time delay is a smooth function of t and its derivative is known to be upper bounded by h, while Case 2 implies that the information of the derivative of $\tau(t)$ is unknown. Note that for most of NSCs the communication delay can be converted to be piecewise continuous but its derivative is unavailable [24], in which situation only Case 2 is effective. Anyway, in the sequel, we will focus on Case 1 unless specified since the results under Case 2 are straightforward from those under Case 1 with special treatments .

Let the full-order dynamic observer-based feedback control be

$$\begin{aligned} \dot{\hat{x}}(t) &= K_a\hat{x}(t) + K_c y(t), \\ u(t) &= K_b\hat{x}(t), & (7) \end{aligned}$$

where $\hat{x} \in R^n$ is the observer state, and the feedback gains K_a, K_b and K_c are to be designed. Denote $\delta(t) = [x(t)^T \hat{x}(t)^T]^T$ and $\rho = \text{diag}\{\rho_1, \dots, \rho_n\}$. Then the closed-loop system of (1) with (4) and (7) is described by

$$\begin{aligned} \dot{\delta}(t) &= M\delta(t) + M_\tau\delta(t - \tau(t)) + B_{\delta w}w(t) \\ &\quad + \sum_{j=1}^{n}(\varphi_j(t) - \rho_j)\left[N\delta(t) + N_\tau\delta(t - \tau(t))\right] \\ z(t) &= M_z\delta(t) + B_{zw}w(t), & (8) \end{aligned}$$

where

$$\begin{aligned} M &= \begin{bmatrix} A & BK_b \\ \rho K_c C & K_a \end{bmatrix}, \ N = \begin{bmatrix} 0 & 0 \\ K_c C & 0 \end{bmatrix}, \\ M_\tau &= \begin{bmatrix} 0 & 0 \\ (I_n - \rho)K_c D & 0 \end{bmatrix}, , \ B_{\delta w} = \begin{bmatrix} B_{xw} \\ K_c B_{yw} \end{bmatrix} \\ N_\tau &= \begin{bmatrix} 0 & 0 \\ -K_c D & 0 \end{bmatrix}, \\ M_z &= \begin{bmatrix} C_z & B_z K_a \end{bmatrix} & (9) \end{aligned}$$

Here, although the dynamic of the closed-loop system requires only initial values of $\hat{x}(0), w(0)$ and $x(t) = \phi(t)(t \in [-\varrho_2, 0])$, for later convenience, we extend the range of the definition of $\phi(t)$ from $[-\varrho_2, 0]$ to $[-2\varrho_2, 0]$ and define a continuous function $\hat{\phi(t)}$ on $[-2\varrho_2, 0]$ such that $\hat{\phi}(t) = \hat{x}(t)$. So, we have $\xi = [\phi(t)^T \hat{\phi}(t)^T]^T$ for $t \in [-\varrho_2, 0]$. We also define $w(t) = 0$ for $t \in [-\tau_0, 0)$.

Stochastic theory has had a wide range of applications in both theory and practice, and many results have appeared tackling various problems ranging from stochastic stabilization, filtering and control, [14]. Let

$$\begin{aligned} f(\delta, t) &:= M\delta(t) + M_\tau\delta(t - \tau(t)) + B_{\delta w}w(t), \\ g(\delta, t) &:= N\delta(t) + N_\tau\delta(t - \tau(t)). & (10) \end{aligned}$$

Since $f(\delta, t)$ and $g(\delta, t)$ in (8) satisfy the local Lipschitz condition and the linear growth condition, the existence and uniqueness of solution to (8) is guaranteed [23]. Moreover, under $v(t) = 0$ for $t \in [-\tau_0, 0)$, it admits a trivial solution (equilibrium) $\delta \equiv 0$. In this work we will follow the definitions of stochastic stability and $H\infty$ performance requirements.

Definition 2.1: System (8) is said to be exponentially mean-square stable (EMS) if there exist constants $a > 0$ and $b > 0$ such that

$$\mathbb{E}\{||\delta(t)||^2\} \quad = \quad ae^{-bt} \sup_{\sigma\in[-2\varrho_2,0]} \mathbb{E}\{||\delta(\sigma)||^2\} \quad (11)$$

Definition 2.2: Given $\eta > 0$, system (8) is said to be EMS with $H\infty$ performance (EMS-η) if under zero-initial conditions, it is EMS and satisfies

$$\int_0^\infty \mathbb{E}\{||z(t)||^2\}dt \quad = \quad \eta^2 \int_0^\infty \mathbb{E}\{||w(t)||^2\}dt \quad (12)$$

Controller for system (8) to be EMS-η will be designed.

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

III. Main Results

Due to the special structure of matrices M_τ and N_τ in system (8), one may choose $[I_n\ 0]\delta = x$ to construct certain terms of Lyapunov functionals in order to establish stability conditions [25]. In this work,the full information of δ is used to construct a suitable functional $J(\delta_t, t)$ and a similar type Lyapunov functional $V(\delta_t, t)$ in our study. In details, motivated by recent construction type for retarded systems in [25], we chose the following type of functionals suitable for system (8) to investigate the $H\infty$ performance analysis:

$$J(\delta_t,t) \quad = \quad J_1(\delta_t,t)+J_2(\delta_t,t)+J_3(\delta_t,t) \qquad (13)$$

where $\delta_t = \delta(t+\sigma), \tau \in [-2\varrho_2, 0]$ and

$$\begin{aligned}
&J_1(\delta_t,t) = \delta^T(t)P\delta(t),\\
&J_2(\delta_t,t) = \int_{t-\tau(t)}^{t} \delta^T(s)Q\delta(s)ds +\\
&\sum_{i=1}^{2}\int_{t-\tau_i}^{t} \delta^T(s)Q_i\delta(s)ds\\
&J_3(\delta_t,t) =\\
&\int_{-\varrho_2}^{0}\int_{t+\theta}^{t} \begin{bmatrix} f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}^T Z \begin{bmatrix} f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix} dsd\theta +\\
&\int_{-\varrho_2}^{-\varrho_1}\int_{t+\theta}^{t} \begin{bmatrix} f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}^T Z_1 \begin{bmatrix} f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix} dsd\theta,
\end{aligned} \qquad (14)$$

in which $\hat{\rho}_j = \sqrt{\rho_j(1-\rho_j)}$, $j = 0,\ldots,n$, $\varphi_0 = \mathrm{diag}\{\hat{\rho}_1,\ldots,\hat{\rho}_n, 0_n\}$, and $P>0, Q>0, Q_1>0, Q_2>0, Z>0$ and $Z_1 > 0$ are to be determined. For system (8) with $w(t) = 0$, we use the following Lyapunov functional to obtain EMS conditions:

$$V(\delta_t,t) \quad = \quad V_1(\delta_t,t)+V_2(\delta_t,t)+V_3(\delta_t,t), \qquad (15)$$

where $V_i(\delta_t,t) = J_i(\delta_t,t)$ with $w(t) = 0$, $i = 1,2,3$. Moreover, we use $\mathcal{L}V$ to denote the infinitesimal operator of V [25], which is defined as

$$\begin{aligned}\mathcal{L}V(\delta_t,t) \quad = \quad &\lim_{\Delta\to 0^+}\frac{1}{\Delta}[\mathbb{E}\{V((\delta_{t+\Delta},t+\Delta)|(\delta_t,t))\}\\ &-V(\delta_t,t)]\end{aligned} \qquad (16)$$

The following lemma is useful in the development, which verifies that $V(\varphi_t, t)$ is a Lyapunov functional and meanwhile shows that certain condition could ensure system (8) to be EMS.

Lemma 3.1: Suppose that $K_a, K_b, K_c, P > 0, Q > 0, Q_i > 0, Z > 0$ *and* $Z_1 > 0$ *are given, and* $V(\varphi_t, t)$ *is chosen as in (15). If there exists a constant* $c > 0$ *such that*

$$\mathbb{E}\{\mathcal{L}V(\delta_t,t)\} \quad \le \quad -c\,\mathbb{E}\{\delta(T)\}$$

holds for all $t \ge 0$*, then system (8) is EMS.*

Proof: By Definition 2.1, the proof is similar to [23].

The next lemma will be used to establish the analysis result for EMS-η.

Lemma 3.2: Let $\Sigma, \Sigma_1 \in R^{p\times p}$ *be symmetric constant matrices. Then,*

$$\Sigma+\tau(t)\Sigma_1 \quad < \quad 0$$

holds for all $\tau(t) \in [\varrho_1, \varrho_2]$ *if and only if the following two inequalities hold:*

$$\begin{aligned}\Sigma+\varrho_1\Sigma_1 \quad &< \quad 0\\ \Sigma+\varrho_2\Sigma_1 \quad &< \quad 0\end{aligned}$$

If this is the case, for any $z(t) \in R^p$*, the following is true*

$$\begin{aligned}&z(t)^T(\Sigma+\tau(t)\Sigma_1)z(t) \le\\ &\max\left\{\lambda_{max}(\Sigma+\varrho_1\Sigma_1), \lambda_{max}(\Sigma+\varrho_2\Sigma_1)\right\}\|z(t)\|^2\end{aligned}$$

Proof: For any $\tau(t) \in [\varrho_1, \varrho_2]$, there exists an $\alpha_t \in [0, 1]$ such that $\tau(t) = \alpha_t\varrho_1 + (1-\alpha_t)\varrho_2$. This gives $\Sigma+\tau(t)\Sigma_1 = \alpha_t(\Sigma+\varrho_1\Sigma_1)+(1-\alpha_t)(\Sigma+\varrho_2\Sigma_1) < 0$. Then

$$\begin{aligned}&z(t)^T(\Sigma+\tau(t)\Sigma_1)z(t)\\ &\le \alpha_t\lambda_{max}(\Sigma+\varrho_1\Sigma_1)\|z(t)\|^2\\ &+\lambda_{max}(\Sigma+\varrho_2\Sigma_1)\|z(t)\|^2\\ &\le \max\left\{\lambda_{max}(\Sigma+\varrho_1\Sigma_1), \lambda_{max}(\Sigma+\varrho_2\Sigma_1)\right\}\|z(t)\|^2\end{aligned}$$

With the aid of *Lemmas* 3.1 and 3.2, the analysis result for system (8) to be EMS-η.

Theorem 3.1: Given $\eta > 0$*, the closed-loop system (8) is EMS-*η *if there exist* $2n\times 2n$ *matrices* $P > 0, Q > 0, Q_1 > 0$ *and* $Q_2 > 0$*,* $4n \times 4n$ *matrices* $Z > 0, Z_1 > 0, L_1 > 0, L_2 > 0$ *and* $L_3 > 0$*,* $(8n+q)\times 2n$ *matrices* F, G *and* H*, such that*

$$\begin{bmatrix} \Theta+\Theta_0 & \sqrt{\varrho_1}F[I,I] & \sqrt{\varrho_1-\varrho_2}H[I,I]\\ \bullet & -L_1 & 0\\ \bullet & \bullet & -L_3\end{bmatrix} < 0 \qquad (17)$$

$$\begin{bmatrix} \Theta+\Theta_0 & \sqrt{\varrho_2}F[I,I] & \sqrt{\varrho_1-\varrho_2}G[I,I]\\ \bullet & -L_1 & 0\\ \bullet & \bullet & -L_2\end{bmatrix} < 0 \qquad (18)$$

$$E_uL_1E_u + E_lL_1E_l - Z \le 0 \qquad (19)$$

$$E_uL_2E_u + E_lL_2E_l - Z_1 \le 0 \qquad (20)$$

$$E_uL_3E_u + E_lL_3E_l - Z - Z_1 \le 0 \qquad (21)$$

where each ellipsis $\bullet$ *denotes a block induced by symmetry, and*

$$\begin{aligned}
&\Theta = [I_{2n}0_{2n\times(6n+q)}]^T P\tilde{M} + \tilde{M}^T P[I_{2n}0_{2n\times(6n+q)}]\\
&+\tilde{M}_z^T\tilde{M}_z + F[I_{2n}, -I_{2n}, 0_{2n\times(4n+q)}]\\
&+[I_{2n}, -I_{2n}, 0_{2n\times(4n+q)}]^T F^T\\
&+\mathrm{diag}\{Q+Q_1+Q_2, (h-1)Q, -Q_1, -Q_2, -\eta^2 I_q\}\\
&+G[0_{2n}, -I_{2n}, I_{2n}, 0_{2n\times(2n+q)}]\\
&+[0_{2n}, -I_{2n}, I_{2n}, 0_{2n\times(2n+q)}]^T G^T\\
&+H[0_{2n}, I_{2n}, 0_{2n}, -I_{2N}, 0_{2n\times q}]\\
&+[0_{2n}, I_{2n}, 0_{2n}, -I_{2N}, 0_{2n\times q}]^T H^T\\
&\Theta_0 = [\tilde{M}^T, \varphi_0\tilde{n}^T](\varrho_2 Z + (\varrho_2-\varrho_1)Z_1)[\tilde{M}^T, \varphi_0\tilde{n}^T]^T,\\
&\tilde{M} = [M, M_\tau, 0_{2n\times 4n}, B_{\delta w}],\\
&\tilde{M}_z = [M_z, 0_{p\times 6n}, B_{zw}], \quad \tilde{N} = [N, N_\tau, 0_{2n\times(4n+q)}],\\
&E_u = \mathrm{diag}\{I_{2n}, 0_{2n}\}, \quad E_l = \mathrm{diag}\{0_{2n}, l_{2n}\},
\end{aligned}$$

Given K_a, K_b, K_c and $\eta > 0$, the conditions of *Theorem* 3.1 are in terms of strict LMIs which could be easily solved using existing LMI solvers. Note that our purpose is to design LMI schemes to seek these feedback gains K_a, K_b and K_c. The maximum tolerant delay bound for ϱ_2 can be searched and the minimum level of η can be computed simultaneously.

Theorem 3.2: Given the delay-interval bounds $\varrho_1 > 0, \varrho_2 > 0$ and $\eta > 0$, the closed-loop system (8) is EMS-η

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

if there exist $n \times n$ matrices $X > 0$ and $Y > 0, 2n \times 2n$ matrices $\tilde{Q} > 0, \tilde{Q}_1 > 0$ and $\tilde{Q}_2 > 0, 4n \times 4n$ matrices $\tilde{Z} > 0, \tilde{Z}_1 > 0, \tilde{L}_1 > 0, \tilde{L}_1 > 0$ and $\tilde{L}_1 > 0$, $(8n+q) \times 2n$ matrices $\tilde{F}, \tilde{G}$ and $\tilde{H}$, $n \times n$ matrix Υ_a, $m \times n$ matrix Υ_b and $n \times r$ matrix Υ_c, such that the following LMIs hold for some scalars $\kappa_1 > 0$ and $\kappa_2 > 0$ (22);

$$E_u \tilde{L}_1 E_u + E_l \tilde{L}_1 E_l - \tilde{Z} < 0, \quad (24)$$
$$E_u \tilde{L}_2 E_u + E_l \tilde{L}_2 E_l - \tilde{Z}_1 < 0, \quad (25)$$
$$E_u \tilde{L}_3 E_u + E_l \tilde{L}_3 E_l - \tilde{Z} - \tilde{Z}_1 < 0, \quad (26)$$

where E_u and E_l are as in *Theorem* 3.1, and other parameters are defined by (27).

In this case, the feedback gains K_a, K_b and K_c are given by

$$\begin{aligned} K_a &= U^{-1}(\Upsilon_a - XB\Upsilon_b)Y^{-1}W^{-T}, \\ K_b &= \Upsilon_b Y^{-1} W^{-T}, \\ K_c &= U^{-1}\Upsilon_c, \end{aligned} \quad (28)$$

where U and W are two invertible matrices satisfying $UW^T = I - XY^{-1}$.

Theorem 3.2 provides an LMI method towards solving the matrix inequalities in (17)-(20), and hence presents controller designs of the form (7) to make the closed-loop system (8) EMS-η. The novelty of the result mainly lies in that an LMI design scheme is proposed for NCSs in continuous-time system settings with random measurements and time delays. Furthermore, the derivation is proceeded using appropriate Lyapunov functionals and matrix decoupling techniques.

In *Theorem* 3.2, we have encountered two conservative steps, that is, the first one is that (52) implies (51), and the second one is in (49) to bound the term $-P\big(\varrho_2 Z + (\varrho_2 - \varrho_1)Z_1\big)^{-1}P$. We give two remarks to address these, respectively.

Remark 3.1: The step of (51) and (52) can be improved by specifying a matrix $K_0 \in R^{m\times n}$ a priori

$$\begin{aligned} &\tilde{\Xi} + \tilde{Y}\tilde{K}_0 + \tilde{K}_0^T \tilde{Y}^T + \kappa_1^{-1}\tilde{Y}\tilde{Y}^T \\ &+\kappa_1(\tilde{K} - \tilde{K}_0)^T(\tilde{K} - \tilde{K}_0) < 0 \quad \Rightarrow (51) \end{aligned} \quad (29)$$

where $\tilde{K}_0 = [BK_0, 0_{n\times(19n+q)}]$. As a result, the conditions (22) and (23) in Theorem 3.2 are replaced by similar ones with Ξ_{11}, Ξ_{12}, and Ξ_{16} replaced by Ξ'_{11}, Ξ'_{12} and Ξ'_{16}, respectively, where

$$\begin{aligned} \Xi'_{11} &= \Xi_{11} + \mathrm{diag}\big\{YBK_0 + K_0^T B^T Y, 0_{7n+q}\big\} \\ \Xi'_{12} &= \Xi_{12} + \big[I_n, 0_{n\times(7n+q)}\big]^T K_0^T B^T Y [I_n, 0_{3n}] \\ \Xi'_{16} &= \Xi_{16} - \big[BK_0, 0_{n\times(7n+q)}\big]^T. \end{aligned} \quad (30)$$

The reason of the resultant improvement with above replacement lies in that when K_0 is chosen close to a computed Υ_b the deduction step of (29) involves no conservatism, and moreover, when $K_0 = 0$ the conditions of (22) and (23) are recovered. However, how to choose such a matrix K_0 involves much difficulty. In case of stabilizable pair (A, B), we could select K_0 such that $A + BK_0$ is Hurwitz.

Remark 3.2: The other conservative step is in (49) $\Rightarrow$ (46) where the inequality (48) is used to bound the term $-\mathrm{diag}\{P, P\}\big(\varrho_2 Z + (\varrho_2 - \varrho_1)Z_1\big)^{-1}\mathrm{diag}\{P, P\}$ This step can be improved by adopting the cone complementary algorithm [21], which is popular in recent control designs. To avoid using algorithms, we can introduce two scaling parameters $\epsilon_1 > 0$ and $\epsilon_2 > 0$ to improve the LMI conditions in Theorem 3.2. That is, we replace (48) by

$$\begin{aligned} &-\mathrm{diag}\{P, P\}\big(\varrho_2 Z + (\varrho_2 - \varrho_1)Z_1\big)^{-1}\mathrm{diag}\{P, P\} \\ &\leq -2\mathrm{diag}\{\epsilon_1 P, \epsilon_2 P\} + \mathrm{diag}\{\epsilon_1 I_{2n}, \epsilon_2 I_{2n}\} \\ &\big[\varrho_2 Z + (\varrho_2 - \varrho_1)Z_1\big]\mathrm{diag}\{\epsilon_1 I_{2n}, \epsilon_2 I_{2n}\}. \end{aligned} \quad (31)$$

As a result, the conditions (22) and (23) in Theorem 3.2 are replaced by similar ones with Υ_{22} replaced by Υ'_{22} where

$$\begin{aligned} \Xi'_{22} = & -2\mathrm{diag}\left\{\epsilon_1 \begin{bmatrix} Y & Y \\ Y & X \end{bmatrix}, \epsilon_2 \begin{bmatrix} Y & Y \\ Y & X \end{bmatrix}\right\} \\ & +\mathrm{diag}\{\epsilon_1 I_{2n}, \epsilon_2 I_{2n}\}\big[\varrho_2 Z + (\varrho_2 - \varrho_1)Z_1\big] \\ & \mathrm{diag}\{\epsilon_1 I_{2n}, \epsilon_2 I_{2n}\}. \end{aligned} \quad (32)$$

It is seen that the resulting conditions with this replacement cover those in Theorem 3.2 as the special choice of $\epsilon_1 = \epsilon_2 = 1$ is not required.

Finally, we would like to remark that the main results in this section imply several results for stability analysis and stabilization via state feedback. We now list two corollaries below for stability tests which are straightforward from Theorem 3.1. Consider the following system of the form (8) with $w(t) = 0$,

$$\begin{aligned} \dot{x}(t) = & M_1 x(t) + M_{1\tau} x(t - \tau(t)) + B_{\delta w} w(t) \\ & +(\varphi(t) - \rho)\left[N_1 x(t) + N_{1\tau} x(t - \tau(t))\right] \end{aligned} \quad (33)$$

where $x \in R^n, M_1, M_{1\tau} N_1$ and $N_{1\tau}$ are constant matrices with appropriate dimensions.

So far, EMS-η conditions for a class of NCSs with probabilistic delays are provided. By constructing a new LKF and incorporating the delay probability distribution function, LMI-based techniques have been developed for achieving delay dependence stability results.

IV. Illustrative Example

To illustrate the theoretical developments, we consider a chemical reactor. The linearized model can be described by the following matrices:

$$\begin{aligned} A &= \begin{bmatrix} -4.931 & -4.886 & 4.902 & 0 \\ -5.301 & -5.174 & -12.8 & 5.464 \\ 6.4 & 0.347 & -11.773 & -1.04 \\ 0 & 0.833 & 11.0 & -3.932 \end{bmatrix}, \\ B^t &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ B_z = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ A_z &= \begin{bmatrix} 1.921 & 1.915 & 0 & 1.908 \end{bmatrix}, \\ C &= D = \begin{bmatrix} 10 & 0 & 0 & 0 \end{bmatrix}, \\ B_{xw} &= \begin{bmatrix} 0.8 & 1 & 0.1 & 0.2 \end{bmatrix}^T, \\ B_{yw} &= 0.01, \begin{bmatrix} 0.8 & 1 & 0.1 & 0.2 \end{bmatrix}^T, \ B_{zw} = 0.4 \end{aligned}$$

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \sqrt{\varrho_1}\tilde{F}[I,I] & \sqrt{\varrho_2-\varrho_1}\tilde{H}[I,I] & \Xi_{15} & \Xi_{16} & \Xi_{17} \\ & \Xi_{22} & 0 & 0 & \Xi_{25} & 0 & 0 \\ & * & -\tilde{L}_1 & 0 & 0 & 0 & 0 \\ & * & * & -\tilde{L}_3 & 0 & 0 & 0 \\ & * & * & * & -\kappa_1 I_n & 0 & 0 \\ & * & * & * & * & -\kappa_1^{-1} I_n & 0 \\ & * & * & * & * & * & -I_p \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \sqrt{\varrho_2}\tilde{F}[I,I] & \sqrt{\varrho_2-\varrho_1}\tilde{G}[I,I] & \Xi_{15} & \Xi_{16} & \Xi_{17} \\ & \Xi_{22} & 0 & 0 & \Xi_{25} & 0 & 0 \\ & * & -\tilde{L}_1 & 0 & 0 & 0 & 0 \\ & * & * & -\tilde{L}_2 & 0 & 0 & 0 \\ & * & * & * & -\kappa_2 I_n & 0 & 0 \\ & * & * & * & * & -\kappa_2^{-1} I_n & 0 \\ & * & * & * & * & * & -I_p \end{bmatrix} < 0 \tag{23}$$

$$\begin{aligned}
&\Xi_{11} = \\
&\begin{bmatrix} YA+A^TY & \left(A^TX+YA+\rho C^T\Upsilon_c^T+\Upsilon_a^T\right) & 0 & 0 & 0 & YB_{xw} \\ & \left(XA+A^TX+\rho\Upsilon_c C+\rho C^T\Upsilon_c^T\right) & (1-\rho)\Upsilon_c D & (1-\rho)\Upsilon_c D & 0 & XB_{xw}+\Upsilon_c B_{yw} \\ & * & 0 & 0 & 0 & 0 \\ & * & * & 0 & 0 & 0 \\ & * & * & * & 0_{4n} & 0 \\ & * & * & * & * & 0_q \end{bmatrix} \\
&+\mathrm{diag}\left\{\tilde{Q}+\tilde{Q}_1+\tilde{Q}_2, (h-1)\tilde{Q}, -\tilde{Q}_1, -\tilde{Q}_2, -\eta^2 I_q\right\} \\
&+\tilde{F}\left[I_{2n}, -I_{2n}, 0_{2n\times(4n+q)}\right] + \left[I_{2n}, -I_{2n}, 0_{2n\times(4n+q)}\right]^T\tilde{F}^T \\
&+\tilde{G}\left[0_{2n}, -I_{2n}, I_{2n}, 0_{2n\times(2n+q)}\right] + \left[0_{2n}, -I_{2n}, I_{2n}, 0_{2n\times(2n+q)}\right]^T\tilde{G}^T \\
&+\tilde{H}\left[0_{2n}, I_{2n}, 0_{2n}, -I_{2n}, 0_{2n\times q}\right] + \left[0_{2n}, I_{2n}, 0_{2n}, -I_{2n}, 0_{2n\times q}\right]^T\tilde{H}^T, \\
&\Xi_{12} = \\
&\begin{bmatrix} A^TY & \left(A^TX+\rho C^T\Upsilon_c^T+\Upsilon_a^T\right) & 0 & \varphi_0 C^T\Upsilon_c^T \\ A^TY & \left(A^TX+\rho C^T\Upsilon_c^T\right) & 0 & \varphi_0 C^T\Upsilon_c^T \\ 0 & (1-\rho)D^T\Upsilon_c^T & 0 & -\varphi_0 D^T\Upsilon_c^T \\ 0 & (1-\rho)D^T\Upsilon_c^T & 0 & -\varphi_0 D^T\Upsilon_c^T \\ 0_{4n\times n} & 0_{4n\times n} & 0_{4n\times n} & 0_{4n\times n} \\ B_{xw}^TY & B_{xw}^TX+B_{yw}^T\Upsilon_c^T & 0 & 0 \end{bmatrix} \\
&\Xi_{22} = -2\mathrm{diag}\left\{\begin{bmatrix} Y & Y \\ Y & X \end{bmatrix}, \begin{bmatrix} Y & Y \\ Y & X \end{bmatrix}\right\} + \varrho_2\tilde{Z} + (\varrho_2-\varrho_1)\tilde{Z}_1, \\
&\Xi_{15} = \begin{bmatrix} Y \\ 0_{(7n+q)\times n} \end{bmatrix}, \quad \Xi_{25} = \begin{bmatrix} Y \\ 0_{3n\times n} \end{bmatrix}, \quad \Xi_{16} = \begin{bmatrix} \Upsilon_b^T B^T \\ 0_{(7n+q)\times n} \end{bmatrix}, \quad \Xi_{17} = \begin{bmatrix} C_z^T + \Upsilon_b^T B_z^T \\ C_z^T \\ 0_{6n\times p} \\ B_{zw}^T \end{bmatrix}.
\end{aligned} \tag{27}$$

Using the LMI toolbox in MATLAB, the ensuing results are summarized by:

$$X = \begin{bmatrix} 0.1448 & -0.0020 & 0.0005 & 0.0002 \\ -0.0020 & 0.1442 & -0.0005 & 0.0009 \\ 0.0005 & -0.0005 & 0.1420 & 0.0001 \\ 0.0002 & 0.0009 & 0.0001 & 0.1463 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0.2142 & -0.0560 & 0.0421 & -0.0153 \\ -0.0560 & 0.0383 & -0.0138 & 0.0515 \\ 0.0421 & -0.0138 & 0.0292 & 0.0435 \\ -0.0153 & 0.0515 & 0.0435 & 0.2597 \end{bmatrix},$$

The corresponding feedback gains are evaluated with $W = I$ and $U = I - XY^{-1}$ to yield

$$\begin{aligned}
K_a &= \begin{bmatrix} 0.7573 & 0.7142 & 0.3973 & 0.8391 \\ 0.2138 & 8.2185 & 13.8882 & -3.4177 \end{bmatrix}, \\
K_b &= \begin{bmatrix} 0.3144 & -0.7983 & -3.8703 & 1.7806 \\ -0.6559 & 7.2776 & 14.8651 & -6.8895 \end{bmatrix}, \\
K_c &= \begin{bmatrix} 0.2634 & -0.1587 & -3.1912 & 1.5713 \\ -1.0803 & 8.5794 & 12.2875 & -3.9658 \end{bmatrix}
\end{aligned}$$

Simulation of the closed-loop system is performed and the ensuing state trajectories are presented in Fig. 1. It is clearly evident that the the closed-loop system is EMSS-η.

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

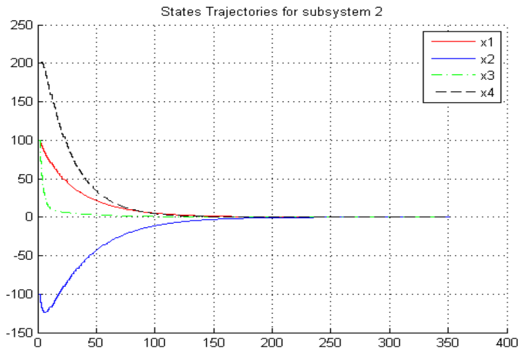

Fig. 1. Closed-loop state trajectories

V. Conclusion

An LMI method has been presented for observer-based H_∞ control of NCSs in continuous-time system settings with random measurements and probabilistic time delays. Improved schemes have been shown for the design method. It has been established that these conditions reduce the conservatism by considering not only the range of the time delays, but also the probability distribution of their variation. A numerical simulation example has been presented to show the merits and advantages of the proposed techniques.

Appendix A
Proof Of Theorem 3.1

The proof is twofold: we first choose a functional J of the form (13) to show that the H_∞ performance requirement (12) is satisfied, and then use the Lyapunov functional V of the form (15) to prove the EMS property. Denote

$$\begin{aligned} \chi(t) &:= [\delta(t)^T, \delta_\tau^T, \delta_1^T, \delta_2^T, w(t)^T]^T, \\ \delta_\tau &:= \delta(t-\delta_t), \quad \delta_i := \delta(t-\delta_i), \quad i=1,2. \end{aligned} \tag{34}$$

From the Newton-Leibniz formula $0 = \delta(t) - \delta_\tau - \int_{t-\tau(t)}^{t} \dot{\delta}(s)ds$, we have that

$$\begin{aligned} \psi_1(t) &:= 2\chi(t)^T F \left[\delta(t) - \delta_\tau - \int_{t-\tau(t)}^{t} \dot{\delta}(s)ds \right] = 0, \\ \psi_2(t) &:= 2\chi(t)^T G \left[\delta_1 - \delta_\tau - \int_{t-\tau(t)}^{t-\varrho_1} \dot{\delta}(s)ds \right] = 0, \\ \psi_3(t) &:= 2\chi(t)^T G \left[\delta_\tau - \delta_2 - \int_{t-\varrho_2}^{t-\tau(t)} \dot{\delta}(s)ds \right] = 0 \end{aligned} \tag{35}$$

hold for any $(8n+q)\times 2n$ matrices F, G and H. Let the functional $J(\delta_t;t)$ be chosen as in (13).

Then, from (16), $\mathcal{L}J$ for the evolution of J is given by [25]

$$\begin{aligned} &\mathcal{L}J(\delta_t;t) = 2\delta(t)^T P f(\delta,t) + \delta(t)^T(Q+Q_1+Q_2)\delta(t) \\ &-(1-\dot{\tau}_t)\delta_\tau^T Q \delta_\tau - \sum_{i=1}^{2} \delta_i^T Q_i \delta_i \\ &+ \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T (\varrho_2 Z + (\varrho_2-\varrho_1)Z_1) \\ &\times \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} \end{aligned} \tag{36}$$

$$\begin{aligned} &- \int_{t-\varrho_2}^{t} \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T Z \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} ds \end{aligned} \tag{37}$$

$$\begin{aligned} &- \int_{t+\varrho_2}^{t+\varrho_1} \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T Z_1 \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} ds \\ &= 2\delta(t)^T P f(\delta,t) + \delta(t)^T(Q+Q_1+Q_2)\delta(t) \\ &-(1-\dot{\tau}_t)\delta_\tau^T Q \delta_\tau - \sum_{i=1}^{2} \delta_i^T Q_i \delta_i \\ &+ \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T (\varrho_2 Z + (\varrho_2-\varrho_1)Z_1) \\ &\times \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} \\ &- \int_{t-\tau(t)}^{t} \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T Z \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} ds \\ &- \int_{t-\tau(t)}^{t-\varrho_1} \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T Z_1 \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} ds \\ &- \int_{t-\varrho_2}^{t-\tau(t)} \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix}^T (Z+Z_1) \begin{bmatrix} f(\delta,s) \\ \varphi_0 g(\delta,s) \end{bmatrix} ds \\ &+\psi_1(t) + \psi_2(t) + \psi_3(t) \end{aligned} \tag{38}$$

Note that, in $\psi_1(t)$, the following inequality holds for any $4n \times 4n$ matrix $L > 0, thus$:

$$\begin{aligned} &-2\chi(t)^T F \int_{t-\tau(t)}^{t} \dot{\delta}(s)ds \\ &= -2\chi(t)^T F \left[I_{2n}, I_{2n}\right] \int_{t-\tau(t)}^{t} \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix} ds \\ &\leq \tau(t)\chi(t)^T F \left[I_{2n}, I_{2n}\right] L_1^{-1} \left[I_{2n}, I_{2n}\right]^T F^T \chi(t) \\ &+ \int_{t-\tau(t)}^{t} \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix}^T L_1 \\ &\times \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix} ds \end{aligned}$$

and, similarly, in $\psi_i(t)$ $(i=2;3)$, the following inequalities hold for any $4n \times 4n$ matrices $L_i > 0$:

$$\begin{aligned} &-2\chi(t)^T G \int_{t-\tau(t)}^{t-\varrho_1} \dot{\delta}(s)ds \\ &\leq (\tau(t)-\varrho_1)\chi(t)^T G \left[I_{2n}, I_{2n}\right] L_2^{-1} \left[I_{2n}, I_{2n}\right]^T G^T \chi(t) \\ &+ \int_{t-\tau(t)}^{t-\varrho_1} \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix}^T L_2 \\ &\times \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix} ds \end{aligned}$$

and

$$\begin{aligned} &-2\chi(t)^T H \int_{t-\tau(t)}^{t-\varrho_1} \dot{\delta}(s)ds \\ &\leq (\varrho_2-\tau(t))\chi(t)^T H \left[I_{2n}, I_{2n}\right] L_3^{-1} \left[I_{2n}, I_{2n}\right]^T H^T \chi(t) \\ &+ \int_{t-\varrho_2}^{t-\tau(t)} \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix}^T L_3 \\ &\times \begin{bmatrix} f(\delta,s) \\ (\varphi(s)-\rho)g(\delta,s) \end{bmatrix} ds \end{aligned}$$

Considering (6), (8), (10) and taking the expectation on

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

(38), we have

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V(\delta_t,t)+\|z(t)\|^2-\eta^2\|w(t)\|^2\}\\&\le\mathbb{E}\{\chi(t)^T\big(\Theta+\Theta_0+\tau(t)\Theta_1+(\tau(t)-\varrho_1)\Theta_2\\&+(\varrho_2-\tau(t))\Theta_3\big)\chi(t)\}+\psi_4(t),\end{aligned}\quad(39)$$

and

$$\begin{aligned}\Theta_1&=F\,[I_{2n},I_{2n}]\,L_1^{-1}\,[I_{2n},I_{2n}]^T\,F^T,\\ \Theta_2&=G\,[I_{2n},I_{2n}]\,L_2^{-1}\,[I_{2n},I_{2n}]^T\,G^T,\\ \Theta_3&=H\,[I_{2n},I_{2n}]\,L_3^{-1}\,[I_{2n},I_{2n}]^T\,H^T,\\ \psi_4(t)&=\int_{t-\tau(t)}^{t}\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}^T\big(E_uL_1E_u\\&+E_lL_1E_l-Z\big)\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}ds\\&+\int_{t-\tau(t)}^{t-\varrho_1}\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}^T\big(E_uL_2E_u\\&+E_lL_2E_l-Z_1\big)\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}ds\\&+\int_{t-\varrho_2}^{t-\tau(t)}\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}^T\big(E_uL_3E_u\\&+E_lL_3E_l-Z-Z_1\big)\begin{bmatrix}f(\delta,s)\\ \varphi_0 g(\delta,s)\end{bmatrix}ds\end{aligned}$$

Applying the Schur complement, conditions (17) and (18) are equivalent to

$$\begin{aligned}\tilde\Theta_1&=\Theta+\Theta_0+\varrho_1\Theta_1+(\varrho_2-\varrho_1)\Theta_3<0,\quad(40)\\ \tilde\Theta_2&=\Theta+\Theta_0+\varrho_2\Theta_1+(\varrho_2-\varrho_1)\Theta_2<0,\quad(41)\end{aligned}$$

From (40),(41), (19)-(20) and *Lemma* 3.2, we deduce from (39) that

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}J(\delta_t,t)+\|z(t)\|^2-\eta^2\|w(t)\|^2\}\\&\le\max\{\lambda_{max}(\tilde\Theta_1),\lambda_{max}(\tilde\Theta_2)\}\mathbb{E}\{\|\chi\|^2\}\le 0\end{aligned}\quad(42)$$

Under zero-initial conditions and noticing $J(\delta_T;T)\ge 0$ for any $T>0$, integrating (42) from 0 to ∞ yields that the H_∞ performance requirement (12) is satisfied. With a procedure similar to the above, we can arrive under the given conditions and by virtue of *Lemma* 3.2 that,

$$\mathbb{E}\{\mathcal{L}V(\delta_t,t)\}\le\max\{\lambda_{max}(\tilde\Theta_1),\lambda_{max}(\tilde\Theta_2)\}\mathbb{E}\{\|\delta(t)\|^2\}$$

Hence, system (8) is EMS from *Lemma* 3.1.

APPENDIX B
PROOF OF THEOREM 3.2

It cen be seen from (22) or (23) that

$$\begin{bmatrix}Y&Y\\Y&X\end{bmatrix}>0$$

which gives $XY>0$, implying that $I--XY^{-1}$ is invertible. Now let U and W be any invertible matrices satisfying $UW^T=I-XY^{-1}$. Choose

$$P=\begin{bmatrix}X&U\\U^T&*\end{bmatrix}>0,\ P^{-1}=\begin{bmatrix}Y^{-1}&W\\W^T&*\end{bmatrix}>0\quad(43)$$

where each ellipsis $*$ denotes a positive definite matrix block that will not influence the subsequent development (of course it makes $PP^{-1}=I$). In the sequel, we show that if (22)-(26) are satisfied, then (17)-(20) hold with $P>0$ chosen as in (39), and thus the result follows immediately from *Theorem* 3.1. Define

$$S=\begin{bmatrix}I&I\\W^TY&0\end{bmatrix}\quad(44)$$

which is invertible and produces

$$S^TP=\begin{bmatrix}Y&0\\X&U\end{bmatrix},\quad S^TPS=\begin{bmatrix}Y&Y\\Y&X\end{bmatrix}.\quad(45)$$

We first show that (22) implies (17). By Schur complement, the matrix inequality (17) holds if and only if (46) In view of

$$\begin{aligned}&\big(\varrho_2Z+(\varrho_2-\varrho_1)Z_1-\mathrm{diag}\{P,P\}\big)\\&\times\big(\varrho_2Z+(\varrho_2-\varrho_1)Z_1\big)^{-1}\\&\times\big(\varrho_2Z+(\varrho_2-\varrho_1)Z_1-\mathrm{diag}\{P,P\}\big)>0\end{aligned}\quad(47)$$

we obtain

$$\begin{aligned}&-\mathrm{diag}\{P,P\}\big(\varrho_2Z+(\varrho_2-\varrho_1)Z_1\big)^{-1}\mathrm{diag}\{P,P\}\\&\le-2\mathrm{diag}\{P,P\}+\varrho_2Z+(\varrho_2-\varrho_1)Z_1\end{aligned}\quad(48)$$

we have that (46) holds if (49) holds

Now, applying the congruence transformation $diag\{S,S,S,S,I_q,S,S,S,S,S,S\}$ to (49) and setting

$$\begin{aligned}\tilde Q&=S^TQS,\quad\tilde Q_1=S^TQ_1S,\quad\tilde Q_2=S^TQ_2S,\\ \tilde Z&=\mathrm{diag}\{S,S\}^TZ\mathrm{diag}\{S,S\},\\ \tilde Z_1&=\mathrm{diag}\{S,S\}^TZ_1\mathrm{diag}\{S,S\},\\ \tilde L_i&=\mathrm{diag}\{S,S\}^TL_i\mathrm{diag}\{S,S\},\quad i=1,2,3\\ \tilde F&=\mathrm{diag}\{S,S,S,S\}^TFS,\\ \tilde G&=\mathrm{diag}\{S,S,S,S\}^TGS,\\ \tilde H&=\mathrm{diag}\{S,S,S,S\}^THS,\\ \Upsilon_a&=XBK_bW^TY+UK_aW^TY,\\ \Upsilon_b&=K_bW^TY\\ \Upsilon_c&=UK_c\end{aligned}\quad(50)$$

we obtain that (49) is equivalent to

$$\tilde\Xi+\tilde Y\tilde K+\tilde K^T\tilde Y^T\quad<\quad 0,\quad(51)$$

where

$$\tilde\Xi=\begin{bmatrix}\Xi_{11}+\Xi_{17}^T\Xi_{17}&\Xi_{12}&\sqrt{\varrho_1}\tilde F[I,I]&\sqrt{\varrho_2-\varrho_1}\tilde H[I,I]\\ \bullet&\Xi_{22}&0&0\\ \bullet&\bullet&-\tilde L_1&0\\ \bullet&\bullet&\bullet&-\tilde L_3\end{bmatrix}$$

$$\tilde Y=\big[\Xi_{15}^T,\Xi_{25}^T,0_{n\times 8n}\big]^T,$$
$$\tilde K=\big[\Xi_{16}^T,0_{n\times 12n}\big].$$

Inequality (51) holds if the following is true for any $\kappa_1>0$,

$$\tilde\Xi+\kappa_1^{-1}\tilde Y\tilde Y^T+\kappa_1\tilde K^T\tilde K\quad<\quad 0,\quad(52)$$

which is equivalent to

$$\begin{bmatrix}\tilde\Xi&\tilde Y&\tilde K^T\\ \tilde Y^T&-\kappa_1I_n&0\\ \tilde K&0&-\kappa_1^{-1}I_n\end{bmatrix}\quad<\quad 0\quad(53)$$

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\begin{bmatrix} \Theta & [\tilde{M}^T, \varphi_0 \tilde{N}^T]\mathrm{diag}\{P,P\} & \sqrt{\varrho_1}F[I,I] & \sqrt{\varrho_2-\varrho_1}H[I,I] \\ \bullet & -\mathrm{diag}\{P,P\}\big(\varrho_2 Z + (\varrho_2-\varrho_1)Z_1\big)^{-1}\mathrm{diag}\{P,P\} & 0 & 0 \\ \bullet & \bullet & -L_1 & 0 \\ \bullet & \bullet & \bullet & -L_3 \end{bmatrix} < 0. \quad (46)$$

$$\begin{bmatrix} \Theta & [\tilde{M}^T, \varphi_0 \tilde{N}^T]\mathrm{diag}\{P,P\} & \sqrt{\varrho_1}F[I,I] & \sqrt{\varrho_2-\varrho_1}H[I,I] \\ \bullet & -2\mathrm{diag}\{P,P\} + \varrho_2 Z + (\varrho_2-\varrho_1)Z_1 & 0 & 0 \\ \bullet & \bullet & -L_1 & 0 \\ \bullet & \bullet & \bullet & -L_3 \end{bmatrix} < 0. \quad (49)$$

The above inequality is, by Schur complement again, exactly that of (22), and we conclude that this implies (17).

Next we show that (23) implies (18). This can be done by using a procedure analogous to the above. As for the verification of other inequalities, applying the congruence transformation diag $\{S, S\}$ to (19)-(20) and setting matrix variables as in (50), it is seen that (19)-(20) are equivalent to (24)- (26). So far, we have proven that (22)-(26) ensure (17)-(20) and thus the closed-loop system (8) is EMS-η. In this case, from (50), the feedback gains are computed as in (28).

ACKNOWLEDGMENTS

The author would like to thank the deanship for scientific research (DSR) at KFUPM for financial support through research group project **RG-1105-1**.

REFERENCES

[1] L. Q. Zhang, S. Yang, T. W. Chen and B. Huang, " A new method for stabilization of networked control systems with random delays," *IEEE Trans. Automat. Control*, vol. 50, no. 8, 2005, pp. 1177–1181.

[2] J. P. Hespanha and Y. G. Xu," A survey of recent results in networked control systems," *Proc. IEEE*, vol. 95, no. 1, 2007, pp. 138–162.

[3] W. Heemels, A. Teel, N. van de Wouw, and D. Nesic. "Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Automat. Control*, vol. 55, no. 8, 2010, pp. 1781–1796.

[4] F. Yang, Z. Wang, D. W. C. Ho, and M. Gani, "Robust $\mathcal{H}_\infty$ control with missing measurements and time delays," *IEEE Trans. Automat. Control*, vol. 52, no. 9, 2007, pp. 1666–1672.

[5] F. Yang, Z. Wang, Y. S. Hung and M. Gani, $\mathcal{H}_\infty$ control for networked systems with random communication delays, *IEEE Trans. Automat. Control*, vol. 51, no. 3, 2007, pp. 511-518.

[6] D. Yue, E. Tian, Z. Wang and J. Lam, Stabilization of systems with probabilistic interval input delays and its applications to networked control systems, *IEEE Trans. Systems, Man and Cybernetics, Part A-Systems and Humans*, vol. 39, no. 42, 2009, pp. 939-945.

[7] M. G. Rivera and A. Barreiro, Analysis of networked control systems with drops and variable delays, *Automatica*, vol. 43, no. 9, 2007, pp. 2054-2059.

[8] J. Xiong and J. Lam, Stabilization of linear systems over networks with bounded packet loss, *Automatica*, vol. 43, no. 1, 2007, pp. 80-87.

[9] L. Zhang, Y. Shi, T. Chen and B. Huang, A new method for stabilization of networked control systems with random delays, *IEEE Trans. Automat. Control*, vol. 50, no. 8, 2006, pp. 1177-1181.

[10] M. S. Mahmoud, Hazem N. Nounou and Yuanqing Xia, "Dissipative control for internet-based switching systems", *J. the Franklin Institute*, vol. 347, no. 1, 2011, pp. 154–172.

[11] Bo Liu, Yuanqing Xia, M. S. Mahmoud and Harris Wu, "New predictive control scheme for networked control systems", *J. Circuits, Systems and Signal Processing*, vol. 31, no. 2, 2012, pp. 945–960.

[12] M. S. Mahmoud and A. W. Saif, "Robust quantized approach to fuzzy networked control systems", *IEEE J. Emerging and Selected Topics in Circuits and Systems (JETCAS)*, vol. 2, no. 1, 2012, pp. 71–81.

[13] M. S. Mahmoud , "Improved networked control systems approach with communication constraint" *IMA J. Mathematical Control and Information*, vol. 29, no. 2, 2012, pp. 215–233.

[14] M. S. Mahmoud, "Robust control and filtering for time- delay systems" (Marcel-Dekker, New-York, 2000).

[15] P. Zhivoglyadov, and R. H. Middleton, "Networked control design for linear systems," *Automatica,* vol. 39, 2003, pp. 743–750.

[16] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Automat. Contr.*, vol. 49, 2004, pp. 1453–1464.

[17] Matveev, A. S., and Savkin, A. V. "Optimal computer control via communication channels with irregular transmission times" *Int. J. Control,* vol. 76, no. 2, 2003, pp. 165–177.

[18] Zhang H S, Xie L H. "Control and estimation of systems with input/output delays." Berlin: Springer, 2007.

[19] Hetel, L., Daafouz, J., and Lung, C. "Analysis and control of LTI and switched systems in digital loops via an event-based modelling," *Int. J. Control,* vol. 81, no. 7, 2008, pp. 1125–1138.

[20] Y. Shi and B. Yu, "Robust mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control of networked control systems with random time delays in both forward and backward communication links"', *Automatica*, vol. 47, no. 4, 2011, pp. 754-760.

[21] L. E. Ghaoui, F. Oustery, and M. Aitrami, A cone complementarity linearization algorithm for static output feedback and related problems, IEEE Trans. Autom. Control, vol. 42, no. 8, 1997, pp. 1171-1176.

[22] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in systems and control theory*." Philadelphia, PA: SIAM, 1994.

[23] M. S. Mahmoud, and A. Y. Al-Rayyah, "Efficient parameterization to stability and feedback synthesis of linear time-delay systems", *IET Control Theory and Applications*, vol. 3, no. 8, 2009, pp. 1107–1118.

[24] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica,* vol. 44, 2008, pp. 39–52.

[25] M. S. Mahmoud, S. Z. Selim and P. Shi, "Global exponential stability criteria for neural networks with probabilistic delays," *IET Control Theory Appl.*, vol. 4, no. 11, 2011, pp. 2405-2415.

[26] L. S. Hu, T. Bai, P. Shi, and Z. Wu, "Sampled-data control of networked linear control systems," *Automatica,* vol. 43, 2007, pp. 903–911.