Proceedings of the World Congress on Engineering 2013 Vol II,
WCE 2013, July 3 - 5, 2013, London, U.K.

Comparative Analysis of Tuning Range of Regulated Cascode Cross Coupled CMOS Oscillators

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***Abstract*— Tuning Range of Oscillator is one of the important specification which is used in phase locked loop. The more wider tuning range, the better the phase lock loop. This paper proposed regulated cascode cross coupled based CMOS oscillator. The analysis and simulation results of tuning range of this type of oscillator are compared with conventional Colpitts based CMOS oscillator as a function of bias current consumption.**

***Index Terms*— Tuning Range, CMOS oscillators, Regulated Cascode cross coupled VCO**

I. Introduction

The conventional method of analysis and design of Colpitts based CMOS Oscillator is not following Barkhausen criterion [1]. By substituting low frequency small signal equivalent circuit of CMOS into single ended Colpitts based CMOS oscillator and injecting input current source into the circuit, we can derive the transimpedance gain of the circuit. Then, we can equate the real part and imaginary part of the function to zero. After that, we can obtain two equations of oscillating frequency as a function of passive components. Then, if we equate both of the equations, substitute the capacitance ratio. We can derive minimum required gain for the Colpitts based circuits.

In contrast to the single ended Colpitts based CMOS oscillator. The cross coupled differential Colpitts based CMOS oscillator can not be designed by the same way as single ended Colpitts based CMOS oscillator but it follows Barkhausen criterion [2]. It is different because cross couple differential Colpitts based CMOS oscillator can be seen as a 2 stage amplifier in cascade while output of the second stage amplifier is fed back into input of the first stage amplifier

Section II describes how to design tuning range of single ended Colpitts based CMOS oscillator by switching the bias resistor. Section III describes how to design tuning range of cross coupled differential Colpitts based CMOS oscillator. Section IV describes how to design cross coupled differential Regulated Cascode based CMOS oscillator.

Manuscript received March 12, 2013
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II. Analysis and Simulation of Single Ended Colpitts Based CMOS Oscillator

The conventional Colpitts based CMOS oscillator was designed in [1]. It was given by

$$
\begin{aligned}
\frac{V_{out}}{I_{in}} &= \frac{-R_P L_P s\left(g_m + C_2 s\right)}{a_3 s^3 + a_2 s^2 + a_1 s + a_o} \\
a_3 &= R_P C_1 C_2 L_P \\
a_2 &= \left(C_1 + C_2\right) L_P \\
a_1 &= g_m R_P + R_P\left(C_1 + C_2\right) \\
a_o &= g_m R_P
\end{aligned}
\tag{1}
$$

where $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \frac{2I_D}{V_{GS} - V_{TH}}$ is small signal transconductance of the MOS transistor.

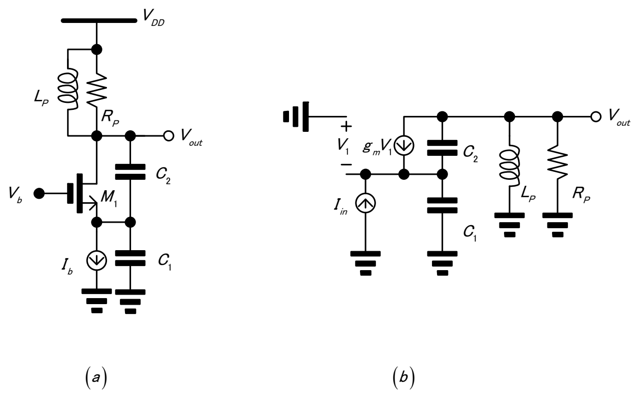

Fig. 1 (a) Single ended Colpitts based CMOS oscillator
(b) Small signal equivalent circuit of (a)

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Oscillating frequency was derived as

$$\omega_R = \pm\sqrt{\frac{1}{L_P\left(\frac{C_1C_2}{C_1+C_2}\right)}} = \pm\sqrt{\frac{g_mR_P}{\left(C_1+C_2\right)L_P}} \tag{2}$$

If we equate two oscillating equation in (2) then we can derived the condition as below.

$$g_mR_P = \frac{C_1}{C_2}\left(1+\frac{C_2}{C_1}\right)^2 \tag{3}$$

From equation (3), if we set capacitance ratio $C_2 / C_1 = 1$ then, the minimum required gain is 4. Supposed that we want to design capacitance, inductance value at oscillating frequency which is equal to 2.1 GHz. For typical design, we can set capacitance to be 1pF. Then, we can use equation (2) to design inductance value as follow.

$$\omega_R = \pm\sqrt{\frac{1}{L_P\left(\frac{1pF\times 1pF}{2pF}\right)}} = 2\pi\left(2.1GHz\right)$$

$$L_P = \frac{2}{\left(2\pi\times 2.1\times 10^{9}\right)^2\left(10^{-12}\right)} = 11nH$$

(4)

Then substitute value of inductance back into equation (2) to see if oscillating frequency is changed by real part condition on the rightmost of equation (2)

$$\omega_R = \sqrt{\frac{4}{\left(2\times 10^{-12}\right)\left(11\times 10^{-9}\right)}} = \left(2\pi\right)2.146GHz \tag{5}$$

The other parameter in the circuit which is still not designed is resistive load R_p, it can be used to set dc output voltage. Assume that if we use supply voltage equal to 3 volt. We can design current to flow 1mA and 2mA, as a result, its oscillating frequency are changed which may be called “tuning range”of Colpitts oscillator. By setting dc output voltage to be half of the supply voltage, 1.5 volt, we can determine resistive load and small signal transconductance to be

$$R_{P1} = \frac{3-1.5}{1mA} = 1.5k\Omega, R_{P2} = \frac{3-1.5}{2mA} = 0.75k\Omega$$
$$g_{m1} = \frac{2I_D}{0.7} = 2.85m\frac{A}{V}, g_{m1} = \frac{2I_D}{0.7} = 5.71m\frac{A}{V} \tag{6}$$

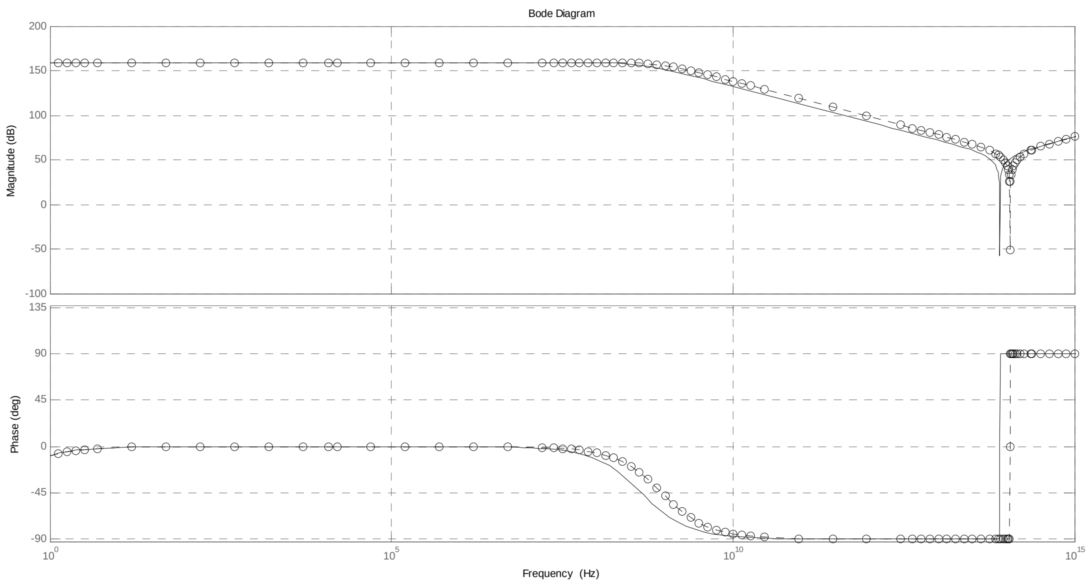

Fig. 2 Magnitude and phase response of the single ended Colpitts based CMOS oscillator
Where solid line is for $R_p = 1.5k\Omega, g_{m1} = 2.85mS$ and circle line is for $R_p = 0.75k\Omega, g_{m1} = 5.71mS$

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

III. Analysis and Simulation of Cross Coupled Differential Colpitts Based CMOS Oscillator

In contrast to the derivation of single ended Colpitts based CMOS oscillator, we can analyze and design cross coupled differential Colpitts based CMOS oscillator according to principle of feedback circuit analysis [3]

General transfer function of amplifier with feedback network can be written as following.

$$H(s) = \frac{A(s)}{1-\beta(s)A(s)}. \tag{7}$$

Because voltage gain of amplifier stage and voltage gain of feedback network in fig. 3 can be derived as

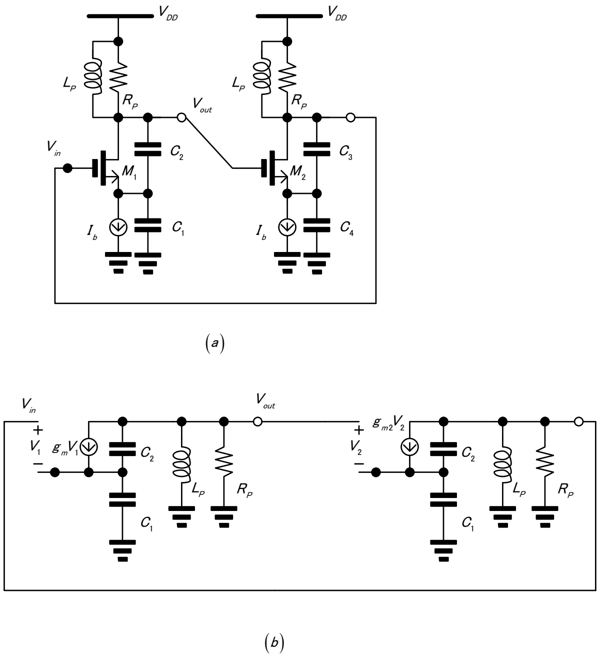

Fig. 3 (a) Cross couple differential Colpitts based CMOS oscillator (b) Small signal equivalent circuit of (a)

$$A(s) = \beta(s) = \frac{V_{out}}{V_{in}} = \frac{s^2\left(g_{m1}C_1L_P\right) + s\left(L_P g_{m1}^2\right)}{s^3 a_3 + s^2 a_2 + s a_1 + a_0}$$

$$a_3 = -C_2C_1L_P$$

$$a_2 = \frac{L_P\left(C_1+C_2\right)}{R_P} - C_2L_Pg_{m1} \tag{8}$$

$$a_1 = C_1 + C_2 + \frac{L_P g_{m1}}{R_P}$$

$$a_0 = g_{m1}$$

After substitute voltage gain of the amplifier and voltage gain of feedback network into equation (7), we can derived

$$H(s) = \frac{s^5b_5 + s^4b_4 + s^3b_3 + s^2b_2 + sb_1 + b_0}{s^6c_6 + s^5c_5 + s^4c_4 + s^3c_3 + s^2c_2 + sc_1 + c_0} \tag{9}$$

The coefficient of the numerator and denominator polynomial can be written as following

$$\begin{aligned}
b_5 &= a_3 g_{m1} C_1 L_P \\
b_4 &= a_3 L_P g_{m1}^2 + a_2 g_{m1} C_1 L_P \\
b_3 &= a_1 g_{m1} C_1 L_P + a_2 L_P g_{m1}^2 \\
b_2 &= a_0 g_{m1} C_1 L_P + a_1 L_P g_{m1}^2 \\
b_1 &= L_P g_{m1}^2 a_0 \\
b_0 &= 0
\end{aligned} \tag{10}$$

$$\begin{aligned}
c_6 &= a_3^2 \\
c_5 &= 2a_3a_2 \\
c_4 &= a_2^2 + 2a_3a_1 - \left(g_{m1}C_1L_P\right)^2 \\
c_3 &= 2a_3a_0 + 2a_2a_1 - \left(2g_{m1}^3L_P^2C_1\right) \\
c_2 &= 2a_2a_0 + a_1^2 - \left(L_P^2 g_{m1}^4\right) \\
c_1 &= 2a_1a_0 \\
c_0 &= a_0^2
\end{aligned} \tag{11}$$

Because denominator polynomial of this circuit has 6th order. Thus, we could not manipulate closed form expression of oscillating frequency anymore.

After substitute the inductance 1 nH, capacitance 1pF, resistance $1.5k\Omega, 0.75k\Omega$ and transconductances (the transistor has the same value as equation (6)) into the coefficients of the polynomial. We can plot magnitude and phase response by bode function in MATLAB as fig 4.

It can be seen from fig 4 that magnitude response seems to have band-pass characteristic. But phase response is not 360 degree phase shifted at resonance frequency. It means this circuit is not oscillate.

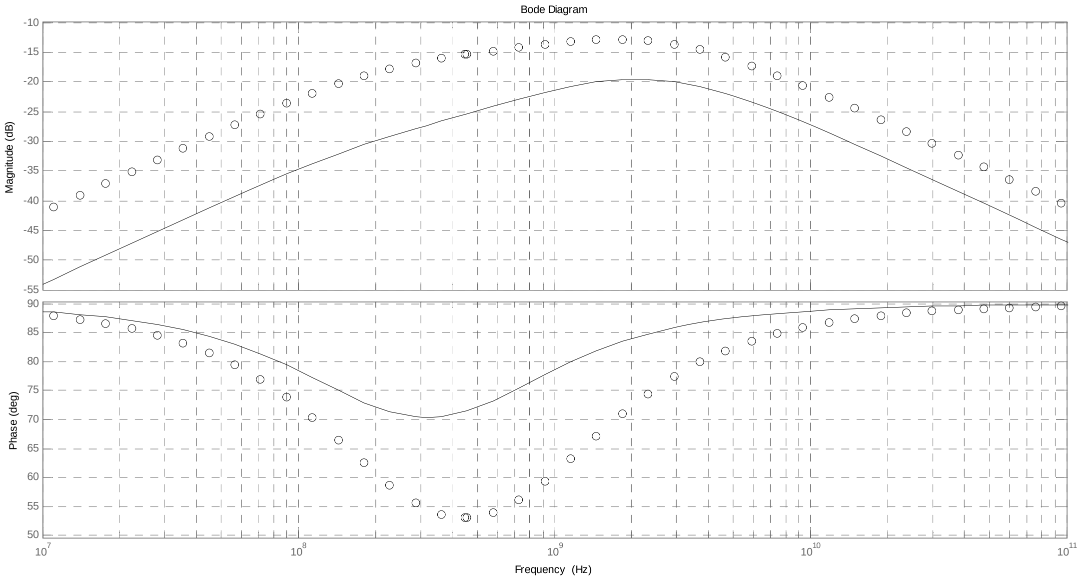

Fig. 4 Magnitude and phase response of cross coupled differential Colpitts based CMOS oscillator.

solid line is for $R_p = 1.5k\Omega, g_{m1} = 2.85mS$ and circle line is for $R_p = 0.75k\Omega, g_{m1} = 5.71mS$

IV. Analysis and simulation of cross coupled differential Regulated cascode based CMOS oscillator

With the cross coupled family. The suitable analysis method should be the same as section III. Voltage gain of cross coupled differential regulated cascode based CMOS oscillator in fig. 5 is derived as following

$$\frac{V_{out}}{V_{in}} = \frac{sL_P g_Y}{s^2\left(L_P C_P\right) + s\left(L_P g_z\right) + 1} = A(s) = \beta(s) \tag{12}$$

$$g_x = g_{m2} + \left(\frac{1}{g_{m2}}\right)\left(\frac{1}{R_B} + \frac{1}{r_{ds3}}\right)\left(\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}} + g_{m2}\right)$$

$$g_y = \frac{g_{m2}g_{m1}}{g_x} - \frac{g_{m1}}{g_x g_{m3}}\left(\frac{1}{R_B} + \frac{1}{r_{ds3}}\right)\left(g_{m2} + \frac{1}{r_{ds2}}\right)$$

$$g_z = \left(\frac{1}{R_P} - \frac{1}{r_{ds2}}\right) - \left(\frac{1}{r_{ds2}g_x g_{m3}}\right)\left(\frac{1}{R_B} + \frac{1}{r_{ds3}}\right)\left(g_{m2} + \frac{1}{r_{ds2}}\right) \tag{13}$$

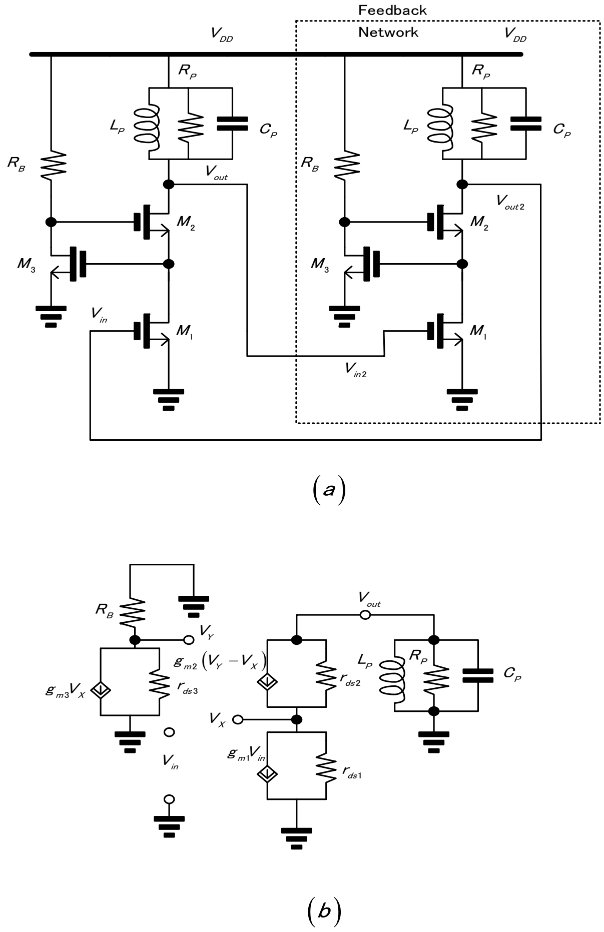

Fig. 5 (a) Cross Couple Differential Regulated Cascode based CMOS oscillator (b) Small signal of Regulated Cascode Bandpass Amplifier

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The coefficient in equation (12) are defined in equation (13) for easy visualization of the whole equation. It can be understood that equation (12) has bandpass response.

Equation (12) can be used to design oscillator by substitute $s = j\omega$ into denominator of equation (12)

Thus, resonance frequency or oscillating frequency can be derived as

$$\omega_R = \sqrt{\frac{1}{L_P C_P}} \tag{14}$$

From equation (14), it can be seen that oscillating frequency of this circuit could not be tune by using bias current. But the bandwidth of the bandpass filter can be tune by bias current.

After substitute equation (12) into equation (7), then we get

$$H(s) = \frac{s^3\left(L_P^2 g_y C_P\right) + s^2\left(L_P^2 g_y g_z\right) + s\left(L_P g_y\right)}{s^4\left(L_P^2 C_P^2\right) + s^3\left(2L_P^2 C_P g_z\right) + s^2\left(2L_P C_P + L_P^2 g_z^2 - L_P^2 g_y^2\right) + s\left(2L_P g_z\right) + 1} \tag{15}$$

To plot magnitude and phase response of equation (15), we typically design current to flow 1mA and 2mA. We use 3 volt supply and we design output voltage to be half of the supply voltage which is equal to 1.5 V. Then $R_{P1} = \frac{1.5}{1mA} = 1.5k\Omega, R_{P2} = \frac{1.5}{2mA} = 0.75k\Omega$

To design current to flow, we use drain current equation as following

$$i_D = \frac{\left(\mu_n C_{ox}\right)}{2}\left(\frac{W}{L}\right)\left(1.5 - 0.7\right)^2 = 1mA \tag{16}$$

For typical process parameters, the process transconductance parameter $\mu_n C_{ox} = 194\mu A / V^2$ is used. Then, aspect ratio of the transistor can be designed to be

$$\left(\frac{W}{L}\right)_1 = \frac{(2)2mA}{\left(194\mu A / V^2\right)\left(1.5 - 0.7\right)^2} = 32.21 \tag{17}$$

Thus , minimum drain to source voltage of the first transistor M_1 is equal to $V_{DS1} = 1.5 - 0.7 = 0.8V$. For convenience in design, transistor M_2 can be designed to have the same aspect ratio, thus the gate to source voltage of transistor M_2 should be design to be 1.5 V. Thus, voltage at the gate terminal of transistor M_2 should be 2.3 V. Again, for convenience in design, the regulated transistor M_3 should have the same aspect ratio with M_1 and M_2

$$\left(\frac{W}{L}\right)_3 = \frac{(2)I_{D3}}{\left(194\mu A / V^2\right)\left(0.8 - 0.7\right)^2} = 32.21 \tag{18}$$

Thus, drain current of transistor M_3 can be computed from equation (18) to be $31.25\mu A$. Bias resistor R_B can be computed to be $R_B = \frac{3 - 2.3}{31.25\mu A} = 22.40k\Omega$

The other parameters use in simulation of cross coupled differential regulated cascode based CMOS oscillator are computed as following

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} = \sqrt{2\left(194\mu A / V^2\right)(32)(2mA)} = 4.98mA / V = g_{m2} \tag{19}$$

$$g_{m3} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_3 I_{D3}} = \sqrt{2\left(194\mu A / V^2\right)(32)(31.25\mu A)} = 0.62mA / V \tag{20}$$

$$r_{ds1} = \frac{1}{\lambda I_{D1}} = r_{ds2} = \frac{1}{\lambda I_{D2}} = \frac{1}{0.01(2mA)} = 50k\Omega \tag{21}$$

$$r_{ds3} = \frac{1}{\lambda I_{D3}} = \frac{1}{0.01(31.25\mu A)} = 3.2M\Omega \tag{22}$$

The capacitance in equation (14) is designed to be 1 pF. If the oscillating frequency is designed to be 2.1 GHz. Then, inductance is computed to be

$$\omega_R = \sqrt{\frac{1}{L_P C_P}} = 2\pi\left(2.1GHz\right) \rightarrow L_P = \frac{1}{\left(1.74 \times 10^{20}\right)\left(10^{-12}\right)} = 5nH \tag{23}$$

After all numerical values in equation (15) are known. Then we can substitute it in MATLAB text file and simulate it. Figure 6 is the result of simulation of equation (15)

ISBN: 978-988-19252-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

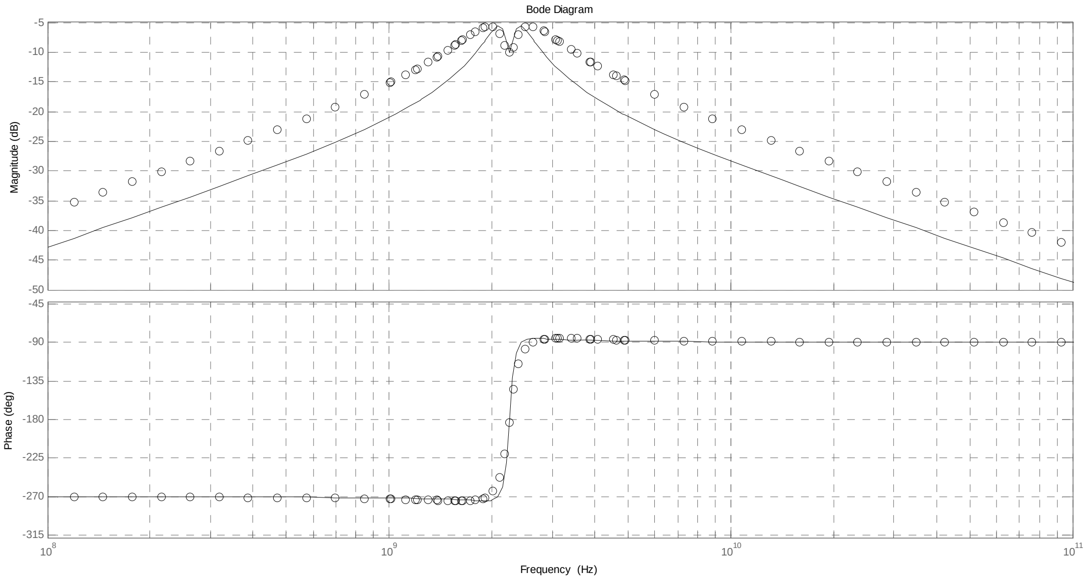

Fig.6 Magnitude and phase response of cross coupled differential regulated cascode based CMOS oscillator

solid line is for $R_p = 1.5k\Omega, g_{m1} = 2.85mS$ and circle line is for $R_p = 0.75k\Omega, g_{m1} = 5.71mS$

V. Conclusion

It is well known that tuning range of cross coupled common source oscillator is limited by minimum and maximum capacitance of the varactor. The tuning range of single ended and differential Colpitts is derived to be dependent on both passive capacitance and transconductance of the transistor. It means that tuning range can be changed by switching of the bias current. The oscillating frequency of cross couple regulated cascode based was derived to be independent with bias current. But the bandwidth depends on bias current.

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