Proceedings of the World Congress on Engineering 2013 Vol I,
WCE 2013, July 3 - 5, 2013, London, U.K.

A Quantity Discount Problem with One Poultry Farmer and Two Retailers for Ameliorating Items

Hidefumi Kawakatsu, Toshimichi Homma and Kiyoshi Sawada

***Abstract*—This study considers a quantity discount problem between a single seller (poultry famer) and two buyers (retailers). The poultry farmer's inventory level increases due to the increase in the weight of the fowls. The retailers purchase fresh chicken meat from the poultry farmer, the inventory levels of the retailers are therefore depleted due to the combined effects of its demand and deterioration. The poultry farmer attempts to increase her/his profit by controlling the retailers' order quantities through a quantity discount strategy. The retailers try to maximize their profits by considering both whether to cooperate with each other and whether to accept the poultry farmer's offer. We formulate the above problem as a Stackelberg game between a single poultry farmer and two retailers to analyze the existence of the poultry farmer's optimal quantity discount pricing policy, which maximizes her/his total profit per unit of time. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed model.**

***Index Terms*—quantity discounts, ameliorating items, total profit, Stackelberg game**

I. INTRODUCTION

Most of Japanese large-scale supermarkets deal in the same items at different sections in the same stores. For instance, *yakitori* are sold both at a meat section and at a ready-made dish section (sozai corner). The *yakitori* consists of bite-sized chicken pieces that are grilled on thin skewers. Managers of these sections independently order the fresh chicken meat as ingredients for the yakitori since they are under competition and are evaluated separately by their superiors. They can possibly reduce their costs if either of them purchases the items for two sections in cooperation with each other. Several super-markets have recently applied this strategy.

This paper presents a model for determining optimal all-unit quantity discount strategies in a channel of one seller (poultry farmer) and two buyers (retailers). Many researchers have developed models to study the effectiveness of quantity discounts[1], [2], [3], [4], [5], [6]. Quantity discounts are widely used by the sellers with the objective of inducing the buyer to order larger quantities in order to reduce their total transaction costs associated with ordering, shipment and inventorying. Our previous study[7] has developed the model to determine an optimal pricing and a ordering policy for ameliorating items with quantity discounts. However, we focused on the quantity discount problem between a single seller and a single buyer.

Manuscript received March 22, 2013; revised April 7, 2013.

H. Kawakatsu is with Department of Economics & Information Science, Onomichi City University, 1600-2 Hisayamadacho, Onomichi 722-8506 Japan (corresponding author to provide phone: +81-848-22-8312 (ex.617); fax: +81-848-22-5460; e-mail: kawakatsu@onomichi-u.ac.jp).

H. Toshimichi is with Faculty of Business Administration of Policy Studies, Osaka University of Economics, Osaka, 533-8533 Japan

K. Sawada is with Department of Policy Studies, University of Marketing and Distribution Sciences, Kobe, 651-2188 Japan

In this study, we discuss the quantity discount problem between the single poultry farmer and the two retailers for ameliorating items. The ameliorating items include the fast growing animals such as broiler in a poultry farm[8], [9], [10]. The poultry farmer purchases chicks from an upper-leveled supplier and then feeds them until they grow up to be fowls. The retailers purchase (raw) chicken meat from the poultry farmer. The stock of the poultry farmer increases due to growth, in contrast, the inventory levels of the retailers are depleted due to the combined effect of its demand and deterioration. The poultry farmer is interested in increasing her/his profi by controlling the retailers' order quantities through the quantity discount strategy. The retailers attempt to maximize their profit considering both whether to cooperate with each other and whether to accept the poultry farmer's proposal. We formulate the above problem as a Stackelberg game between the poultry farmer and the two retailers to analyze the existence of the poultry farmer's optimal quantity discount pricing policy which maximizes her/his total profi per unit of time. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed formulation.

II. NOTATION AND ASSUMPTIONS

The poultry farmer uses a quantity discount strategy in order to improve her/his profit The poultry farmer proposes, for the retailers, an order quantity per lot along with the corresponding discounted price, which induces the retailers to alter their replenishment policies. We consider the two options throughout the present study as follows:

Option V_1**:** The retailer i ($i = 1, 2$) does not adopt the quantity discount proposed by the poultry farmer. When the retailer i chooses this option, she/he purchases the products from the poultry farmer at an initial price in the absence of the discount, and she/he determines her/himself an optimal order quantity which maximizes her/his own total profi per unit of time.

Option V_2**:** The retailer i accepts the quantity discount proposed by the poultry farmer.

The main notations used in this paper are listed below:

$Q_i^{(j)}$**:** the order quantity per lot for the retailer i under Option $V_j (i = 1, 2,\ j = 1, 2)$.

$T_i^{(j)}$**:** the length of the order cycle for the retailer i under Option V_j.

h_i**:** the inventory holding cost for the retailer i per item and unit of time.

a_i**:** the ordering costs per lot for the retailer i.

θ_i**:** the deterioration rate of the retailer i's inventory.

p_b**:** the retailer i's unit selling price, i.e., unit purchasing price for her/his customers.

μ_i**:** the constant demand rate of the product for the retailer i.

ISBN: 978-988-19251-0-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

c_s**:** the poultry farmer's unit acquisition cost (unit purchasing cost from the upper-leveled supplier).

p_s**:** the poultry farmer's initial unit selling price, i.e., each of the retailers' unit acquisition costs in the absence of the discount.

y**:** the discount rate for the price proposed by the poultry farmer. The poultry farmer therefore offers a unit discounted price of $(1-y)p_s$ $(0 \leq y < 1)$.

$S^{(j)}$**:** the poultry farmer's order quantity per lot under Option V_j.

h_s**:** the poultry farmer's inventory holding cost per item and unit of time.

a_s**:** the poultry farmer's ordering cost per lot.

α**,** β**:** the parameters of the Weibull distribution whose probability density function is given by

$$f(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}. \tag{1}$$

The assumptions in this study are as follows:

1) The poultry farmer's inventory increases due to growth during the prescribed time period $[0, T_{\max}]$.
2) The retailers' inventory levels are continuously depleted due to the combined effects of its demand and deterioration.
3) The rate of replenishment is infinit and the delivery is instantaneous.
4) Backlogging and shortage are not allowed.
5) The quantity of the item can be treated as continuous for simplicity.
6) Both the poultry farmer and the retailers are rational and use only pure strategies.
7) The period when chicks grow up to be fowls is a known constant, and therefore, this feeding period can analytically be regarded as zero.
8) The length of the poultry farmer's order cycle is given by $N^{(j)}T_i^{(j)}$ under Option V_j $(j = 1, 2)$, where $N^{(j)}$ is a positive integer. This is because the poultry farmer can possibly improve her/his total profi by increasing the length of her/his order cycle from $T_i^{(j)}$ to $N^{(j)}T_i^{(j)}$.
9) The instantaneous rate of amelioration of the on-hand inventory at time t is denoted by $r(t)$ which obeys the Weibull distribution[8], [9], [10], i.e.,

$$r(t) = \frac{g(t)}{1-F(t)} = \alpha\beta t^{\beta-1} \ (\alpha > 0,\ \beta > 0), \tag{2}$$

where $F(t)$ is the distribution function of Weibull distribution.

III. Retailers' Total Profits

This section formulates the retailer i's $(i = 1, 2)$ total profi per unit of time for the Option V_1 and V_2 available to the two retailers.

Figure 1 shows the two retailers' transitions of inventory level under Option V_j $(j = 1, 2)$.

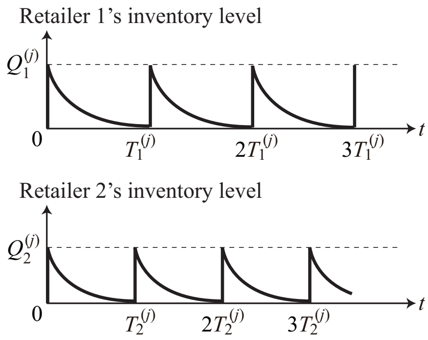

Fig. 1. Transition of Retailers' Inventory Level

A. Under Option V_1

If the retailer i chooses Option V_1, her/his order quantity per lot and her/his unit acquisition cost are respectively given by $Q_i^{(1)} = Q\left(T_i^{(1)}\right)$ and p_s, where p_s is the unit initial price in the absence of the discount. In this case, she/he determines her/himself the optimal order quantity $Q_i^{(1)} = Q\left(T_i^{(1)*}\right)$ which maximizes her/his total profi per unit of time.

Since the inventory level of the retailer i is depleted due to the combined effect of its demand and deterioration, the inventory level, $I_B(t)$, at time t during $[0, T_1)$ can be expressed by the following differential equation:

$$dI_B(t)/dt = -\theta_i I_B(t) - \mu_i. \tag{3}$$

By solving the differential equation in Eq. (3) with a boundary condition $I_B(T_i^{(1)}) = 0$, the retailer's inventory level at time t is given by

$$I_B(t) = \rho_i\left\{e^{\theta_i[T_i^{(1)}-t]} - 1\right\}, \tag{4}$$

where $\rho_i = \mu_i/\theta_i$.

Therefore, the initial inventory level, $I_B(0)$ $(= Q_i^{(1)} = Q\left(T_i^{(1)}\right))$, in the order cycle becomes

$$Q\left(T_i^{(1)}\right) = \rho_i\left[e^{\theta_{b_i}T_i^{(1)}} - 1\right]. \tag{5}$$

On the other hand, the cumulative inventory, $A(T_1)$, held during $[0, T_1)$ is expressed by

$$\begin{aligned} A\left(T_i^{(1)}\right) &= \int_0^{T_i^{(1)}} I_B(t)dt \\ &= \rho_i\left\{\frac{\left[e^{\theta_i T_i^{(1)}} - 1\right]}{\theta_i} - 1\right\}. \end{aligned} \tag{6}$$

Hence, the retailer i's total profi per unit of time under Option V_1 is given by

$$\begin{aligned} &\pi_i^{(1)}\left(T_i^{(1)}\right) \\ &= \frac{p_b\int_0^{T_i^{(1)}}\mu_i dt - p_s Q\left(T_i^{(1)}\right) - h_i A\left(T_i^{(1)}\right) - a_i}{T_i^{(j)}} \\ &= \rho_i(p_b\theta_i + h_i) - \frac{\left(p_s + \frac{h_i}{\theta_i}\right)Q\left(T_i^{(1)}\right) + a_i}{T_i^{(1)}}. \end{aligned} \tag{7}$$

In the following, the results of analysis are briefl summarized:

There exists a unique finit $T_i^{(1)} = T_i^{(1)^*}$ (> 0) which maximizes $\pi_i^{(1)}(T_i^{(1)})$ in Eq. (7). The optimal order quantity is therefore given by

$$Q_i^{(1)^*} = \rho_i \left[e^{\theta_i T_i^{(1)^*}} - 1 \right]. \quad (8)$$

The total profi per unit of time becomes

$$\pi_i^{(1)^*} = \rho_i \left[(p_b\theta_i + h_i) - \theta_i \left(p_s + \frac{h_i}{\theta_i} \right) e^{\theta_i T_i^{(1)^*}} \right]. \quad (9)$$

B. Under Option V_2

If the retailer i chooses Option V_2, the order quantity and unit discounted price are respectively given by $Q_i^{(2)} = Q\left(T_i^{(2)}\right) = \rho_i \left[e^{\theta_i T_i^{(2)}} - 1 \right]$ and $(1-y)p_s$. The retailer i's total profi per unit of time can therefore be expressed by

$$\begin{aligned} &\pi_i^{(2)}\left(T_i^{(2)}, y\right) \\ &= \rho_i(p_b\theta_i + h_i) \\ &\quad - \frac{\left[(1-y)p_s + \frac{h_i}{\theta_i}\right] Q\left(T_i^{(2)}\right) + a_i}{T_i^{(2)}}. \end{aligned} \quad (10)$$

Let $p^{(1)}$ and $p^{(2)}$ be define by $p^{(1)} = p_s$ and $p^{(2)} = (1-y)p_s$, respectively, then $\pi_i^{(1)}\left(T_i^{(1)}\right)$ in Eq. (7) and $\pi_i^{(2)}\left(T_i^{(2)}, y\right)$ in Eq. (10) can be rewritten as follows:

$$\pi_i^{(j)} = \rho_i(p_b\theta_i + h_i) - \frac{\left[p^{(j)} + \frac{h_i}{\theta_i}\right] Q\left(T_i^{(j)}\right) + a_i}{T_i^{(j)}}. \quad (11)$$

IV. RETAILERS' OPTIMAL POLICIES UNDER THE COOPERATIVE GAME

This section discusses a cooperative game between two retailers. In this study, we focus on the situation where there are two sections in the same store, and therefore we assume that the transportation cost of the product from one retailer to the other is zero. This signifie that the retailers can possibly reduce their costs by adopting the strategy that either of the retailers purchases the products from the poultry farmer and stocks them, and then she/he distributes the products to the other retailer.

The joint profi function per unit of time can therefore be expressed by

$$\begin{aligned} J\left(T_i^{(j)}\right) &= \frac{\mu_1 + \mu_2}{\theta_i}(p_b\theta_i + h_i) \\ &\quad - \frac{\left(p_s + \frac{h_i}{\theta_i}\right) Q\left(T_i^{(j)}\right) + a_i.}{T_i^{(j)}}. \end{aligned} \quad (12)$$

A. Under Option V_1

Under Option V_1, we can prove that there exist a unique finit positive $T_i^{(1)} = T_i^{(1)^*}$, which maximizes $J\left(T_i^{(j)}\right)$ in Eq. (12), and the maximum joint profi becomes

$$J^{(1)^*} = \max_{i=1,2} \hat{J}_i^{(1)}, \quad (13)$$

where

$$\begin{aligned} \hat{J}_i^{(1)} &= \frac{\mu_1 + \mu_2}{\theta_i} \\ &\quad \times \left[(p_b\theta_i + h_i) - (p_s\theta_i + h_i)\, e^{\theta_i T_i^{(1)^*}} \right]. \end{aligned} \quad (14)$$

Equation (14) signifie a local maximum value of the joint profi when the retailer i is in charge of ordering and inventory management.

Let R denote the retailer who is in charge of ordering and inventory control and bargains with the poultry farmer on behalf of two retailers, and then R is given by

$$R = \begin{cases} 1, & \text{if } \hat{J}_1^{(1)} \geq \hat{J}_2^{(1)}, \\ 2, & \text{if } \hat{J}_1^{(1)} < \hat{J}_2^{(1)}. \end{cases} \quad (15)$$

The analysis with respect to comparing $J_1^{(1)}$ with $J_2^{(1)}$ becomes considerably complicated since Eq. (14) includes the term $T_i^{(1)^*}$ which is determined by a nonlinear equation solution. Neglecting higher order terms of θ_i in the expansion of $e^{\theta_i T_i^{(1)}}$, we have $e^{\theta_i T_i^{(1)}} \approx 1 + \theta_i T_i^{(1)} + [\theta_i T_i^{(1)}]^2/2$.

In this case, $J^{(1)^*}$ in Eq. (13) can be expressed as

$$J^{(1)^*} = \begin{cases} \hat{J}_1^{(1)}, & \text{if } a_1(p_s\theta_1 + h_1) \leq a_2(p_s\theta_2 + h_2), \\ \hat{J}_2^{(1)}, & \text{if } a_1(p_s\theta_1 + h_1) > a_2(p_s\theta_2 + h_2). \end{cases} \quad (16)$$

It can also be shown in this case that $\hat{J}_R^{(1)} > \sum_{i=1}^{2} \pi_i^{(1)^*}$. We therefore focus on the case where $\hat{J}_R^{(1)} > \sum_{i=1}^{2} \pi_i^{(1)^*}$ in the following sections.

B. Under Option V_2

Under Option V_2, the retailer R accepts the quantity discount offered by the poultry farmer, which is described in the previous subsection.

Under this option, the retailer R's joint profi per unit of time can be expressed by

$$\begin{aligned} &J^{(2)}\left(T_R^{(2)}, y\right) \\ &= \frac{\mu_1 + \mu_2}{\theta_R}(p_b\theta_R + h_R) \\ &\quad - \frac{\left[(1-y)p_s + \frac{h_R}{\theta_R}\right] Q\left(T_R^{(2)}\right) + a_R}{T_R^{(2)}}. \end{aligned} \quad (17)$$

V. RETAILERS' OPTIMAL RESPONSE AND SHAPLEY VALUE IMPUTATION

A. Retailers' optimal response

This subsection discusses the retailer R's optimal response. The retailer R prefers Option V_1 over Option V_2 if $J^{(1)^*} > J^{(2)}$, but when $J^{(1)^*} < J^{(2)}\left(T_R^{(2)}, y\right)$, she/he prefers V_2 to V_1. The retailer R is indifferent between the two options if $J^{(1)^*} = J^{(2)}\left(T_R^{(2)}, y\right)$, which is equivalent to

$$\begin{aligned} y &= \frac{1}{p_s Q\left(T_R^{(2)}\right)} \\ &\quad \times \Bigg\{ \left[Q\left(T_R^{(2)}\right) - \rho\theta_R T_R^{(2)} e^{\theta_R T_R^{(1)*}} \right] \\ &\quad \times \left(p_s + \frac{h_R}{\theta_R} \right) + a_R \Bigg\}. \end{aligned} \quad (18)$$

Let us denote, by $\psi\left(T_R^{(2)}\right)$, the right-hand-side of Eq. (18). It can easily be shown from Eq. (18) that $\psi(T_2)$ is increasing in $T_R^{(2)}$ $(\geq T_R^{(1)*})$.

ISBN: 978-988-19251-0-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

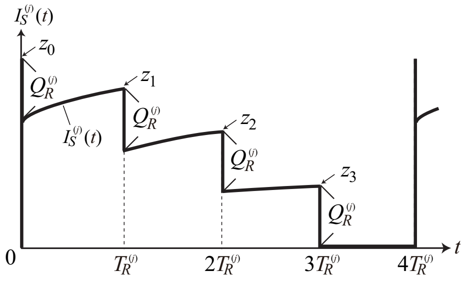

Fig. 2. Transition of the Poultry Farmer's Inventory Level under Option V_j ($N^{(j)} = 4$)

The maximum value of the joint profi is given by

$$J^* = \begin{cases} J^{(1)^*}, & \text{if } J^{(1)^*} \geq J^{(2)}\left(T_R^{(2)}, y\right), \\ J^{(2)}\left(T_R^{(2)}, y\right), & \text{if } J^{(1)^*} < J^{(2)}\left(T_R^{(2)}, y\right). \end{cases} \quad (19)$$

B. Shapley value imputation

We focus on the case where two retailers maximize their joint profi and share their cooperative profi according to the Shapley value[11], [12]. In this subsection, we determine the retailers' allocations of cooperative profi based on the concept of Shapley value. The Shapley value is one of the commonly used sharing mechanisms in static cooperation games with transferable payoff[11], [12].

Some additional notations used in this subsection are listed below.

x_i**:** the retailer i's allocation of the cooperative profi ($i = 1, 2$).

v**:** a characteristic function of the coalition, i.e., $v(1) = \pi_1^{(1)^*}$, $v(2) = \pi_2^{(1)^*}$ and $v(1,2) = J^*$.

Vector $\boldsymbol{x} = (x_1, x_2)$ is called an imputation if it satisfie the following two conditions:

(1) Individual rationality: $x_i \geq \pi_i^{(1)^*}$ ($i = 1, 2$)

(2) Group rationality: $x_1 + x_2 = J^*$

The Shapley value gives an imputation rule for retailer i ($i \in [1, 2] \equiv K$) described by Eq. (20).

$$\phi_i = \sum_{S \subset K} \frac{(s-1)!(n-s)!}{n!} \left[v(S) - v(S \setminus \{i\})\right], \quad (20)$$

where $|S| = s$. In this study, therefore, the imputation $x_1 = \phi_1$ and $x_2 = \phi_2$ are respectively given by

$$\phi_1 = \frac{\pi_1^{(1)^*} + J^* - \pi_2^{(1)^*}}{2}, \quad (21)$$

$$\phi_2 = \frac{J^* - \pi_1^{(1)^*} + \pi_2^{(1)^*}}{2}. \quad (22)$$

VI. POULTRY FARMER'S TOTAL PROFIT AND OPTIMAL POLICY

The retailers adopt the cooperative strategy to increase their profit as mentioned in Section IV. The poultry farmer can therefore regard the retailers as a single retailer since either of the retailers is in charge of ordering and inventory management. In this case, the poultry farmer's total profi per unit of time can be formulated in the same manner as our previous formulation[7]. For this reason, in the following we briefl summarize the results associated with the poultry farmer's profit under Option V_1 and V_2 and her/his optimal policy.

Figure 2 shows the poultry farmer's transitions of inventory level in the case of $N^{(j)} = 4$.

The length of the poultry farmer's order cycle is given by $N^{(j)} T_R^{(j)}$ under Option V_j ($j = 1, 2$), where $N^{(j)}$ is a positive integer. This is because the poultry farmer can possibly improve her/his total profi by increasing the length of her/his order cycle from $T_R^{(j)}$ to $N^{(j)} T_R^{(j)}$.

The poultry farmer's inventory increases due to growth during $[0, T_{\max}]$. Therefore, the poultry farmer's inventory level, $I_S(t)$, at time t can be expressed by the following differential equation:

$$dI_S(t)/dt = r(t) I_S(t) \ (0 \leq t \leq T_{\max}). \quad (23)$$

with a boundary condition $I_S(jT_R^{(j)}) = z_k(T_R^{(j)})$ under Option V_j, where $z_k(T_R^{(j)})$ denotes the remaining inventory at the end of the kth shipping cycle.

In this case, the poultry farmer's total profi per unit of time under Option V_1 is given by

$$\begin{aligned} &P^{(1)}\left(N^{(1)}, T_R^{(1)*}\right) \\ &= \frac{p_s Q\left(T_R^{(1)*}\right) - a_s / N^{(1)}}{T_R^{(1)*}} \\ &\quad - \frac{Q\left(T_R^{(1)*}\right)}{N^{(1)} T_R^{(1)*}} \left\{ \sum_{k=1}^{N^{(1)}-1} e^{-\alpha (k T_R^{(1)*})^\beta} \right. \\ &\quad \left. \times \left[c_s + h_s \int_0^{k T_R^{(1)*}} e^{\alpha t^\beta} dt \right] + c_s \right\}. \end{aligned} \quad (24)$$

In contrast, under Option V_2, the poultry farmer's total profi per unit of time becomes

$$\begin{aligned} &P^{(2)}\left(N^{(2)}, T_R^{(2)}, y\right) \\ &= \frac{(1-y) p_s Q\left(T_R^{(2)}\right) - a_s / N^{(2)}}{T_R^{(2)}} \\ &\quad - \frac{Q\left(T_R^{(2)}\right)}{N^{(2)} T_R^{(2)}} \left\{ \sum_{k=1}^{N^{(2)}-1} e^{-\alpha (k T_R^{(2)})^\beta} \right. \\ &\quad \left. \times \left[c_s + h_s \int_0^{k T_R^{(2)}} e^{\alpha t^\beta} dt \right] + c_s \right\}, \end{aligned} \quad (25)$$

where

$$Q\left(T_R^{(j)}\right) = \rho \left(e^{\theta_b T_R^{(j)}} - 1\right), \quad (26)$$

$$S\left(N^{(j)}, T_R^{(j)}\right) = Q\left(T_R^{(j)}\right) \sum_{k=0}^{N^{(j)}-1} e^{-\alpha (k T_R^{(j)})^\beta}. \quad (27)$$

The poultry farmer's optimal values for $T_R^{(2)}$ and y can be obtained by maximizing her/his total profi per unit of time considering the retailer R's optimal response which was discussed in Subsection V-A. Henceforth, let Ω_j ($j = 1, 2$) be define by

$$\Omega_1 = \left\{ \left(T_R^{(2)}, y\right) \mid y \leq \psi\left(T_R^{(2)}\right) \right\},$$

$$\Omega_2 = \left\{ \left(T_R^{(2)}, y\right) \mid y \geq \psi\left(T_R^{(2)}\right) \right\}.$$

ISBN: 978-988-19251-0-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

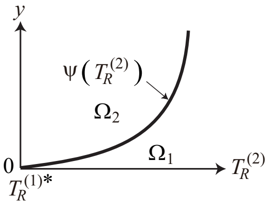

Fig. 3. Characterization of retailer R's optimal responses

Figure 3 depicts the region of Ω_j $(j = 1, 2)$ on the $\left(T_R^{(2)}, y\right)$ plane.

A. Under Option V_1

If $\left(T_R^{(2)}, y\right) \in \Omega_1 \setminus \Omega_2$ in Fig. 3, the retailer will naturally select Option V_1. In this case, the poultry farmer can maximize her/his total profi per unit of time independently of T_2 and y on the condition of $\left(T_R^{(2)}, y\right) \in \Omega_1 \setminus \Omega_2$. Hence, the poultry farmer's locally maximum total profi per unit of time in $\Omega_1 \setminus \Omega_2$ becomes

$$P^{(1)^*} = \max_{N^{(1)} \in N} P^{(1)} \left(N^{(1)}, T_R^{(1)^*}\right), \tag{28}$$

where N signifie the set of positive integers.

B. Under Option V_2

On the other hand, if $\left(T_R^{(2)}, y\right) \in \Omega_2 \setminus \Omega_1$, the retailer's optimal response is to choose Option V_2. Then the poultry farmer's locally maximum total profi per unit of time in $\Omega_2 \setminus \Omega_1$ is given as

$$P^{(2)^*} = \max_{N^{(2)} \in N} \hat{P}^{(2)} \left(N^{(2)}\right), \tag{29}$$

where

$$\hat{P}^{(2)} \left(N^{(2)}\right) = \max_{\left(T_R^{(2)}, y\right) \in \Omega_2 \setminus \Omega_1} P^{(2)} \left(N^{(2)}, T_R^{(2)}, y\right). \tag{30}$$

More precisely, we should use "sup" instead of "max" in Eq. (30).

For a given $N^{(2)}$, we show below the existence of the poultry farmer's optimal quantity discount pricing policy $\left(T_R^{(2)}, y\right) = \left(T_R^{(2)^*}, y^*\right)$ which attains Eq. (30). It can easily be proven that $P^{(2)} \left(N^{(2)}, T_R^{(2)}, y\right)$ in Eq. (25) is strictly decreasing in y, and consequently the poultry farmer can attain $\hat{P}^{(2)} \left(N^{(2)}\right)$ in Eq. (30) by letting $y \to \psi\left(T_R^{(2)}\right) + 0$. By letting $y = \psi\left(T_R^{(2)}\right)$ in Eq. (25), the total profi per unit of time on $y = \psi\left(T_R^{(2)}\right)$ becomes

$$\begin{aligned} & P^{(2)} \left(N^{(2)}, T_R^{(2)}\right) \\ & = (\mu_1 + \mu_2) \left(p_s + \frac{h_R}{\theta_R}\right) e^{\theta_R T_R^{(1)*}} - \frac{1}{N^{(2)} T_R^{(2)}} \\ & \quad \times \Bigg\{ Q\left(T_R^{(2)}\right) \Bigg[\textstyle\sum_{k=1}^{N^{(2)}-1} e^{-\alpha (k T_R^{(2)})^\beta} \\ & \qquad \times \left(c_s + h_s \int_0^{k T_R^{(2)}} e^{-\alpha t^\beta} dt\right) \\ & \quad + \left(N^{(2)} \frac{h_b}{\theta_R} + c_s\right) \Bigg] + \left(N^{(2)} a_R + a_s\right) \Bigg\}. \end{aligned} \tag{31}$$

By differentiating $P^{(2)} \left(N^{(2)}, T_R^{(2)}\right)$ in Eq. (31) with respect to $T_R^{(2)}$, we have

$$\begin{aligned} & \frac{\partial}{\partial T_R^{(2)}} P^{(2)} \left(N^{(2)}, T_R^{(2)}\right) \\ & \left\{ \begin{aligned} & \left[(\mu_1 + \mu_2) T_R^{(2)} e^{\theta_R T_R^{(2)}} - Q\left(T_R^{(2)}\right)\right] \\ & \times \Bigg[\left(N^{(2)} \tfrac{h_R}{\theta_R} + c_s\right) \\ & + \textstyle\sum_{k=1}^{N^{(2)}-1} e^{-\alpha (k T_R^{(2)})^\beta} \\ & \quad \times \left(c_s + h_s \int_0^{k T_R^{(2)}} e^{-\alpha t^\beta} dt\right) \Bigg] \\ & + Q\left(T_R^{(2)}\right) T_R^{(2)} \Bigg[h_s \tfrac{N^{(2)}(N^{(2)}-1)}{2} \\ & \quad - r\left(T_R^{(2)}\right) \textstyle\sum_{k=1}^{N^{(2)}-1} k^\beta e^{-\alpha (k T_R^{(2)})^\beta} \\ & \quad \times \left(c_s + h_s \int_0^{k T_R^{(2)}} e^{-\alpha t^\beta} dt\right) \Bigg] \end{aligned} \right\} \\ & - \left(N^{(2)} a_R + a_s\right) \\ & = - \frac{\qquad\qquad}{N^{(2)} \left(T_R^{(2)}\right)^2}. \end{aligned} \tag{32}$$

Let $L\left(T_R^{(2)}\right)$ express the terms enclosed in braces { } in the right-hand-side of Eq. (32).

We here summarize the results of analysis in relation to the optimal quantity discount policy which attains $\hat{P}^{(2)} \left(N^{(2)}\right)$ in Eq. (30) when $N^{(2)}$ is fi ed to a suitable value.

1) $N^{(2)} = 1$:

In this subcase, there exists a unique finit T_o $(> T_R^{(1)*})$ which maximizes $P^{(2)} \left(N^{(2)}, T_R^{(2)}\right)$ in Eq. (31), and therefore $\left(T_R^{(2)^*}, y^*\right)$ is given by

$$\left(T_R^{(2)^*}, y^*\right) \to \left(\tilde{T}_R^{(2)}, \tilde{y}\right), \tag{33}$$

where

$$\tilde{T}_R^{(2)} = \begin{cases} T_o, & T_o \le T_{\max}/N^{(2)}, \\ T_{\max}/N^{(2)}, & T_o > T_{\max}/N^{(2)}, \end{cases} \tag{34}$$

$$\tilde{y} = \psi\left(T_R^{(2)}\right). \tag{35}$$

The poultry farmer's total profi then becomes

$$\begin{aligned} \hat{P}^{(2)} \left(N^{(2)}\right) = & (\mu_1 + \mu_2) \big[(p_s + h_R/\theta_R) e^{\theta_R T_R^{(1)*}} \\ & - (c_s + h_R/\theta_R - \alpha) e^{\theta_R T_R^{(2)*}} \big]. \end{aligned} \tag{36}$$

2) $N^{(2)} \ge 2$:

Let us defin $T_R^{(2)} = \tilde{T}_R^{(2)}$ $(> T_R^{(1)*})$ as the unique solution (if it exists) to

$$L\left(T_R^{(2)}\right) = \left(a_R N^{(2)} + a_s\right). \tag{37}$$

In this case, the optimal quantity discount pricing policy is given by Eq. (33).

C. Under Option V_1 and V_2

In the case of $\left(T_R^{(2)}, y\right) \in \Omega_1 \cap \Omega_2$, the retailer is indifferent between Option V_1 and V_2. For this reason, this study confine itself to a situation where the poultry farmer does not use a quantity discount policy $\left(T_R^{(2)}, y\right) \in \Omega_1 \cap \Omega_2$.

ISBN: 978-988-19251-0-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE I
SENSITIVITY ANALYSIS

(a) Under Option V_1

c_s	$S^{(1)*}$	$N^{(1)*}$	$P^{(1)*}$	$Q_R^{(1)*}$	$x_1^{(1)}$	$x_2^{(1)}$
35	109.37	1	653.33	109.37	482.61	365.83
40	110.29	2	614.44	109.37	482.61	365.83
45	110.29	2	584.95	109.37	482.61	365.83
50	110.29	2	555.47	109.37	482.61	365.83

(b) Under Option V_2

a_s	$S^{(2)*}$	$N^{(2)*}$	$P^{(2)*}$	$Q_R^{(2)*}$	$x_1^{(2)}$	$x_2^{(2)}$
35	188.83	1	691.82	188.83	482.61	365.83
40	184.61	1	630.95	184.61	482.61	365.83
45	120.45	2	586.02	119.73	482.61	365.83
50	119.91	2	556.44	119.19	482.61	365.83

VII. NUMERICAL EXAMPLES

Table I reveals the results of sensitively analysis in reference to $S^{(j)^*}(= S(N^{(j)*}, T_R^{(j)*}))$, $N^{(j)^*}$, $P^{(j)^*}$, $Q_R^{(j)^*}(= Q(T_R^{(j)*}))$, $x_1^{(j)}$ and $x_2^{(j)}$ under Option V_j $(j = 1, 2)$ for $(p_b, p_s, h_s, a_s) = (200, 100, 20, 1, 1000)$, $(h_1, \theta_1, a_1, \mu_1) = (1, 0.013, 1200, 6)$, $(h_2, \theta_2, a_2, \mu_2) = (1.5, 0.015, 1300, 5)$ and $(\alpha, \beta, T_{max}) = (0.8, 0.8, 30)$ when $c_s = 35$, 40, 45 and 50. In this case, we obtain $\pi_1^{(1)^*} = 412.88$ and $\pi_2^{(1)^*} = 296.11$, which are independent of c_s.

In Table I(a), we can observe that $Q_R^{(1)*}$ takes a constant value ($Q_R^{(1)*} = 109.37$). Under Option V_1, the retailer R does not adopt the quantity discount offered by the poultry farmer. The poultry farmer cannot therefore control the retailer R's ordering schedule, which signifie that $Q_R^{(1)*}$ is independent of c_s. Table I(a) also shows that the values of both $S^{(1)*}$ and $N^{(1)*}$ jump up when c_s increases from 35 to 40 (more precisely, at the moment when c_s increases from 36.622 to 36.623). In the case of $N^{(1)*} = 2$, the poultry farmer ships the items to the retailer R twice in the farmer's single order cycle. The fowls in the second shipment are raised by the poultry farmer for relatively long time. Under this option, when c_s increases, the poultry farmer should make up for the loss by means of increasing the length of her/his order cycle, i.e., increasing the period of feeding.

Table I(b) indicates that, under Option V_2, $Q_R^{(2)*}$ is greater than $Q_R^{(1)*}$ (compare with Table I(a)). Under Option V_2, the retailer R accepts the quantity discount proposed by the poultry farmer. The poultry farmer's lot size can therefore be increased by stimulating the retailer R to alter her/his order quantity per lot through the quantity discount strategy.

Table I reveals that we have $P_1^* < P_2^*$. This indicates that using the quantity discount strategy can increase the poultry farmer's total profi per unit of time. We can notice in Table I that $x_i^{(1)} = x_i^{(2)}$ $(i = 1, 2)$ for each value of a_s. This signifie that the retailers' profit do not increase if they accept the quantity discount proposed by the poultry farmer.

VIII. CONCLUSION

In this study, we have discussed a quantity discount problem between a single poultry farmer and two retailers for ameliorating items. These items include the fast growing animals such as broiler in poultry farm. The poultry farmer purchases chicks from an upper-leveled supplier and then feeds them until they grow up to be fowls. The retailers purchase (raw) chicken meat from the poultry farmer. The stock of the poultry farmer increases due to growth, in contrast, the inventory levels of the retailers are depleted due to the combined effect of its demand and deterioration. The poultry farmer is interested in increasing her/his profi by controlling the retailers' order quantity through the quantity discount strategy. The retailers attempt to maximize their profit considering both whether to cooperate with each other and whether to accept the poultry farmer's proposal. The analysis with respect to comparing the cooperative solution with non-cooperative one becomes considerably complicated since the local maximum values of the players' total profi per unit of time cannot be expressed as closed form expressions. For this reason, we have shown that the retailers can increase their profit by means of adopting the cooperative strategy in the case where higher order terms of the deterioration rate in the expansion of the exponential can be ignored. Focusing on such a situation, the poultry farmer can regard the retailers as a single retailer since either of the retailers is in charge of ordering and inventory management. In this case, we can formulate the above problem as a Stackelberg game between the poultry farmer and the retailers in the same manner as our previous formulation[7]. It should be pointed out that our results are obtained under the situation where the inventory holding cost is independent of the value of the item. The relaxation of such a restriction is an interesting extension.

REFERENCES

[1] J. P. Monahan, "A quantity discount pricing model to increase vendor's profit" *Management Sci.*, vol. 30, no. 6, pp. 720–726, 1984.
[2] M. Data and K. N. Srikanth, "A generalized quantity discount pricing model to increase vendor's profit" *Management Sci.*, vol. 33, no. 10, pp. 1247–1252, 1987.
[3] M. J. Rosenblatt and H. L. Lee, "Improving pricing profitabilit with quantity discounts under fi ed demand," *IIE Transactions*, vol. 17, no. 4, pp. 338–395, 1985.
[4] H. L. Lee and M. J. Rosenblatt, "A generalized quantity discount pricing model to increase vendor's profit" *Management Sci.*, vol. 32, no. 9, pp. 1177–1185, 1986.
[5] M. Parlar and Q. Wang, "A game theoretical analysis of the quantity discount problem with perfect and incomplete information about the buyer's cost structure," *RAIRO/Operations Research*, vol. 29, no. 4, pp. 415–439, 1995.
[6] S. P. Sarmah, D. Acharya, and S. K. Goyal, "Buyer vendor coordination models in supply chain management," *European Journal of Operational Research*, vol. 175, no. 1, pp. 1–15, 2006.
[7] H. Kawakatsu, T. Homma, and K. Sawada, "An optimal quantity discounting pricing policy for ameliorating items," *Lecture Notes in Engineering and Computer Science: Proceedings of the International MultiConference of Engineers and Computer Scientists 2012, IMECS 2012*, pp. 1549–1554, 2012.
[8] H. S. Hwang, "A study on an inventory model for items with weibull ameliorating," *Computers & industrial engineering*, vol. 33, pp. 701–704, 1997.
[9] B. Mondal and A. K. Bhunia, "An inventory system of ameliorating items for price dependent demand rate," *Computers & industrial engineering*, vol. 45, pp. 443–456, 2003.
[10] S. Y. Chou and W. T. Chouhuang, "An analytic solution approach for the economic order quantity model with weibull ameliorating items," *Mathematical and computer modeling*, vol. 48, pp. 1868–1874, 2008.
[11] M. J. Osborne and A. Rubinstein, *A Course in game theory*. The MIT Press, Massachusetts, 1994.
[12] J. Eichberger, *Game theory for economists*. Academic Press, California, 1993.