NARMAX_OLS Representation of a Semi-active Dynamic Leg Joint Model for a Paraplegic Subject Using Functional Electrical Stimulation

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Abstract—**A nonlinear autoregressive moving average with exogenous input (NARMAX) model structure is used to develop paraplegic dynamic leg joint model and the result is compared to a model built using artificial neural network (ANN). A series of experiments employing Functional Electrical Stimulation (FES) with varying pulse width are conducted to determine the impact of the leg swing angle. The data obtained is used to develop the paraplegic leg joint model. Seven sets of data with 3300 data in each are used to develop the joint model. The joint model thus developed is validated by experimental data from one paraplegic subject. It is revealed that ANN and NARMAX models both can represent the joint dynamics but NARMAX joint model is found to be highly accurate and able to predict the leg movement with better precision as compared to the ANN joint model. In order to improve the performance and the precision of the model, NARMAX is used with orthogonal least square (OLS) algorithm. The established model is then used to predict the behaviour of the underlying system and will be employed in future for the design and evaluation of various control strategies.**

Index Terms— **Functional electrical stimulation; joint mode; NARMAX; NARMAX_OLS; ANN; paraplegic; spinal cord injury.**

I. Introduction

Paraplegic is impairment in motor and/or sensory function of the lower extremities. It is usually the result of spinal cord injury (SCI), which affects the neural elements of the spinal canal. In United Kingdom, incidents of SCI are 10 to 15 per million people per annum, resulting in 600 to 900 new cases each year [1]. Sisto et al [2] have reported that more than 200,000 people in the United States suffer from SCI and each year 10,000 new cases occur.

Functional electrical stimulation (FES) is a means of producing contraction in paralyzed muscle due to central nervous system lesion by utilizing electrical stimulation [3]. However, due to the reverse recruitment order, the paralyzed muscles tend to fatigue rapidly and the force decline is steeper. In a FES activity muscle fatigue is affected by several factors including stimulation parameters, pattern of stimulation, method of stimulation and the task to be performed [4]. To ensure safety in open -loop control of FES muscle is normally over stimulated which results in greater fatigue [5].

To improve the quality of control method and reduce the effect of muscle fatigue, closed-loop control methods have been extensively reviewed [6-7]. Moreover, modeling of muscle behavior under electrical stimulation considering fatigue and recovery has gained a lot of momentum. To realize an accurate closed-loop control system, a precise dynamic model of the muscle or the joint is needed to recalculate the best trajectory to complete a desired task. The controller then would be able to adjust the controlled parameters in order to finish the task by reduced fatigue. Therefore, development of a joint model to predict the behavior of the system and use it as a feedback is an important step prior to the implementation of the movement synthesis and associated control strategy.

The human knee joint has been modeled extensively [8-11]. Most of the reported models have limited prediction capabilities, since they describe the knee joint under very restricted conditions and contain too many parameters, making them unidentifiable when only the state variables are known. Eom et al. [9] have developed their joint model considering joint recruitment feature, activation dynamics and contraction dynamics, where all the muscle fiber parameters have to be estimated. These values are difficult to measure and require special equipment and experimental procedures. They estimated these values to calculate their joint model. Franken [10] introduced a simpler method using least square algorithm in combination with Levenburg-Marquardt algorithm to develop a joint model. However, developed model failed to track the leg joint for movements above 10Hz [10].

Ibrahim [11] has a similar approach for their joint model. The joint model is developed using fuzzy logic by optimizing the model parameters and genetic algorithm to fit the model output to the experimental leg motion. The developed model is not generalized and is completely subject based. Moreover, it is not been cost effective and the technique is limited and not reliable.

This paper presents the development of a dynamic leg joint model using NARMAX_OLS ERR [12] method to represent the actual joint behavior of paraplegics and it will be used in computer simulation, also it can be used for feedback or feed forward control in practical applications before FES can be applied in a practical. The sitting joint model is developed using input/output data sets. As mentioned before, two modeling strategies are used to model the leg joint and the results achieved with these strategies are discussed.

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II. Material

A. Subject

A 1.73 meter paraplegic subject weighing 80 Kg and aged 45 years, was involved in the data collection after informed consent. The subject was injured at T2/T3 level and suffered from the lesion for 29 years.

B. Apparatus and Setup

Throughout the experiments, paraplegic subject is placed in a semi-upright sitting position (45º) with the thigh hanging using thigh support on a frame to avoid any constraint on the leg movement. Velcro straps are used to stabilise the subject's upper trunk, waist and thigh. The knee angle of the leg movement is recorded via a goniometer (Biometrics Ltd, UK), Fig 1 shows the position of goniometer. The position of the leg is recorded instantaneously using Matlab software through analogue to digital converter (ADC) card and serial connection.

C. Data Collection

Electrical stimulation is delivered via two MultiStick™ gel surface electrodes (Pals platinum, Axelgaard Mfg. Comp, USA, 50mm x 90mm). The cathode is positioned over the upper thigh, covering the motor point of rectus femoris and vastus lateralis. The anode is placed over the lower aspect of thigh, just above patella. Prior to each test, the electrodes are tested for suitable placement on the muscle by moving the electrode about the skin over the motor point, looking for the maximum muscular contraction using identical stimulation signal for the entire trials. A RehaStim Pro 8 channels (Hasomed GmbH, Germany) stimulator receives stimulation pulses generated in Matlab software through USB connection for application to the muscle. More than 1200 stimulation pulses with 10 Hz frequency and pulse widths varying from 200µsec to 350sec are used to develop the joint model.

III. Methodology

Most of the natural phenomena are highly nonlinear. There are two approaches to model dynamics of these types of systems. The first method is to use physical laws in order to design a model for the system where the second is to utilize system identification techniques. However, the first method is not easy to implement due to three main reasons [13]: Setting the right values for physical parameters in order to have specific model, difficulties in identification of physical parameters from data samples and complexity of the model.

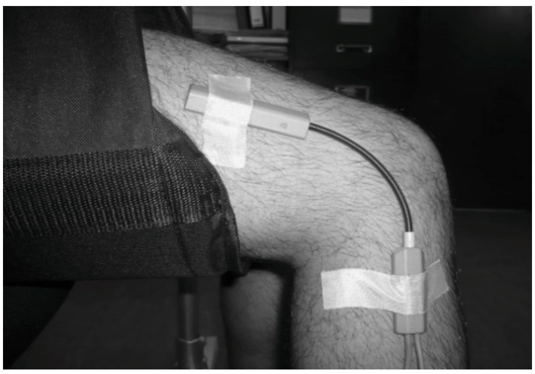

Fig 1. Experimental Setup

For the second approach, a number of methods have been produced, some of which assume that the whole system is like a black box. In these methods, there is no need to have a lot of information about the physics of the system. These methods utilize mapping between the inputs and output [14].

A. NARMAX and NARMAX_OLS

The nonlinear autoregressive moving average with exogenous input (NARMAX) model structure is commonly used to model nonlinear dynamics and is a good candidate of the black box modeling approach. It is able to represent a wide range of functions around their equilibrium point. Moreover, for improved performance and precision, the orthogonal least squares (OLS) algorithm [15] is employed. In this approach the output can be considered as a nonlinear function *f(.)* of inputs, output and error. The greatest challenge in writing this function is to find the most important terms that should be included. The OLS algorithm searches for the most important terms, that should be considered in *f(.)* as the most significant terms on an orthogonal basis [16].

The NARMAX-OLS algorithm includes three different procedures: parameter estimation, structure selection and model validation. Finding the most significant terms, which affect the output, is a part of structure selection procedure, and this is the most important and complicated part of the NARMAX-OLS algorithm. It depends on different parameters, such as sampling frequency, prior knowledge of the system etc. In this article, the error reduction ratio (ERR) method, which is commonly employed and is very useful in terms of system identification, is used [12].

B. Polynomial NARMAX Model Estimation

The nonlinear relationship between output and input can be expressed as:

$$\begin{aligned} y(t) = f(&y(t-1),\ldots,y(t-n_y),u_1(t-1),\\ &\ldots,u_1(t-n_{u_1}),\ldots,u_m(t-1),\ldots,u_m(t-n_{u_m}),\\ &e(t-1),\ldots e(t-n_e))+e(t) \end{aligned} \quad (1)$$

Where y, u and e are the output, input and error respectively, and n_u, n_y, n_e represent the maximum lags in the output, input and error, respectively. Noise terms included in this model can avoid bias and take into consideration uncertainties and un-modeled dynamics. Billings and Tsang [17] showed that the NARMAX algorithm is able to present a satisfactory model for every system around an equilibrium point, if these conditions are met: 1- the system should have a finite and realizable response; 2- A linear model should exist around the system's equilibrium point.

It is shown that the final model will be:

Long calculations to reach the final estimated model result in:

$$\hat{y}(t) = \phi_{yu}^T(t-1)\hat{\Theta}_{yu} + \phi_{yu\xi}^T\hat{\Theta}_{yu\xi} + \phi_{\xi}^T(t)\hat{\Theta}_{\xi} + \xi(t) \quad (2)$$

Where ϕ_{yu}^T is a matrix includes all the linear and nonlinear combinations of input and output terms up to and including time (*t-1*) with the maximum degree of *l*. $\hat{\Theta}_{yu}$ represents the parameters corresponding to these terms. $\xi(t)$ stands for residuals which can be defined as:

$$\xi(t) = y(t) - \hat{y}(t) \tag{3}$$

To clarify 2, it is better to rewrite this equitation as:

$$\hat{y}(t) = \phi^T(t-1)\hat{\Theta} + \xi(t) \tag{4}$$

To solve this equation, $\hat{\Theta}$ value has to be estimated, meaning that a cost function must be defined. It is clear that cost function should depend on $\xi(t)$. A good choice for cost function is:

$$\mathrm{c}(\hat{\Theta}) = \left\| y(t) - \phi^T(t-1)\hat{\Theta} \right\| \tag{5}$$

In equation 5 $\| \;\; \|$ denotes the Euclidean norm.

Chen et al. [18] suggested three possible approaches to solving this problem. The most common method is the orthogonal estimation algorithm.

C. Orthogonal Estimation Algorithm

Complexity of a large scale system which has an *m*-dimensional space, can be reduced by transforming the m-dimensional space into *m* different one-dimensional spaces. This idea can be applied into estimation problems. It makes processing easier and results in an independent computation of each coefficient [19].

y(t) can be expressed as:

$$y(t) = \sum_{i=0}^{n_\theta} \theta_i p_i(t) + \xi(t) \tag{6}$$

Where n_θ, p_i, θ_i represent the number of coefficients, regressors and coefficients respectively. This equation can be rewritten in other forms, including orthogonal terms. Therefore a transformation capable of mapping equation 6 onto the orthogonal form is required. The general form of transformed equation, should appear as:

$$y(t) = \sum_{i=0}^{n_\theta} g_i \omega_i(t) + \xi(t) \tag{7}$$

Where $\omega_i(t)$'s are orthogonal regressors and g_i 's are coefficients. All the $\omega_i(t)$ values should satisfy orthogonality over the allotted time. Therefore the following conditions should be met:

$$\frac{1}{N}\sum_{t-1}^{N} \omega_i(t)\omega_{j+1}(t) = 0, \qquad i = 1,2,....j \tag{8}$$

Where *N* represents the number of data points. To find ω_m, these relationships can be used:

$$\omega_0(t) = p_0(t) = 1$$
$$m = 1,2,....,n_\theta$$
$$\omega_m(t) = p_m(t) - \sum_{i=0}^{m-1} a_{im}\omega_i(t) \tag{9}$$

Here a_{im} is:

$$a_{im} = \frac{\sum_{t=1}^{N} p_m(t)\omega_i(t)}{\sum_{t=1}^{N} \omega_i(t)^2}, \qquad 0 \le i \le m-1 \tag{10}$$

To find the coefficients in equation 7, the following relationships are useful [17, 20].

$$\hat{g}_i = \frac{1}{N}\sum_{t=1}^{N} \frac{y(t)\omega_i(t)}{\omega_i^2(t)}, \qquad i = 1,2,....n_\theta \quad \omega_i^2(t) \ne 0 \tag{11}$$

The following can transform back the coefficients to the original form:

$$\hat{\theta}_i = \sum_{i=m}^{n_\theta} \hat{g}_i v_i, \qquad m = 1,2,....,n_\theta$$
$$v_m = 1$$
$$v_i = -\sum_{r=m}^{i-1} a_{ri} v_r, \qquad i = m+1,....,n_\theta$$
$$a_{ij} = \frac{1}{N}\sum_{t=1}^{N} \frac{\omega_i(t) p_j(t)}{\omega_i^2(t)}$$
$$i = 1,2,....,j-1, \quad j = 2,....,n_\theta \tag{12}$$

Chen et al. [18] studied four different methods for orthogonal decompositions and suggested the modified Gram-Schmidt model. This model was preferred over the others available because of its greater precision.

D. Structure Selection

One of the drawbacks of the polynomial NARMAX model is the large number of regressors, which are produced after the first step. Even when all the regressors for the small values of n_y, n_u, n_e and *l* are calculated, there are plenty of terms that can be used. For example, if there are just 14 terms (regressors), then 16384 (2^{14}) different models can be produced. It is not possible to check all these models to find the most appropriate one. One does not know how many terms are necessary to model the principle dynamics of the system. This is similar to the problem that occurs when neural networks are used to model a system. Usually, there is a tradeoff between generalization and over-fitting. The network should be able to predict the behavior of the system for an unseen input, and it should also model the small dynamics of the system. In order to model the small dynamics, as many terms as possible should be included. However, this leads to the problem of over-fitting. In addition, this may produce some dynamic regimes, which do not exist in the original system. This network would also not have a good performance when using unseen data. The same problem occurs when using the polynomial NARMAX model [21]. It is rare that all the terms are necessary to form a nonlinear model. Usually, no more than 10 terms are

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required for creating a good model. Even after finding out how many terms are necessary, the question remains as to which terms should be selected. The terms with the most significant effects should be selected and the others eliminated. Several methods have been suggested for solving this problem [22]. They can be grouped into two major categories: 1- Constructive techniques, 2- Elimination techniques. Both groups have the same goal but use different approaches. Constructive techniques start by gradually making and improving the model while elimination techniques try to eliminate less important terms from the initial all-encompassing model. Most of these techniques use statistical analysis to decide which terms should be added or eliminated.

Draper and Smith [23] and Billings and Voon [24] suggested a stepwise method, which became a basis for future research. Korenberg et al. [20] and Billings et al. [17] introduced a new method that became very popular in subsequent years, called the error reduction ratio (ERR) method. This method has considerable advantages and is able to facilitate analysis without the need for all the terms to be included, and is compatible with the orthogonal estimation algorithm.

Following a short review, the ERR test can be explained as below:
Multiplying equation 7 by itself and calculating the average of the whole equation gives:

$$\frac{1}{N}\sum_{t=1}^{N} y^2(t) = \frac{1}{N}\sum_{t=1}^{N}\{\sum_{i=0}^{n_\theta} g_i^2\omega_i^2(t)\} + \sum_{t=1}^{N}\xi^2(t) \qquad (13)$$

Assuming that $\xi(t)$ is the zero mean average and the orthogonality of $\omega_i(t)$, gives:

$$\frac{\frac{1}{N}\sum_{t=1}^{N}\{\sum_{i=0}^{n_\theta} g_i^2\omega_i^2(t)\}}{\frac{1}{N}\sum_{t=1}^{N} y^2(t)} \le 1 \qquad (14)$$

Which can be rewritten as:

$$\sum_{i=1}^{n_\theta}\frac{\frac{1}{N}\sum_{t=1}^{N} g_i^2\omega_i^2(t)}{\frac{1}{N}\sum_{t=1}^{N} y^2(t)} \le 1 \qquad (15)$$

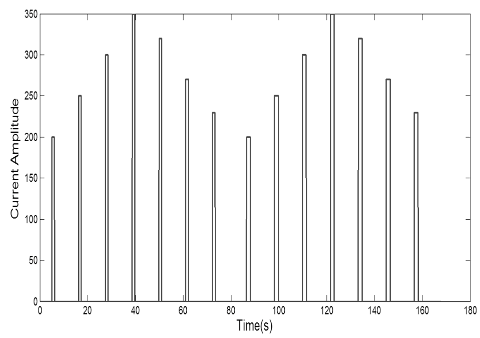

Fig 2. Input to the muscle

$$\text{If} \quad ERR_i = \frac{\frac{1}{N}\sum_{t=1}^{N} g_i^2\omega_i^2(t)}{\frac{1}{N}\sum_{t=1}^{N} y^2(t)} \qquad (16)$$

Then equation 15 can be rewritten as:

$$\sum_{i=1}^{n_\theta} ERR_i \le 1 \qquad (17)$$

In fact, when no terms are put into the model, this results in the maximum squared error and its value is:

$$\frac{1}{N}\sum_{t=1}^{N} y^2(t) \qquad (18)$$

When one term is added to the model, it reduces the total error by:

$$\frac{1}{N}\sum_{t=1}^{N} g_i^2\omega_i^2(t) \qquad (19)$$

Therefore, to find the significance of each term in comparison to the other terms, it is necessary to find its error reduction value as a fraction of the total squared error.

IV. Results

Knee modeling provides a better perception of the mechanisms of muscular and knee functions. Linear approaches are not appropriate for knee modeling due to the nonlinear behavior of muscle. In order to model the knee joint, experimental data were obtained from an SCI subject and used. The subject was seated on a chair, and his foot had enough space to move freely. A functional electrical stimulator stimulated the quadriceps muscles of his right leg, and the knee angle was recorded by a goniometer. One of the drawbacks of recording swinging leg by tracking goniometer angle is drifting from set value after some time due to the leg movement. Angle of 90 degree between the shank and foot was considered as zero degree. Displacements in the dorsiflexion direction were considered as positive and those in the plantar flexion direction as negative. The experimental data was sampled every 0.05s using Hasomede device. To avoid muscle spasm, the electrical input current was limited to 40 mA for maximum period of 1s. The input to the muscle stimulator is shown in Fig 2, which is electrical current applied to the subject's muscle. The higher amplitude means higher current stimulation level, and resulting in broader leg movement. Fig 3 shows the knee angle recorded by the goniometer.

The NARMAX-OLS-ERR method and an MPL neural network were used to find a model for the knee angle. The experimental data used in these studies were not filtered or prepossessed. Several different types of neural networks were trained in order to find the optimum model [25]. Finally, a two layer MLP network including four neurons in each layer with five inputs was selected from the trained neural networks. To model the knee dynamics *u(t), u(t-1), u(t-2), y(t-1)* and *y(t-2)* were used to form the input data structure to the neural network. The network was

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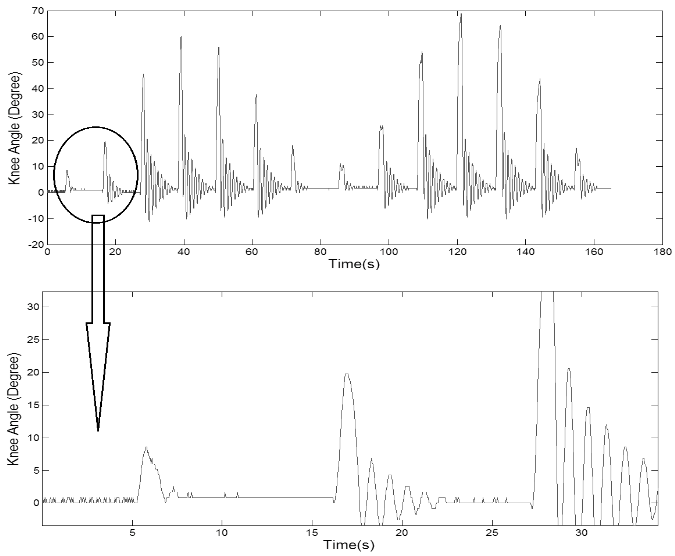

Fig 3. Recorded data from subject's knee angle

trained using back propagation method [25]. Fig 4 compares the neural network output and the experimental data. It is clear that the neural network has modeled the system dynamics with high accuracy. In order to run the simulation, *y(t-1)* and *y(t-2)* were stored offline and fed to the network during the simulation. The major problem with neural network model is that it is unable to predict the knee angle accurately as shown in Fig 5. In this figure *y(t-1)* and *y(t-2)* were generated online by delaying the predicted output from the neural network. It is obvious that the neural network cannot be used for long horizon predictions.

To overcome this limitation and find a simpler model that can predict the output correctly, NARMAX-OLS-ERR algorithm was used. The following model was suggested by this method:

$$y(t) = K_1 y(t-1) + K_2 y(t-2) + K_3 u(t-2)^2 + K_4 y(t-1)^2 + K_5 y(t-1) y(t-2) \quad (20)$$

The terms and coefficients are summarized in Table 1.

Table I. NARMAX parameters

$(t-1)$	k1= 1.78
$y(t-2)$	k2= 8.57e-01
$u(t-2)^2$	k3 = 2.947665e-06
$y(t-1)^2$	k4= 3.222674e-02
$y(t-1)y(t-2)$	k5= 2.974369e-02

The main advantage of using NARMAX-OLS-ERR method is that it has good prediction error. Fig 5 shows the predicted output of the nonlinear model. Similar to neural network model, *y(t-1)* and *y(t-2)* were generated online by delaying the predicted output from the nonlinear model. It is obvious that the system had very good prediction performance, although there was accumulation error. A delay was noted between the real-time data and the NAMAX-OLS-ERR model's prediction after 50s. As shown in Fig 5, the NARMAX model has very good robustness to the system noise. The experimental data that was used for NARMAX-OLS-ERR algorithm was not filtered or processed. It is clear from the equation 20 that the NARMAX-OLS-ERR model has just few terms, which makes the implementation of the model remarkably easy in experimental studies.

V. DISCUSSION AND CONCLUSION

The human leg muscles have been modelled extensively. Knee joint is also considered by some methods such as mathematical models or genetic algorithm [11]. Prediction of knee dynamics using simple model and identifiable parameters is the most significant focus of all studies. In this paper, a ANN joint model, and NARMAX_OLS_ERR have been studied and their parameters are compared using FES data recorded from paraplegic muscle.

Most of the reported models have limited prediction capabilities, since they describe the knee joint under very restricted conditions and contain too many parameters, our results show that knee modeling using NARMAX_OLS_ERR generates a robust and simple model which has much better prediction horizon. This make this model suitable for predictive control methods where having a robust model with an accurate prediction is necessary. The generated model is easy to linearize and easy to implement compared with the models generated by neural networks or fuzzy methods. It can also be integrated with linear control methods like model predictive control.

It has been demonstrated that NARMAX_OLS_ERR could improve the prediction and follow the dynamics pattern, at the expense of a delay. So shortening this delay, extending the final equation to include parameters to model fatigue would be future studies. However, the developed model is robust to long-range prediction for the subject, but with more data sets from different paraplegics the model may be generalized.

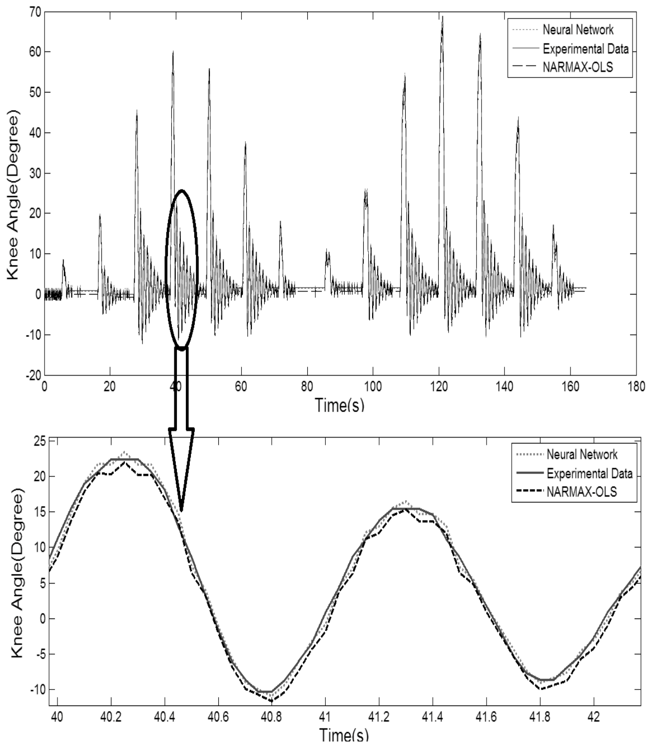

Figure 4. Trained Neural Network and NARMAX

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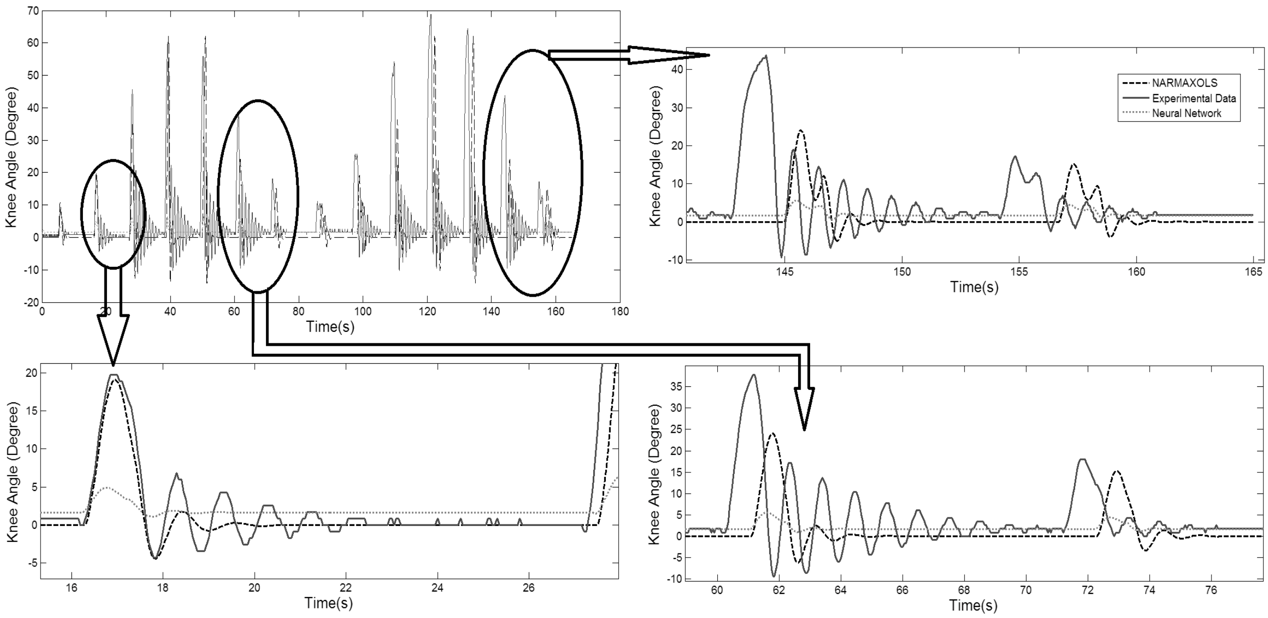

Figure 5. Comparison between long-range prediction by Neural Network model and NARMAX model

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