A Non-linear Robust Control of a Multi-purpose Earthquake Simulator

S. Strano and M. Terzo

Abstract— **A non-linear robust control of a multi-purpose earthquake simulator has been designed and experimentally tested. The test rig is characterized by a double functionality based on two configurations of the hydraulic actuation system. Due to the several operating conditions, the system is affected by structured and unstructured uncertainties that require a robust approach for the control of the position. Starting from a non-linear dynamic model, a sliding control is developed taking into account the incomplete knowledge of the system. The experimental results highlight the goodness of the proposed control in terms of stability and tracking error.**

Index Terms— **Earthquake simulator, shaking table, hydraulic actuator, seismic isolator**

I. INTRODUCTION

THIS paper concerns a new hydraulically actuated multi-purpose earthquake simulator finalized to execute both shaking table test and seismic isolator characterization. The versatile earthquake simulator is essentially constituted by a hydraulically actuated shaking table and a suitable reaction structure.

The hydraulic actuation system exhibits significant non-linear behaviour [1] due to the pressure-flow rate relationship, the dead band of the control valve and frictions [2]; these non-linearities make the mathematical model more complex and, at the same time, highly limit the performance achieved by the classical linear controller [3 – 6]. Aside from the non-linear nature of the dynamics, the hydraulic systems also have a large extent of model uncertainties. The uncertainties can be classified into two categories: structured (parametric uncertainties) and unstructured. Examples of parametric uncertainties include the large changes in load seen by the system in industrial use and the large variations in the hydraulic parameters (e.g., bulk modulus) due to the change of temperature and component wear (Whatton, 1989). The unstructured uncertainties are typically caused by a simplified representation of the system dynamics. Indeed, actions due to external disturbances cannot be modelled exactly or can be neglected. These model uncertainties can lead to an unstable behaviour of the controlled system or a very degraded performance.

The earthquake simulator is characterized by two different hydraulic schemes that can be selected in dependence of the test that has to be executed. This determines an induced structured uncertainty. Moreover, the seismic isolator and the test structure can largely influence the controlled system performances due to very large restoring force or neglected dynamics (e.g. structural modes) that cause an induced unstructured uncertainty. Consequently, differently from a common approach substantially based on the parametric imprecision, the present paper presents the design and the experimental testing of a sliding control starting from a specific experimental need based on a variable structure system with strongly different operating conditions that determine a variability in terms of system structure and parameters (geometric and not) characterized by a large extent.

In order to meet this requirement, the basic idea is the choice of the non-linear hydraulic actuator model, with friction and dead band, as nominal one and characterized by external disturbances caused by seismic isolator or test structures.

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II. THE EARTHQUAKE SIMULATOR

The earthquake simulator consists of movable and fixed parts made in structural steel. Particularly, it is constituted by:

- fixed base;
- hydraulically actuated sliding table with dimensions 1.8 m x 1.6 m;
- hydraulic actuator.

The table motion is constrained to a single horizontal axis by means of recirculating ball-bearing linear guides.

The hydraulic power unit consists of a variable displacement pump powered by a 75 kW AC electric motor and able to generate a maximum pressure of 210 bar and a maximum flow rate equal to 313 l/min. A pressure relief valve is located downstream of the pump.

The hydraulic circuit consists of a four way-three position proportional valve and a hydraulic cylinder. The cylinder is constituted by two equal parts separated by a diaphragm and contains two pistons which rods are connected to the base; so, the actuator is characterized by a mobile barrel and fixed pistons. The maximum horizontal force is 190kN, the maximum speed 2.2m/s and the maximum stroke 0.4 m (± 0.2 m). Three way valves allow to select different configurations of the test rig activating different thrust area (active surface) and control volume. The selection of the maximum thrust configuration and the installation of suitable reaction structures allow the bench to be employed as seismic isolator test rig; conversely, the removal of the reaction structures, together with the selection of the maximum speed configuration, allows the earthquake simulator to be used as shaking table.

III. Nominal Model Derivation

The modelling refers to the testing machine in which no isolator or test structure is installed: the hydraulic cylinder has to move the sliding table only.

The modelling procedure is based on the following hypothesis: a) fluid properties not depending on the temperature; b) equal piston areas; c) equal chamber volume for each side in the case of barrel in the centred position; d) negligible internal and external fluid leakages.

The test rig is modelled as a single DOF system subjected to both actuation and friction force and can be considered equivalent to a double-ended hydraulic actuator, driven by a four-way spool valve, and with trust area and control volume depending on the selected configuration (Fig. 1).

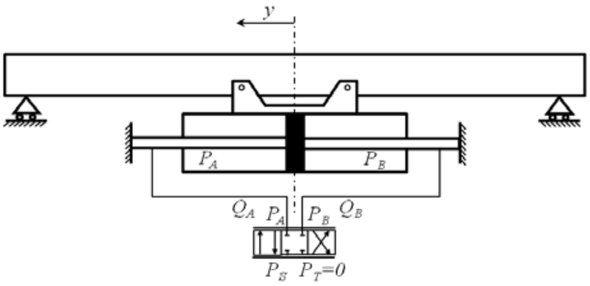

Fig. 1. Scheme of the hydraulic actuator adopted for the mathematical model.

The dynamics of the sliding table can be described by:

$$m\ddot{y} + \sigma\dot{y} + F_c \operatorname{sgn}(\dot{y}) + \mu N \operatorname{sgn}(\dot{y}) = A_p P_L \tag{1}$$

where

y is the table displacement;

m is the movable mass;

F_f is the friction force due to the hydraulic actuator and linear guides;

$P_L = P_A - P_B$ is the load pressure;

P_A and P_B the pressures in the two chambers;

σ is the viscous friction coefficient;

F_c is the Coulomb friction force in the hydraulic actuator;

μ is the Coulombian friction coefficient of the linear guides;

N is the vertical load on the linear guides.

The actuator dynamics can be written as [1]:

$$\frac{V_0}{2\beta}\dot{P}_L = -A_p\dot{y} + Q_L \tag{2}$$

where

V_0 is the volume of each chamber for the centred position of the barrel;

β the effective bulk modulus;

A_p is the ram area;

$Q_L = (Q_A+Q_B)/2$ is the load flow;

Q_A and Q_B are, respectively, the supplied flow rate and the return flow rate of the proportional valve.

An overlapped four-way valve is considered: this kind of valve is typically characterized by the lands of the spool greater than the annular parts of the valve body. Consequently, the flow rate is zero (dead band) when the spool is in the neighbourhood of its central position. Moreover, since the adopted valve is characterized by a high response, it is assumed that the control applied to the spool valve is directly proportional to the spool position. Under the assumption of a tank pressure P_T equal to zero, the load flow depends on the supply pressure, the load pressure and the input voltage in accordance with the following:

$$Q_L = DZ(u)\sqrt{P_s - \tilde{v}_e P_L} \tag{3}$$

where u is the input voltage, $DZ(u)$ is the dead band function and $\tilde{v}_e$ defined as

$$\tilde{v}_e = \begin{cases} 1 & if \quad u > 0 \\ 0 & if \quad u = 0 \\ -1 & if \quad u < 0 \end{cases} \tag{4}$$

Without loss of generality, the slope of the static map beyond the dead band region can be assumed equal and the analytical expression of $DZ(u)$ is:

$$DZ(u) = \begin{cases} K_Q(u-u^+) & if \quad u > u^+ \\ 0 & if \quad u^- \le u \le u^+ \\ K_Q(u-u^-) & if \quad u < u^- \end{cases} \tag{5}$$

where u^+ and u^- are the limits of the dead band and K_Q the slope.

Defined the state vector as $\mathbf{x}=[\dot{y} \quad y \quad P_L]^T$, the system (sliding table + hydraulic actuator) is given by the following third order model non-linear in the state:

$$\begin{cases} m\ddot{y} = -(\mu N + F_C)\operatorname{sgn}(\dot{y}) - \sigma\dot{y} + A_p P_L \\ \dfrac{V_0}{2\beta}\dot{P}_L = -A_p\dot{y} + DZ(u)\sqrt{P_s - \tilde{v}_e P_L} \end{cases} \tag{6}$$

IV. Sliding Mode Control

The nominal model (6) has been adopted for the sliding control design.

The discontinuous nonlinearities in the friction force can be smoothly approximated taking into account that:

$$\operatorname{sgn}(\dot{y}) = \frac{2}{\pi}\arctan(\gamma\dot{y}) \tag{7}$$

where γ is the approximation parameter.

Differentiating the first equation of (6) and taking into account that the dead band can be expressed as

$$DZ(u) = K_Q u + S(u) \tag{8}$$

with

$$S(u)=\begin{cases}-K_Q u^+ & if \quad u>u^+\\ -K_Q u & if \quad u^- \le u \le u^+\\ -K_Q u^- & if \quad u<u^-\end{cases} \tag{9}$$

the following single expression is obtained for the nominal plant:

$$\begin{aligned}\dddot{y}=&-\left(\frac{2\gamma}{\pi}\frac{\mu N+F_C}{m(1+\gamma^2\dot{y}^2)}+\frac{\sigma}{m}\right)\ddot{y}-\frac{2A_P{}^2\beta}{mV_0}\dot{y}+\\ &+\frac{2A_P\beta K_Q}{mV_0}\sqrt{P_S-\tilde{v}_e P_L}\,u+\frac{2A_P\beta}{mV_0}\sqrt{P_S-\tilde{v}_e P_L}\,S(u)\end{aligned} \tag{10}$$

At this step the plant model can be synthetically expressed as:

$$\dddot{y}=-\alpha_1\ddot{y}-\alpha_2\dot{y}+\alpha_3 u+\frac{\alpha_3}{K_Q}S(u) \tag{11}$$

where

$$\alpha_1=\frac{2\gamma}{\pi}\frac{\mu N+F_C}{m(1+\gamma^2\dot{y}^2)}+\frac{\sigma}{m} \tag{12}$$

$$\alpha_2=\frac{2A_P{}^2\beta}{mV_0} \tag{13}$$

$$\alpha_3=\frac{2A_P\beta K_Q}{mV_0}\sqrt{P_S-\tilde{v}_e P_L} \tag{14}$$

So the single-input dynamic system (11) is not exactly known but affected by uncertainties.

Taking into account the realistic hydraulic system together with the practical seismic isolator and test structures, the following realistic assumption is made.

Assumption. The modelling uncertainties (intrinsic and induced) in (11) are all bounded.

Given the table target displacement y_T, the objective is to design a bounded control input u so that the current table displacement y tracks as closely as possible the desired motion in spite of various model uncertainties, including parametric uncertainties and neglected dynamics due to both physical changing in the plant configuration and induced disturbances (i.e. seismic isolator or test structure).

The sliding mode control design procedure starts from the definition of a suitable sliding surface. With the intention of keeping stability conditions and improving closed loop system performance, the following sliding surface is defined [7]:

$$s=\left(\frac{d}{dt}+\lambda\right)^2 e=\ddot{e}+\lambda^2 e+2\lambda\dot{e} \tag{15}$$

being λ a strictly positive constant and $e=y-y_T$ the tracking error.

The dynamics in sliding mode can be written as:

$$\dot{s}=\dddot{e}+\lambda^2\dot{e}+2\lambda\ddot{e}=\dddot{y}-\dddot{y}_T+\lambda^2\dot{e}+2\lambda\ddot{e}=0 \tag{16}$$

Once the sliding surface is reached, the motion should continue on this surface with the application of the equivalent control law u_{eq} that is determined solving formally equation (16) referred to the nominal system

$$u_{eq}=\frac{\dddot{y}_T+\hat{\alpha}_1\ddot{y}+\hat{\alpha}_2\dot{y}-\lambda^2\dot{e}-2\lambda\ddot{e}}{\hat{\alpha}_3}-\frac{S(u_{ND})}{\hat{K}_Q} \tag{17}$$

where the superscript $\hat{\bullet}$ refers to the nominal parameter and $S(u_{ND})$ is the S function (9) evaluated for the equivalent control action with no dead band u_{ND} given by:

$$u_{ND}=\frac{\dddot{y}_T+\hat{\alpha}_1\ddot{y}+\hat{\alpha}_2\dot{y}-\lambda^2\dot{e}-2\lambda\ddot{e}}{\hat{\alpha}_3} \tag{18}$$

In reality, system uncertainties make the state trajectories to oscillate in the neighbourhood of the ideal sliding mode and consequently and additional robust term u_r has to be considered for the control action in order to ensure the attractivity of the sliding surface:

$$u_r=-k(\mathbf{x})\,\mathrm{sgn}(s) \tag{19}$$

So, the robust control action is characterized by a term discontinuous across the sliding surface.

Taking into account a Lyapunov based design approach, the following Lyapunov function is selected:

$$V(s)=\frac{1}{2}s^2 \tag{20}$$

which is a measure of the squared distance to the sliding surface and generates the following sliding condition:

$$s\dot{s}<0 \tag{21}$$

In order to guarantee that the system trajectories reach the sliding surface in a finite time, the sliding condition is modified to:

$$s\dot{s}<-\eta|s| \tag{22}$$

in which η is a strictly positive constant.

The sliding condition constraints the system subjected to the following control action

$$u=u_{eq}-k(\mathbf{x})\,\mathrm{sgn}(s) \tag{23}$$

to point towards the sliding surface in spite of uncertainties.

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The robust control gain $k(\mathbf{x})$ can be derived taking into account the sliding condition (22):

$$k(\mathbf{x}) = |\alpha - 1|\left|\dddot{y}_T - \lambda^2 \dot{e} - 2\lambda\ddot{e} + \hat{\alpha}_1\ddot{y} + \hat{\alpha}_2\dot{y}\right| + \\ + \alpha(F + \eta + \rho\alpha_{3_{MAX}}) \tag{24}$$

where F is the additive error bound given as $\left|(\hat{\alpha}_1 - \alpha_1)\ddot{y} + (\hat{\alpha}_2 - \alpha_2)\dot{y}\right| \le F$, α the multiplicative error bound given as $\sqrt{\alpha_{3_{MIN}}{}^{-1}\alpha_{3_{MAX}}}$ and ρ the dead band error bound $\left(\left|S(u)/K_Q\right| \le \rho\right)$. The subscript *MIN*, *MAX* indicate the bounds of the uncertainty ranges.

In order to counteract the chattering phenomenon caused by the discontinuous nature of the robust control action (19), a boundary layer is introduced around the sliding surface and the robust control action can be modified to:

$$u_r = -k(\mathbf{x})sat\left(\frac{s}{\phi}\right) \tag{25}$$

where ϕ represents the width of the boundary layer and *sat* is the saturation function defined as:

$$\begin{cases} sat\left(\frac{s}{\phi}\right) = \frac{s}{\phi} & \text{if } \left|\frac{s}{\phi}\right| \le 1 \\ sat\left(\frac{s}{\phi}\right) = \text{sgn}\left(\frac{s}{\phi}\right) & \text{otherwise} \end{cases} \tag{26}$$

V. Experimental results

A variability is considered for all the parameters of the hydraulic actuator, including the geometric parameters and the supply pressure. The nominal values are set as:

$\hat{m} = 540$ kg, $\hat{\sigma} = 22500 \ \frac{\text{Ns}}{\text{m}}$, $\hat{F}_C = 950$ N, $\hat{\mu} = 0.03$, $\hat{N} = 67000$ N, $\hat{A}_p = 0.0055 \text{ m}^2$, $\hat{V}_0 = 0.0035 \text{ m}^3$, $\hat{\beta} = 0.75e9$ Pa, $\hat{K}_Q = 2.1082e-7 \ \frac{\text{m}^3}{\text{sVPa}^{\frac{1}{2}}}$, $\hat{u}^+ = 0.65$ V, $\hat{u}^- = -0.65$ V, $\hat{P}_S = 110e5$ Pa.

These nominal values contribute to determine the $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$ values.

To demonstrate the robustness of the proposed controller, the following large bounds of the uncertain parameters are considered. They refer to the same measurement units previously adopted.

$[m_{MIN}, m_{MAX}] = [440,\ 640]$, $[\sigma_{MIN}, \sigma_{MAX}] = [20000,\ 25000]$, $[F_{C_{MIN}}, F_{C_{MAX}}] = [900,\ 1000]$, $[\mu_{MIN}, \mu_{MAX}] = [0.01,\ 0.05]$, $[N_{MIN}, N_{MAX}] = [4316,\ 130000]$, $[A_{P_{MIN}}, A_{P_{MAX}}] = [0.003,\ 0.009]$, $[V_{0_{MIN}}, V_{0_{MAX}}] = [0.003,\ 0.004]$, $[\beta_{MIN}, \beta_{MAX}] = [5e8,\ 1e9]$, $[K_{Q_{MIN}}, K_{Q_{MAX}}] = [1.5811e-7,\ 2.6352e-7]$, $[u^+_{MIN}, u^+_{MAX}] = [0.3,\ 1]$, $[u^-_{MIN}, u^-_{MAX}] = [-1,\ -0.3]$, $[P_{sMIN}, P_{sMAX}] = [20e5,\ 200e5]$.

Taking into account the above ranges, the bounds of α_1, α_2 and α_3 are determined.

With reference to the seismic isolator configuration (Fig. 2), a common elastomeric isolation bearing (Fig. 3) has been adopted to test the sliding control.

Fig. 2. The earthquake simulator in the seismic isolator configuration

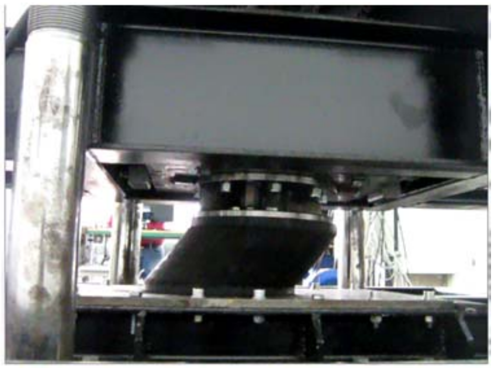

Fig. 3. Detail of the strained seismic isolator

The isolator has been vertically loaded ($1.25\text{x}10^5$ N) by means of a hydraulic jack and subjected to the horizontal actuation force. A supply pressure P_s of 100 bar has been imposed and a target displacement of amplitude 0.1 m and a frequency of 0.5 Hz has been adopted.

The result in terms of table displacement is showed in Fig. 4. The performance of the controlled system is fully insensitive respect to the external disturbance due to the seismic isolator. Indeed, the table displacement in presence of the isolator and the same one with no isolator are practically superimposed. Moreover, the performance is fully appreciable for both tracking error and stability.

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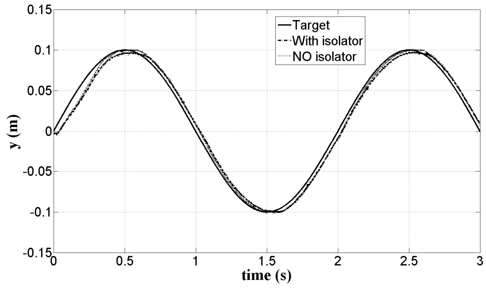

Fig. 4. Table displacement in the seismic isolator configuration

Fig. 5 illustrates the control action with and without the seismic isolator. It has to be highlighted that the signal is not affected by the undesired chattering phenomenon.

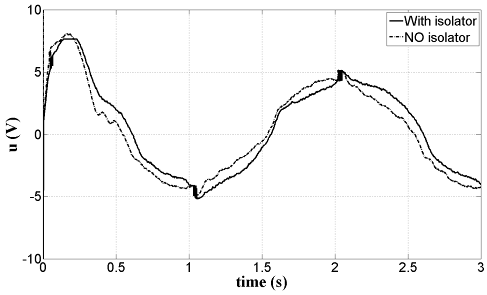

Fig. 5. Control action in the seismic isolator configuration

The sliding mode control has been then experimentally tested on the earthquake simulator in the shaking table configuration. To this aim, a suspended structure has been fixed on the sliding table (Fig. 6) [8, 9].


Fig. 6. The earthquake simulator in the shaking table configuration

The structure consists of a rigid cabin (200 kg) equipped with suspensions, and is characterized by a 1.2 Hz resonant mode. A sinusoidal target displacement has been assigned (amplitude 0.01 m and frequency 1.2 Hz) together with a supply pressure P_s of 30 bar. The selection of the target frequency allows to evaluate the goodness of the proposed approach under the condition in which the unmodeled dynamics of the suspended structure highly perturbs the controlled system.

The controlled system is stable and shows (Fig. 7) a substantial robust performance, as can be observed focusing on the results obtained with and without the test structure. In this test, the test structure is characterized by an acceleration increased of one order of magnitude respect to the table acceleration, and the excited additional dynamics doesn't cause effects on the controlled displacement. The target displacement amplitude is guaranteed, confirming the attitude to be employed as shaking table.

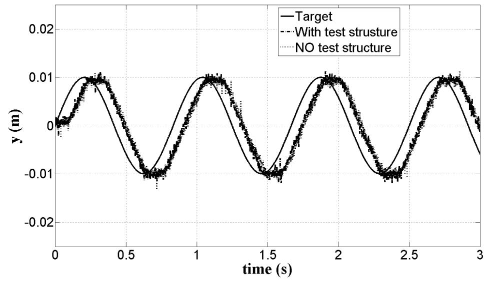

Fig. 7. Table displacement in the shaking table configuration

A contained influence of the test structure can be seen in the control action (Fig. 8) that is not contaminated by chattering.

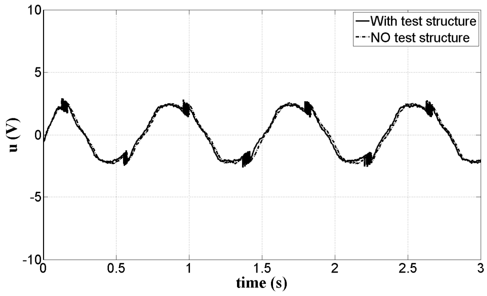

Fig. 8. Control action in the shaking table configuration

VI. Conclusion

A robust control has been designed for a multi-purpose earthquake simulator. A sliding mode approach has been followed starting from a third order non-linear dynamic model in presence of dead-band. The experimental results highlight the effectiveness in terms of stability, tracking error and robustness of the controlled earthquake simulator for both the configurations.

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