Finite Design for Critical Stresses of Compressed Biomaterials under Transportation

Christopher C. Ihueze, Christian E. Okafor and Peter O. Ogbobe

Abstract—**This paper focuses on the evaluation of critical stresses of biomaterials of spherical shape under radial and axial compression employing Hertz contact theory and finite element method. The mechanical properties of UTC tomatoes were determined with a compression test rig for radial and axial compression contacts with metal plate. The maximum contact pressure P_{max} was evaluated as 0.063MPa while the maximum shear stress was 0.022MPa. The finite element analysis gave the von mises stresses for axial and radial contacts as 0.001699MPa and 0.012271MPa. The design yield stress was predicted for UTC tomatoes and specified as 0.0012MPa for UTC containers design for transportation. The results of this study will therefore be relevant in the design of transportation for fruits of spherical shape.**

Index Terms—**contact stress, hertzian stresses, principal stresses, shear stress, sphericity**

I. INTRODUCTION

FRUIT and vegetables are highly susceptible to mechanical damage during harvesting, handling, transportation and storage. The damage causes them to rot quickly, reduces quality and increases loss [1]. In order to minimise mechanical damage the handling and transportation stresses must be kept under a certain value, it therefore became necessary that during design and optimization of machine for handling, cleaning, transporting, and storing, the physical attributes of improved UTC tomato variety and their relationships must be known [2]. Designing such equipment without consideration of these properties may yield poor results. Therefore the determination and consideration of these properties have an important role [3].

Initial work on tomato based on transportation and handling cultures has been undertaken using compression experiments and it has been shown that force- deformation data can be obtained up to cell bursting. Reference [4] gave the mechanical properties of two tomato varieties at various stages of maturity by whole fruit compression test. Reference [5] also reported the effects of compression on the structure of red tomato using magnetic resonance imaging while [6] characterized the mechanical behavior of single tomato fruit cells using high strain-rate micro-compression testing.

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Reference [7] reported compression tests results for transverse and longitudinal directions of tomato fruit at different ripening phases and tests of bending and stretching on tomato peduncle. In general, there exist some studies on mechanical behavior of some agricultural products as found in [8]-[12]. But despite an extensive search, no published literature was found on the detailed physical and mechanical behavior under compression loading of improved UTC tomato variety.

Reference [13] established 11 models for the prediction of orange mass based upon dimensions, volume and surface areas. Reference [14] also examined some orange parameters, such as coefficient of sphericity, mean geometrical diameter, apparent specific mass, an orange pile specific mass, rind ratio, and packing coefficient. Also [15] subjected orange fruit of Tarocco variety to conventional parallel plate compression tests while assessing precisely the contact area of the fruit under squeezing at different deformation levels via two different visual methods, and successfully converted the typical force–deformation curves into true stress–strain relationships, as an attempt to assess the real mechanical properties of Tarocco orange fruit and to develop more efficient on-line non-destructive sorting rules.

In citrus fruits the relationship between puncture force and firmness is concealed by the differences in the tissue types directly under the puncture probe. Moreover, such tests are generally inadequate for fruit sorting and should be replaced with another one capable of assessing the mechanical properties of citrus fruits in a more objective and reproducible way [16], hence this study targets a holistic bridge of the gap by utilizing compression test rig [17] in determining some relevant mechanical properties of UTC tomatoes fruit under axial and radial loading.

Other studies related to this study are those of [18] who worked on elastic and visco-elastic properties of radially compressed corn cob and that of [19] who worked on the hardness and elastic properties of tropical seed grains and tomato fruit.

II. THEORETICAL RELATIONS RELAVANT TO STUDY

The following relations necessary for the computation of the mechanical and biological properties of material are adapted from [20]-[23].

$$\mu = \frac{Transverse\ strain}{Axial\ Strain} = \frac{\left\{\frac{(A_i - A_0)}{A_0}\right\}}{Axial\ Strain} \qquad (1)$$

$$E = \frac{(FL_0)}{A_0 \Delta L} \qquad (2)$$

$$\varphi = \frac{\left(D_L D_{T_{max}} D_{T_{min}}\right)^{\frac{1}{3}}}{D_L} \quad (3)$$

$$D = \sqrt{AB} \quad (4)$$

$$G = \frac{E}{2(1+\mu)} \quad (5)$$

$$\sigma = \frac{F}{A} \quad (6)$$

For large deformations characteristic of loaded biological materials, the contact area between the material and the loading device do not remain constant [18]. Therefore the calculation of stress as force per unit cross-sectional area is not valid. Hence Hertz contact stress theory for calculating contact stresses in spherical and cylindrical materials can be employed for biological materials.

Contact stresses relations can be evaluated in line with the experimental procedures of [24] such that the moving platen has diameter $d_1 = \infty$ (flat surface) and the test piece has d_2 (curved surface) during loading with a force of magnitude F when the biological material is deformed. Hertz contact stresses relations are presented as follows by [23] for spherical material as

$$\mathrm{a} = \sqrt[3]{\frac{3\mathrm{F}}{8}\frac{(1-\mu_1^2)/\mathrm{E}_1 + (1-\mu_1^2)/\mathrm{E}_2}{1/\mathrm{d}_1 + 1/\mathrm{d}_2}} \quad (7)$$

Equation (7) reduces to

$$\mathrm{a} = \sqrt[3]{\frac{3\mathrm{F}}{8}\frac{(1-\mu_1^2)/\mathrm{E}_1 + (1-\mu_1^2)/\mathrm{E}_2}{1/\mathrm{d}_2}} \quad (8)$$

When the moving surface is a plate as applicable in this study,

$$\mathrm{P}_{max} = \frac{3F}{2\pi a^2} \quad (9)$$

$$\sigma_x = \sigma_y = -P_{max}\left[\left(1 - \frac{z}{a}\tan^{-1}\frac{1}{\frac{z}{a}}\right)(1+\mu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right] \quad (10)$$

$$\sigma_z = \frac{-\mathrm{P}_{\max}}{1+\frac{\mathrm{z}^2}{\mathrm{a}^2}} \quad (11)$$

By employing the concept of Mohr's circle and equation (10) and equation (11) the value of shearing stress is expressed as

$$\tau_{xz} = \tau_{yz} = \frac{\sigma_x - \sigma_z}{2} = \frac{\sigma_y - \sigma_z}{2} \quad (12)$$

Reference [23] also reported relations for computation of hertzian stresses of cylindrical shaped surfaces in contact. Hertz contact stress theory is applicable on the assumption that the stress response behaviour of material is essentially that of linear isotropic material.

III. MATERIAL AND METHODS

The improved UTC tomatoes were used in this study. The first step of this experimentation is to determine the approximate shape of the materials by computing the sphericity of the material. The equivalent diameter D (for transverse loading) values were calculated with (1) and (2) of [26] and [27]. Subsequently, the original cross-sectional area through which the force is applied A_0 and final cross-sectional area A_i were computed. The sphericity is a shape index of fruit, which indicates the difference between the actual shape of fruit and the sphere [22]. The replicated samples of tomatoes were radially loaded as shown in Fig. 1 and Fig. 2 using compression test rig of [17].

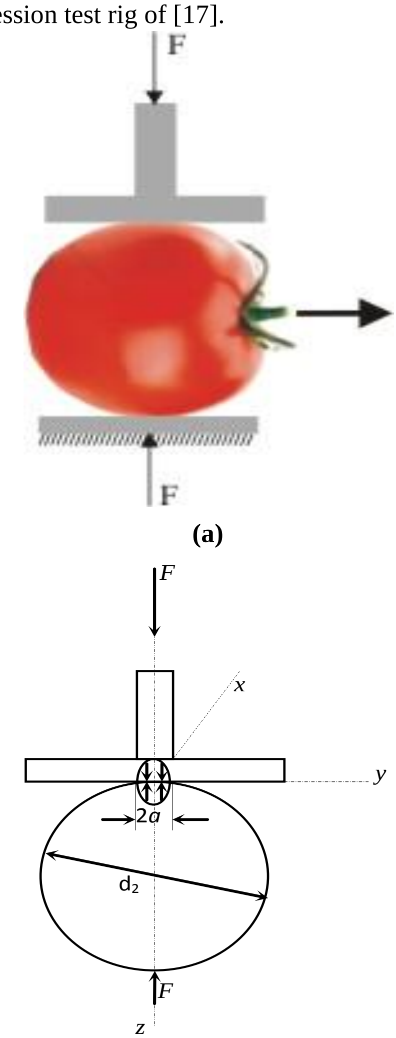

Fig. 1(a), (b) Depiction of Radial Loading

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IV. EXPERIMENTAL RESULTS

Engineering Property Results of UTC Tomato under Radial and axial compression are presented in Table I. The experimentations and computations gave an average contact radius and maximum contact pressure of 0.011m and 0.063MPa respectively using appropriate values in (8) and (9) and proportionality limit range force of 16N. An approximate elastic modulus of 0.2MPa and a Poisson's ratio of 0.3 were evaluated at the proportionality limit of the material. The radial loading gave an elastic modulus of 0.2MPa and a Poisson's ratio of approximately 0.3.

TABLE I
ENGINEERING PROPERTY RESULTS OF UTC TOMATOES UNDER RADIAL AND AXIAL COMPRESSION

Loading	Elastic Modulus MPa	Poisson's Ratio μ	S_y MPa	S_u MPa	Fracture Strength S_f MPa	Bulk Modulus MPa	Maximum force (N)
Axial compression	0.1628	0.26	0.009	0.015	0.014	0.06479	19
Radial compression	0.2158	0.26	0.009	0.017	0.012	0.08566	20

A. *Estimation of sphericity*

Sphericity is considered important in the designs for biological products. Equation (13) of [28] expressed as

$$\psi = \frac{\pi^{\frac{1}{3}}(6V_p)^{\frac{2}{3}}}{A_p} \qquad (13)$$

was used with Table II to evaluate the sphericity of the UTC tomato fruit after the computation of the volume and surface area of the tomato fruit with relations of Table II. The approximate surface area relation of ellipsoid of Table II credited to Knud Thomsen was cited by [29] and can be found in en.wikipedia.org/wiki/Ellipsoid. The sphericity value of 0.996 obtained with (13) suggests that the shape of UTC tomato can be approximated to sphere. Similar computation with (3) also gave sphericity value of 1.17 with $D_L = 45.29mm$, , $D_{Tmax} = 63.95mm$ $D_{Tmin} = 61.96mm$.

TABLE II
ESTIMATION OF ROUNDNESS FOR IMPROVED UTC TOMATO VARIETY: A = RADIUS OF MINOR AXIS OF TOMATO (TRANSVERSE) = B = R, C = RADIUS OF MAJOR AXIS OF TOMATO (LONGITUDINAL), H = 2C (AXIAL DIMENSION)

Assumed shape	Volume (V_p)	Surface area(A_p)	Sphericity(ψ)
Ellipsoid	$\frac{4}{3}\pi abc$	$4\pi\left(\frac{a^{1.6}b^{1.6}+a^{1.6}c^{1.6}+b^{1.6}c^{1.6}}{3}\right)^{1/1.}$	0.919
Cylinder	$\pi r^2 h$	$2\pi r(r+h)$	0.856
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$	0.996

B. *Computations for Contact Stresses for Radial Loaded UTC Tomatoes*

The Hertz theory variables were computed with (7)-(12) and with material properties data reported in [30] as in Table I after establishing the shape of UTC tomato as sphere and presented in Table III.

TABLE III
EVALUATED HERTZIAN STRESSES OF RADIALLY COMPRESSED TOMATO

z/a	σ_x (MPa)	σ_z (MPa)	τ_{max} (MPa)	σ_x/P_{max} (MPa)	σ_z/P_{max} (MPa)	τ_{max}/P_{max} (MPa)
0	-0.0589	-0.0736	0.0074	-0.8	-1	0.1
0.5	-0.0133	-0.0589	0.0228	-0.1803	-0.8	0.3098
1	-0.0021	-0.0368	0.0174	-0.0289	-0.5	0.2355
1.5	3.32208E-05	-0.0227	0.0114	0.0005	-0.3076	0.1541
2	0.0004	-0.0147	0.0076	0.0055	-0.2	0.1027
2.5	0.0004	-0.0102	0.0053	0.0056	-0.1379	0.0718
3	0.0004	-0.0074	0.0039	0.0048	-0.1	0.0524

The results of Table III are presented as a ratio of the maximum contact compressive pressure in Fig. 2.

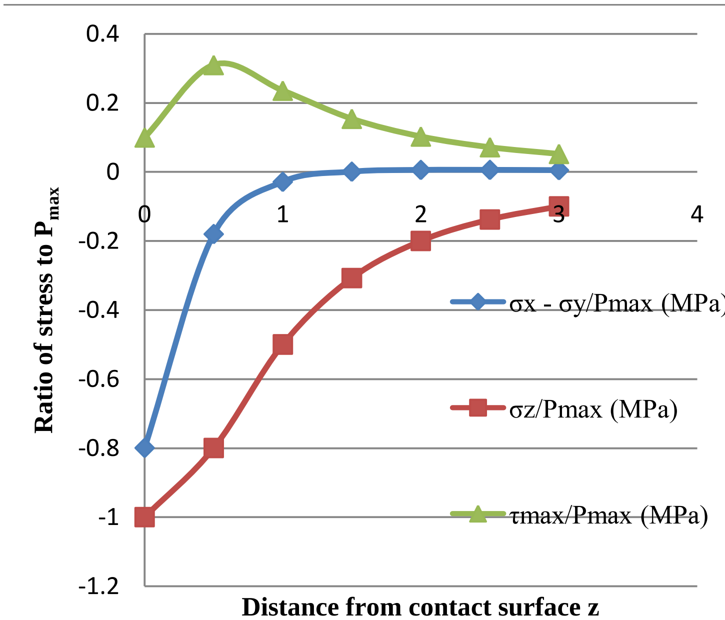

Fig. 2 Magnitude of the stress components below the surface as a function of the maximum pressure for radially compressed tomato fruit

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V. FINITE ELEMENT ANALYSIS OF CRITICAL STRESSES

The ANSYS finite element software was applied to obtain the maximum surface stresses and the yield

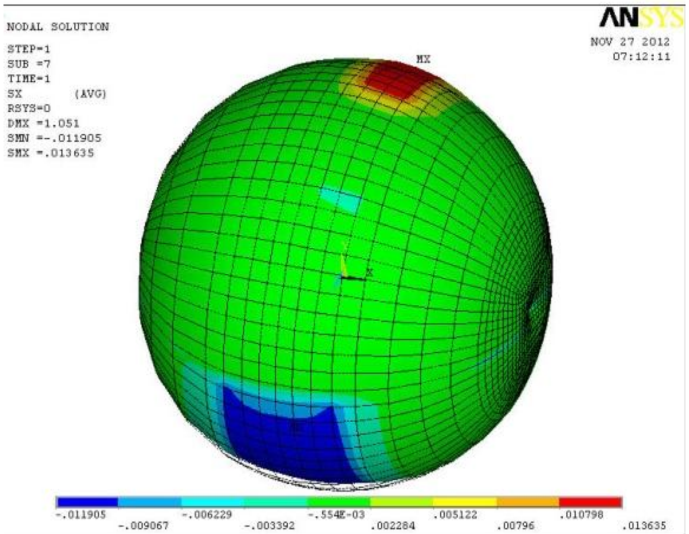

Fig. 3 X-component orthogonal stress distibution for radial compression

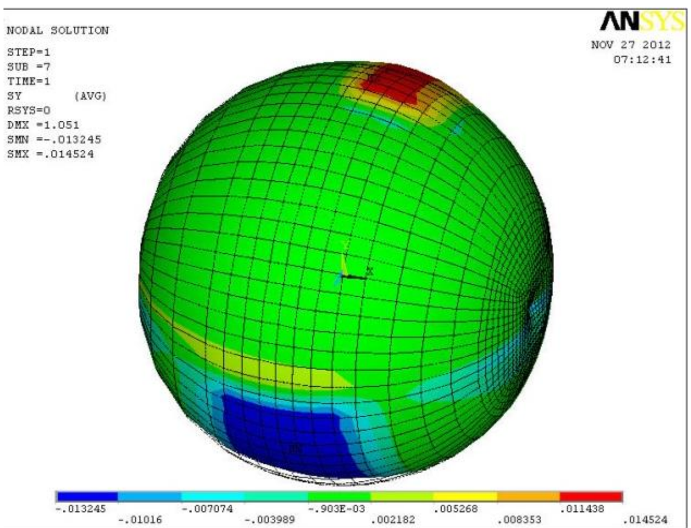

Fig. 4 Y-component orthogonal stress distibution for radial compression

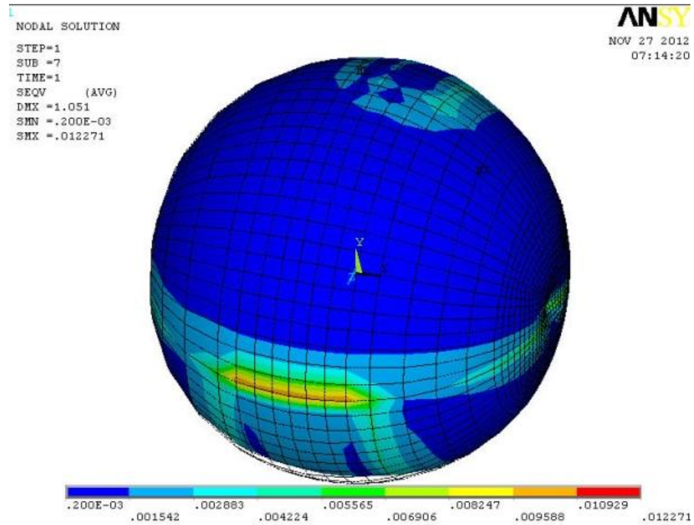

Fig. 5 von Mises stress distribution for UTC tomatoes under radial compression

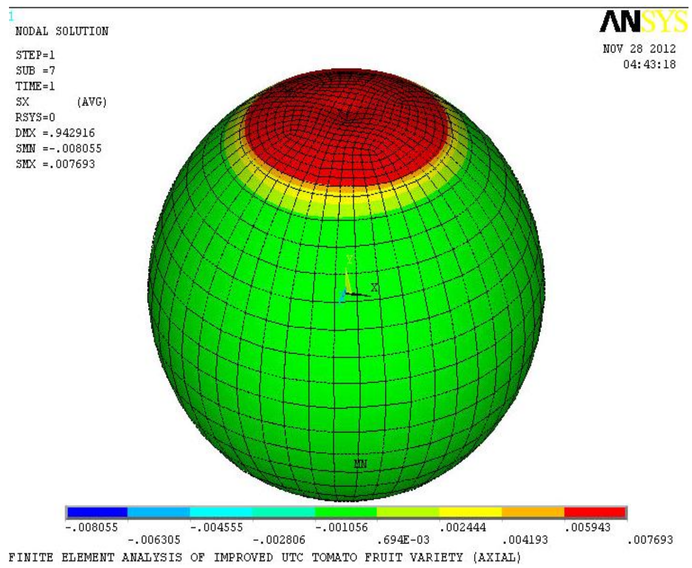

Fig. 6 X-component orthogonal stress distibution for axial compression

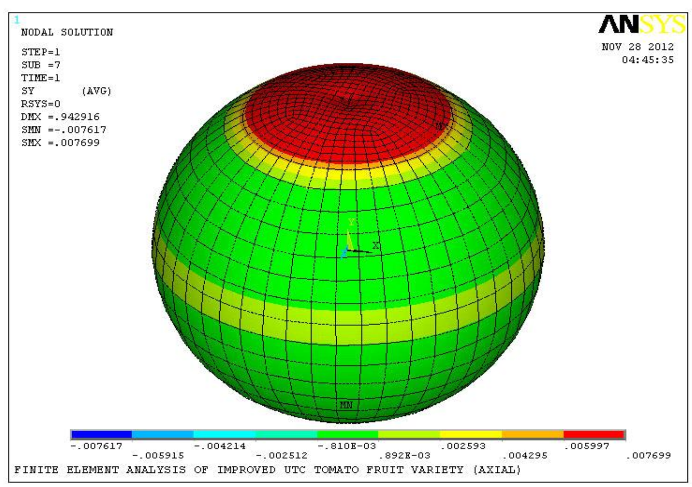

Fig. 7 Y-component orthogonal stress distribution for axial compression

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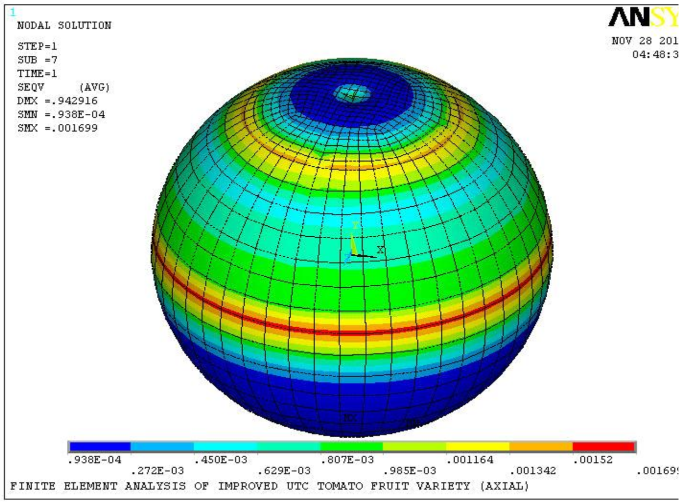

Fig. 8 von Mises stress distribution for UTC tomatoes under axial compression

The stresses of radially and axially compressed UTC tomatoes experimentally determined are reported in Table I. The ANSYS analysis gave results of surface stresses of compressed UTC tomatoes in Fig. 3-Fig. 8.

VI. DISCUSSION OF RESULTS

Table I exhibits the mechanical properties of compressed UTC tomatoes. Table III and Fig. 2 also show that the failure of radially compressed tomato fruit is governed by the shearing stress which has maximum value of 0.31Pmax slightly below the contact surface. The maximum contact pressure P_{max} was computed as 0.063MPa for UTC tomatoes.

Hertz theory was used to evaluate the maximum shear stress as 0.022MPa as depicted in Fig. 2 and Table III. The normal stress in the radially compressed UTC tomatoes is greater in transverse direction where the shear stress is maximum as shown in Fig. 2 and Table 3. Fig. 3 and Fig. 4 show the maximu stress distribution in x and y direction as 0.013635MPa and 0.014524MPa for radially compressed UTC tomatoes. Fig. 5 describes the possible axial failure of sample with the red portions while Fig. 6 and Fig. 7 depict x and y maximum stress components distribution as 0.0067693MPa and 0.007699MPa for axially compressed UTC tomatoes. These are the principal normal stresses of the axially compressed biomaterial. Also Fig. 8 describes circumferential radial failure of axially compressed biomaterial.

The ANSYS finite element method evaluated the yield stress as 0.001699MPa and 0.012271MPa for axial and radial contacts as found in Fig. 8 and Fig. 5 through the application of von Mises stress criteria and these values are smaller than the maximum principal stress of -0.073622694MPa presented in Table III. The ANSYS predicted yield stress corresponds to the the bioyield stress of 0.009MPa predicted by [30] for radial compression test of UTC tomatoes whose stress- strain plot is shown in Fig. 9. Fig. 9 also shows that the biomaterial is ductile as fracture strain was more than 5%. Hence failure prediction by ductile failure theory of von Mises criterion is correct.

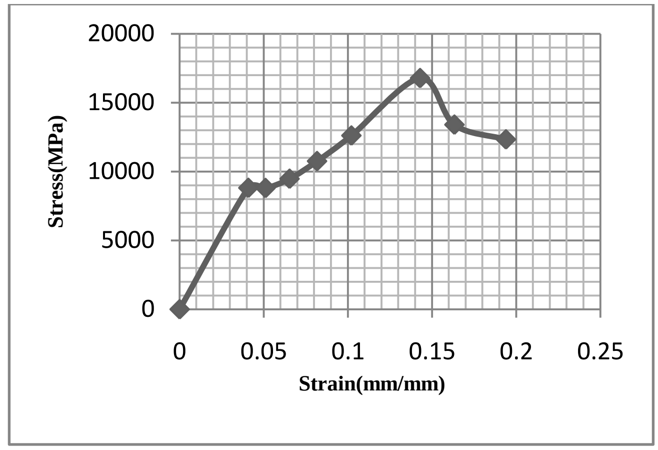

Fig. 9 Stress-strain plot depiction for mechanical properties of UTC tomatoes under radial compression

It is the opinion of many authorities that this maximum shear stress is responsible for the surface fatigue failure of contacting elements. The explanation is that a crack originates at the point of maximum shear stress below the surface and progresses [23]. The design yield stress is therefore predicted for UTC tomatoes and specified as 0.0012MPa for design of UTC containers for transportation and storage.

The reports of the study complied with the application of the distortion energy theory or the von Mises criterion which suggests that the safe stresses or the stresses bellow the yield stresses must be less than the principal stresses as shown in Fig.10 and as reported in [31]. The stress intensity for radially compressed UTC tomatoes is shown in Fig. 11 which is the classical understanding of von Mises stress theory. The von Mises stresses are found below the principal stresses as depicted in Fig. 5, Fig. 8 and Fig. 11.

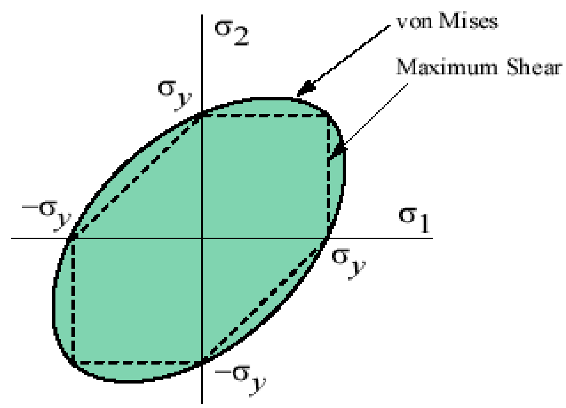

Fig.10 Depiction of principal stress ellipse for von Mises failure prediction:σ_1 = maximum stress at principal plane 1,σ_2 = maximum stress at principal plane 2, σ_y = yield stress equivalent to von Mises stress.

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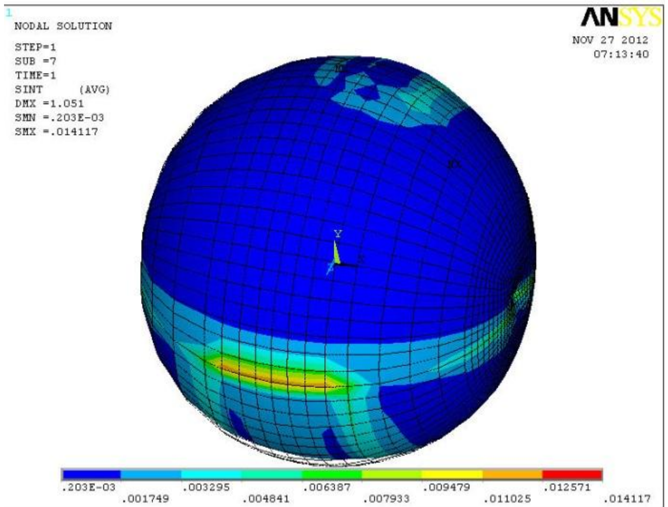

Fig. 11 ANSYS depiction of intensity of stresses (principal stresses) for radially compressed UTC tomatoes

VII. CONCLUSION

The maximum pressure due to the metal container contact with the fruits on transportation was evaluated and found to be less than the compressive strength of the fruits. The shearing stresses was also found to be greater than the other principal stresses associated to geometric shape of the fruit for containerization and transportation design. The maximum pressure, maximum principal stresses and maximum shearing stresses were found to be a maximum near the contacting surfaces of the container with the fruits on transportation. The results of this study will therefore be relevant in the design of structures for storage and transportation for fruits.

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