3D Periodic Foundation-based Structural Vibration Isolation

Z.B. Cheng, Y.Q. Yan, Farn-Yuh Menq, Y.L. Mo, H.J. Xiang, Z.F. Shi and Kenneth H. Stokoe, II

***Abstract*—Guided by the recent advances in solid-state physics research, a 3D periodic foundation-based structural vibration isolation is studied. Using construction materials, concrete, rubber and steel, the three-component periodic foundation is developed. Frequency band gaps for the specimens are found to be in low-frequency region (<20Hz). A parametric study is also conducted to illustrate the influences of the geometrical and material parameters on the frequency band gaps. Based on the frequency band gaps analysis, numerical simulations are performed to verify the efficiency of the periodic foundation. Harmonic analysis results show periodic foundation can reduce vibrations in the frequency band gap. Further, a transient analysis shows that 3D periodic foundations can also isolate seismic wave effectively.**

***Index Terms*—frequency band gap, finite element analysis, periodic foundation, vibration isolation.**

I. INTRODUCTION

A commonly accepted method for the design of seismic-resistant buildings and structures, has been one of the research focus by engineers for many decades. However, the problem has not been solved up to the present time. Various methods have been proposed to resist seismic loading. Traditionally, a popular strategy is to use base isolation technology, which is to add an extra isolation system between the isolated structure and the foundation [1-3]. The method gives the isolated structure a fundamental frequency much lower than both its fixed-base frequency and the predominant frequencies of the ground motion. Therefore, it can reduce the acceleration response significantly. Besides, damping devices are usually added to mitigate the horizontal displacement response of the structure.

Recently, some investigations in the field of solid-state physics indicated that *phononic crystals*, a new type of composite with different materials arranged in a periodic manner, exhibits a unique dynamic property, the so-called frequency band gap[4]. When the frequency contents of a wave fall within the range of the frequency band gap of the periodic structure, the wave, and hence its energy, cannot propagate through the periodic structure. The special dynamic property of periodic materials draws a great deal of attention over the last several decades. It has numerous potential engineering applications, such as noise suppression and the control or isolation of vibrations[5, 6].

Guided by the recent advances in solid-state physics research and the concept of frequency band gaps in periodic materials, Shi and his co-workers utilized this periodic materials as a new and innovative method in seismic base isolation to mitigate the potential damage to structures[6-11]. It is hoped that the band gaps of periodic foundation can cover the main frequency region of seismic waves, and can cover the main characteristic frequency of the upper structures. Therefore, the seismic responses of upper structure will be lowered from two aspects: 1, less energy input as seismic waves obstructed or changed when it reaches the periodic foundation of the structural system; 2, smaller structure responses as the resonance phenomenon of upper structure is prevented. There are three types of periodic foundation: one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D).

Focused on 1D periodic structure, Xiang *et al.*[7] studied the feasibility of layered periodic foundation, composite of concrete layers and rubber layers. Using the same model, Bao *et al.*[8] presented the influences of the incident angle on the band gaps and studied the effectiveness of the layered periodic foundations to isolate the seismic load in the horizontal and vertical direction through numerical simulations. A systematic experimental study on the vibration attenuation of the layered periodic foundation has also been conducted by Xiang *et al.*[9]. In the studied, a model on a periodic foundation was fabricated and shake table tests(Fig. 1) were performed, in which great attenuations were found when the exciting frequencies fell into the frequency band gaps. As shown in Fig.2, for the frame on the periodic foundation (i.e. specimen A), the peak horizontal acceleration is reduced by as much as 50% compared to that of the frame without the periodic foundation (i.e. specimen B).

By considering a two-dimensional three-component cell composite of concrete, rubber and Pb, Jia and Shi[6] conducted the feasibility study of this seismic isolation system. In this study, through comprehensive parametric study the frequency band gaps are also achieved in the range of 2.49 and 3.72Hz, which can be used in foundation to effectively isolate seismic waves with frequencies in this range. Cheng and Shi[10,11] studied the frequency band gaps for two-component and three-component two dimensional period

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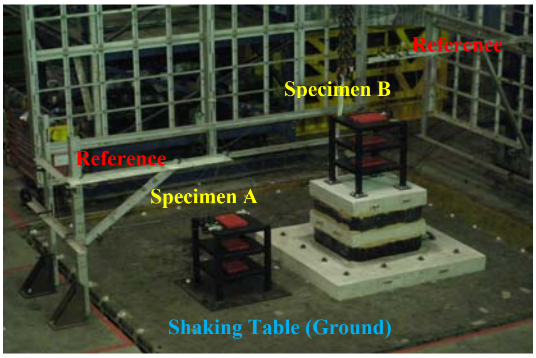

Fig.1 Test setup for one-dimensional periodic foundation

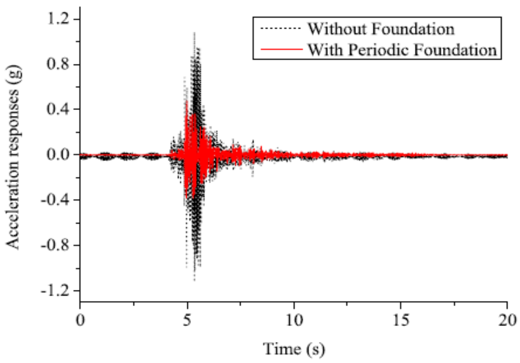

Fig. 2. Horizontal acceleration responses at the tops of the frames under the 1975 Oroville earthquake [9]

Fig. 3. Test setup for two-dimensional periodic foundaiton

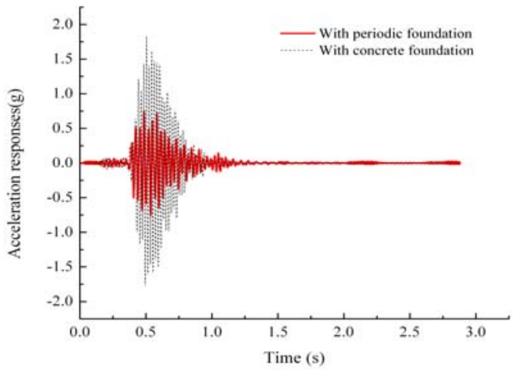

Fig. 4. Horizontal acceleration responses at the tops of the frames under the modified 1984 Bishop earthquake [12]

materials using concrete, rubber and steel. In the study, numerical simulations were also conducted to investigate the efficiency of two dimensional periodic foundations. Recently, an experimental study on the two dimensional periodic foundation was conducted[12]. As shown in Fig.3, steel frames were set on two types of foundation: namely, concrete foundation (i.e. specimen C) and periodic foundation (i.e. specimen D). Vibrations and seismic wave inputs generated by the truck shake (T-Rex) were induced for on specimens. Fig. 4 shows the acceleration responses in horizontal direction at the top of steel frames under the 1984 Bishop earthquake record, whose main frequency was modified into the frequency band gap of 50Hz. Comparatively speaking, for the case with periodic foundation (i.e. specimen D) acceleration response at the top of the steel frame is much lower than that of the frame with concrete foundation (i.e. specimen C).

Based on all of these previous studies, this research aims to investigate the feasibility of three-dimensional (3D) periodic foundations. Theoretically speaking, the frequency band gaps of three dimensional periodic structures are the *absolute* frequency gap, which means that vibrations/waves in the gap cannot propagate in any direction. Therefore, three dimensional periodic foundations are much applicable for multi-dimensional structure vibration isolation.

In the present paper, the feasibility of 3D seismic isolation method by using a periodic foundation is studied based on the theory of elasto-dynamics. In section II, two types of 3D three-component periodic structure, cubic lattice with cube or sphere, are investigated. Frequency band gaps is found in a low frequency region <20Hz. The influences of the material parameters and the geometrical parameters on the frequency band gaps are studied. Numerical simulations about the finite periodic structures are reported afterwards. Finally, the main conclusions are given in section V.

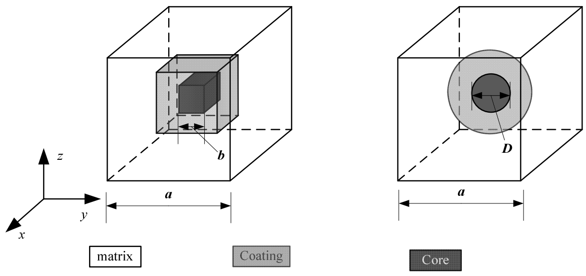

Fig. 5. Illustration of the unit cell of a three-dimensional periodic foundation

II. Basic Theory

A. Governing Equations

Consider 3D periodic foundations with hard core coated by soft rubber set into reinforced concrete in a periodic manner. Fig.5 shows two typical unit cells. The coated core is set at the center of the cube matrix. The side length of the cubic unit is a. For the case with cube, the side length of the cube core is b. For the case with sphere, the diameter of the core is d. For both cases, the thickness of the coating layer is t. To search possible engineering application, the matrix, coating and the core are made of reinforced concrete, rubber, and steel, respectively. Material properties are listed in Table I.

For infinite periodic structure system, the unit cell is arranged infinitely in three dimensions. So, the structure is highly symmetrical. According to the theory of periodic structure, the periodicity of the structure makes it possible to obtain the frequency band gaps by studying one periodic unit.

TABLE I Material Parameters

Materials	Young Modulus E (Pa)	Poisson Ratio υ (1)	Mass Density ρ (kg/m^3)
Concrete	4.00×10^{10}	0.2	2500
Rubber	1.37×10^{5}	0.463	1300
Steel	2.09×10^{11}	0.275	7890

Let $u_{i(i=1,2,3)}$ be displacement in each of the x, y and z directions, respectively. Under the assumption of continuous, isotropic, perfectly elastic and small deformation as well as without consideration of damping, the governing equation of motion [4] is:

$$\rho(\mathbf{r})\frac{\partial^2\mathbf{u}}{\partial t^2}=\nabla\left\{\left[\lambda(\mathbf{r})+2\mu(\mathbf{r})\right](\nabla\cdot\mathbf{u})\right\}-\nabla\times\left[\mu(\mathbf{r})\nabla\times\mathbf{u}\right] \tag{1}$$

where $\mathbf{u}=\{u_x, u_y, u_z\}$ is the displacement vector and $\mathbf{r}=\{x, y, z\}$ the coordinate, λ and μ the Lamé's constants, ρ the mass density.

For isotropic material, Lamé coefficients can be expressed in terms of the Young's modulus E and the Poisson's ratio ν as:

$$\lambda=\frac{\nu E}{(1+\nu)(1-2\nu)};\ \mu=\frac{E}{2(1+\nu)} \tag{2}$$

According to the Bloch's theory [4], the solutions of Eq. (1) can be expressed as:

$$\mathbf{u}(\mathbf{r},t)=e^{i(\mathbf{K}\cdot\mathbf{r}-\omega t)}\mathbf{u}_{\mathbf{K}}(\mathbf{r}) \tag{3}$$

where $\mathbf{K}$ denotes the wave vector in the reciprocal space; ω is the angular frequency. $\mathbf{u}_{\mathbf{K}}(\mathbf{r})$ is the wave amplitude, which is a periodic function:

$$\mathbf{u}_{\mathbf{K}}(\mathbf{r})=\mathbf{u}_{\mathbf{K}}(\mathbf{r}+\mathbf{a}) \tag{4}$$

$\mathbf{a}$ is the periodic constant vector.

Substituting Eq. (4) into Eq. (3), periodic boundary conditions can be obtained:

$$\mathbf{u}(\mathbf{r}+\mathbf{a},t)=e^{i\mathbf{K}\cdot\mathbf{a}}\mathbf{u}(\mathbf{r},t) \tag{5}$$

Given a special Bloch wave vector **K**, the eigen-frequencies of the system can be found by the modal analysis. To ensure the accuracy of the results, the mesh size is set according to shortest wave length in calculations [6].

Owing to the high symmetry for the considered periodic structures, it is sufficiently accurate to calculate the eigen-frequencies for wave vector varying along the boundary of the first irreducible *Brillouin* zone (the pyramid R-M-Γ-X-M) as shown in Fig. 6. Relationship between eigen-frequency and the wave vector is the so called dispersion structure.

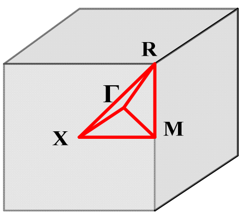

Fig. 6. First irreducible *Brillouin* zone of periodic structures

B. Dispersion relationships

Fig.7 shows the dispersion relationship of a coated square inclusion periodic structure with a=1m, t=0.1m and b=0.6m. An absolute band gap is in the region between 9.10Hz and 13.80Hz, which means waves/vibrations in the region cannot propagate in the infinite periodic structure. Different from the dispersion relationship of the case without the coating, shown in [13], the dispersion relationship in this case is flat.

For the case with sphere core, similar result is given in Fig.8. Comparatively, the thickness of the coating t=0.1m and

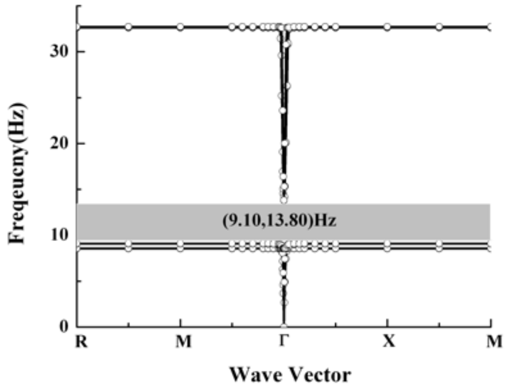

Fig. 7. Dispersion relationship of a coated cube inclusion periodic structure with a=1.0m b=0.6m, and t=0.1m

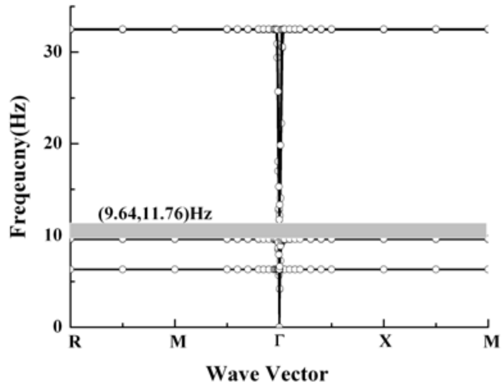

Fig. 8. Dispersion relationship of a coated sphere inclusion periodic structure with a=1.0m d=0.6m, and t=0.1m

the diameter of the sphere is d=0.6m. An absolute frequency band gap is also found between 9.64Hz and 11.76Hz.

Obviously, all of the frequency band gaps are below 20Hz, which is much useful for structure isolation. Further, the frequency band gap for the case with sphere inclusion is smaller than that of the case with cube inclusion when the side length of the cube core is equal to the diameter of the sphere core. Therefore, in the following analysis, our attention will be focused on the case with coated cube core.

III. Parametric Study

To isolate the external vibration effectively, it is always hoped that the frequency band gaps of periodic structure can be wider and lower for seismic design. To obtain better design for application, parametric study is needed to investigate the influences of the geometrical and materials parameters on the frequency band gap. For simplicity, the lower and upper bound frequency of the band gap, and the width of the band gaps is replaced by LBF, UBF and WBG, respectively.

A. Geometrical Parameters

First, Fig.9 shows the influences of the side length of the unit cell on the band gap as taking the ratios of t/a and b/a constants. Obviously, with the increase of the side length the LBF and UBF are lowered dramatically. Because the UBF decreases faster than the LBF, the WBG become smaller. Besides, it is found that the band gaps will be <20Hz for the side length in (1~2)m.

Influences of the thickness of the rubber coating on the band gap are investigated as taking the side length of the core b=0.6m and the side length of the unit cell a=1.0m as const. Fig.10 shows the changes of the first band gap with the thickness of the rubber coating (t). The LBF and UBF are

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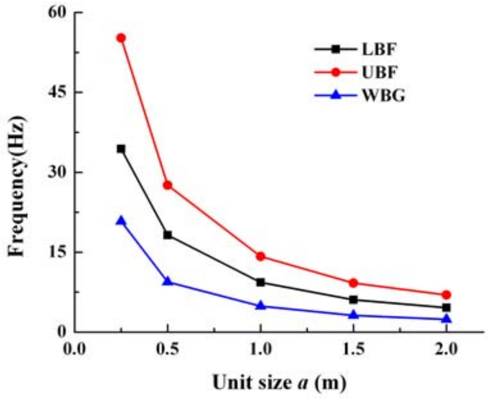


Fig. 9. First band gaps versus the side length of the unit cell (*a*)

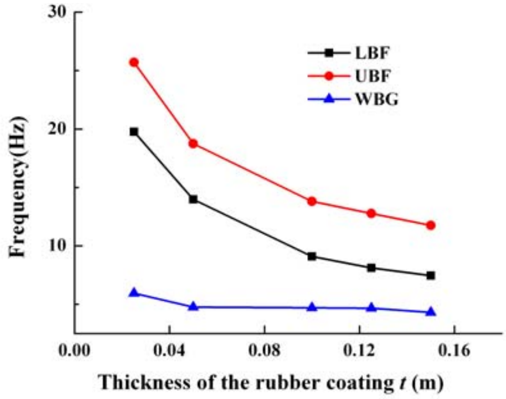


Fig. 10. First band gap as a function of *t,* the thickness of the rubber coating

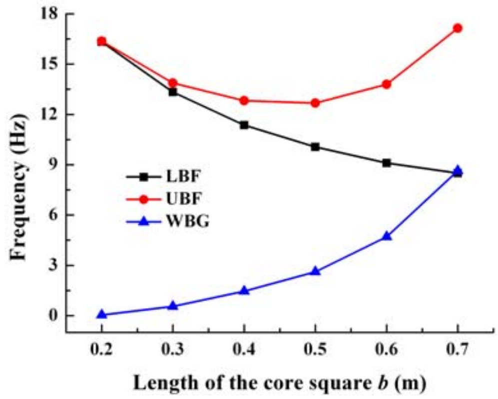


Fig. 11. First band gap as a function of *b*, the side length of the square core

lowered as the thickness of the rubber coating increases. Different with Fig.6, the WBG changes small in this case.

The relationship between the frequency band gap and the side length of the core square is studied. In the case, the thickness of the rubber coating is t=0.1m, the side length of the unit cell is a=1.0m. As shown in Fig.11, a small band gap is found for the case b=0.2m. With the increase of the side length of the cube core, the LBF decreases. The UBF decreases for b<0.5m and it increases for b>0.5m. Further, it is found that the LBF lowers faster than the UBF for b<0.5m. Therefore, one can see the WBG increases continuously. For b=0.7m, the WBG is about 9Hz (i.e. from 8.5 to17.15Hz).

B. *Material Parameters*

The three component periodic structure considered here is a type of local resonant phononic crystal, proposed by Liu *et al.* [14]. The frequency band gap of local resonant phononic crystal is governed by the local resonant oscillator. Therefore, materials parameters also play important role in frequency band gaps. Take the model with the side length of the cube matrix a=1.0m, the side length of the square core b=0.6m and the thickness of the coating t=0.1m for an example. Two main parameters: the Young's modulus of the coating and the mass density of the core are considered here.

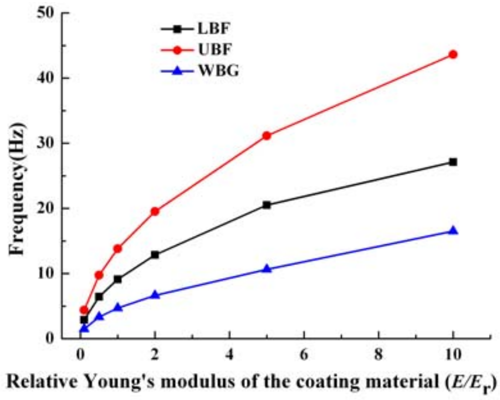


Fig. 12. Influences of the Young's modulus of the coating material on the first band gap

Fig.12 presents the relationship between the relative Young's modulus of the coating material and the first band gap. The LBF and UBF go up with the increase of the relative Young's modulus. Also, the WBG goes up in the same manner. Therefore, it means that softer coating materials will give lower and narrower frequency band gap.

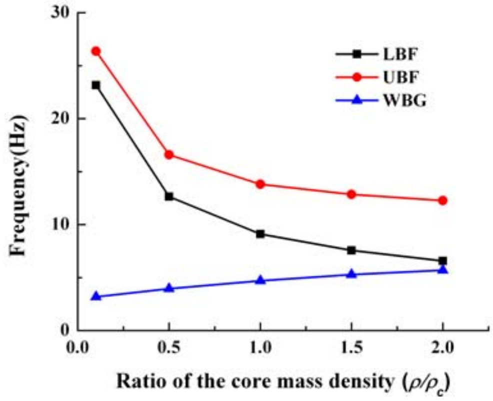


Fig. 13. First band gap versus the mass density of the core material

Fig.13 gives the variation of the frequency band gap as varying the mass density of the core materials. In a similar way, the ratio of the core mass density ρ/ρ_c is considered as the variation. The LBF and UBF are lowered with the increase of the relative mass density parameter. Interestingly, the LBF decreases much faster than the UBF, and the WBG goes up along with the increment of the relative mass density. In other word, the heavier core material is beneficial to get lower and wider frequency band gap.

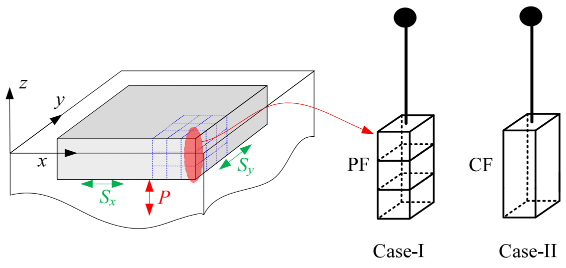
IV. Finite Periodic Structures

All of the periodic structures used in actual design are finite periodic structures. Therefore, it is necessary to analyze the dynamic properties of finite periodic structure. Specially, for structural isolation application the dynamic responses of upper-structure are very important.

Fig.14 shows the layout of a 3D periodic foundation system. Superior to 1D and 2D periodic foundation, 3D periodic foundation is a multi-dimensional isolation system. The absolute frequency band gaps can isolate all types of vibrations, the so-called *P* mode, S_x mode and S_y mode waves. In the present analysis, the complex 3D periodic foundation system is simulated by using a simple model, in which three unit cells are set in *z* direction. The upper structure is simplified as a lumped mass with a beam. The simplified upper structure set on the top surface of the periodic foundation is considered (i.e. PF). For comparison, the simplified upper structure with concrete foundation is also considered (i.e. CF).

The side length of the cube unit cell is 1m, the side length of the cube core is 0.7m, and the thickness of the rubber coating is 0.1m. As given in Section III, the first frequency band gap for the infinite periodic structure composited by this unit cell, is in the region from 8.50Hz to 17.15Hz. For the upper-structure, the lumped masses are m=120kg, and the radius of circle section for the concrete columns is 0.15m.

Our simulations are conducted by using the commercial software ANSYS 10.0. The solid foundation is simulated by using the element SOLID-45. For the simplified upper-structure, the concentrated mass and the beam are simulated by using the element MASS-21 and element BEAM-189, respectively. Additionally, the interaction between the element Beams-189 (6 degrees of freedom) and the element Solide-45 (3 degrees of freedom) are simulated by three node coupling constraints and three constraint equations.



Layout of periodic foundation   Simplified models

Fig. 14. Layout of periodic foundation and simplified models

A. Frequency response function

To obtain the frequency response function in one direction, fixed displacement boundary conditions will be set on the bottom surface of foundations in the other two directions. And a displacement input in the direction with unit amplitude and different frequency is added on every node in the bottom surface of foundations. The frequency of the input varies in (8.5~18) Hz, with a interval Δf = 0.5Hz. The steady-state responses for every node are obtained.

In our numerical simulation, frequency response functions, in z direction (vertical direction) and in y direction (horizontal directions), are considered. As the model is symmetrical in x and y directions, results in x direction are the same with those in y direction.

The frequency response function is defined as:

$$FRF = 20\lg(x_{rep} / x_{inp}) \qquad (6)$$

where x_{rep} is the response and x_{inp} the input.

Fig.15 presents the displacement FRF of the top node with the model under the displacement input in z direction, the so-called P mode wave input. Obviously, for the case with periodic foundation responses are lower than that of the case with concrete foundation at the beginning of the band gap. However, as f>11.5Hz responses of the top node with PF are larger. The result is lined with the so-called FANO-like phenomenon, analyzed by Goffaux *et al.*[15]. Similarly, the displacement FRF for the point of the lumped mass with the displacement input in y direction, the so-called S_y mode wave inputs are shown in Fig.16. For the case with periodic foundation responses for the lumped mass is smaller than that with concrete foundation. The FANO-like feature is not obvious in this case.

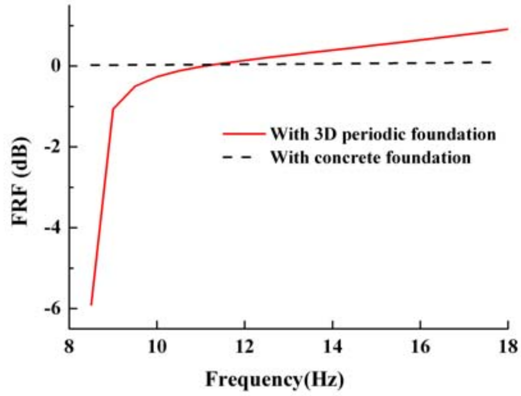


Fig. 15. Displacement frequency responses function of the lumped mass node in vertical direction

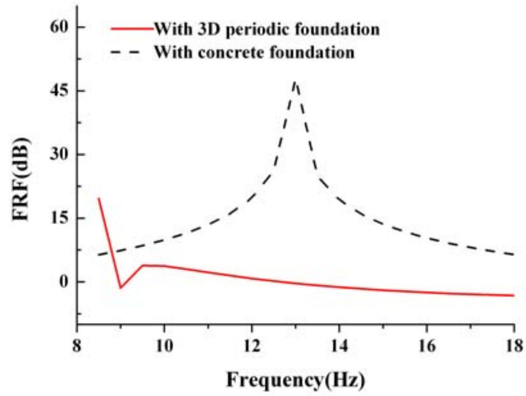


Fig. 16. Displacement frequency responses function for the lumped mass node in horizontal direction

B. Seismic isolation analysis

In this part, the efficiency of the periodic foundation to isolate seismic waves under multi-dimensional directions is analyzed. Damping effect is neglected in our analysis. By using the Big-Mass-Method, the acceleration records are applied in z and y directions on all nodes at the bottom surface. According to the symmetrical property of the model, vibration properties in z (vertical) direction and y (horizontal) direction are considered only. The 1984 Bishop (Rnd Val) seismic acceleration records MCG-UP and MCG360 are useed for every node at the bottom surface of foundation in z and y directions, respectively. The main frequencies of the seismic records MCG-UP and MCG360 are modified to 8.9Hz and 9.0Hz, respectively. The seismic records are obtained from the PEER Ground Database [16].

Fig.17 and Fig.18 show the acceleration responses of the lumped mass node in z (vertical) and in y (horizontal) directions, respectively. Obviously, for the case with PF acceleration response of the lumped mass node is smaller than that of the case with CF. The results indicate that the 3D periodic foundation can isolate the seismic waves in all three directions.

V. Future Work

Based on the results obtained in this study, in the near future the attention should be focused on the optimization methods to obtain wider and lower band gaps of periodic structures and the experimental verification of 3D periodic foundation.

Studies will be conducted to investigate the optimized

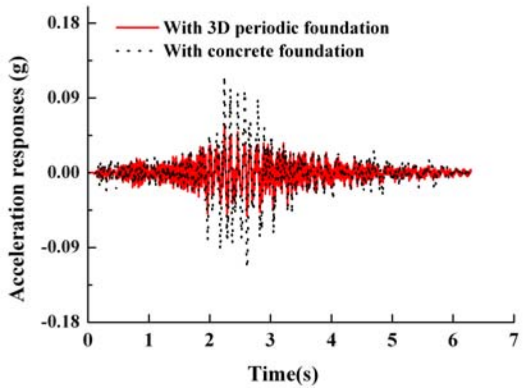


Fig. 17. Acceleration responses for the lumped mass node in vertical direction under the 1984 Bishop MCG-UP seismic record

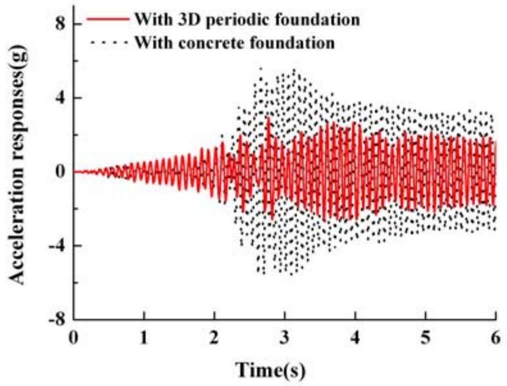


Fig. 18. Acceleration responses for the lumped mass mode in horizontal direction under the 1984 Bishop MCG360 seismic record

periodic foundation with lower and wider band gaps. On one hand, for actual structural application, the length of the unit cell in the vertical direction will be reduced. In this manner, directional band gaps need to be analyzed. On the other hand, based on the local resonant theory, the band gaps will be lowered by optimizing the inner oscillators, like replacing the soft coating with springs, using the high-density concrete as the core masses, and so on.

For experimental study, a 3D periodic foundation model, with an steel frame as a upper-structure, will be designed and fabricated to verify the efficiency of the 3D periodic foundation. In the study, the experiment with real seismic input will be investigated. Also, for engineering structural isolation application, shake table tests for full-scale 3D periodic foundations are also in our schedule.

VI. Conclusions

This work studied the feasibility of a 3D periodic foundation-based structural vibration isolation. From the elasto-dynamic theory, the absolute frequency band gap for infinite periodic structure is found. Using numerical simulations the efficiency of seismic isolation for the periodic structure is investigated. Some conclusions can be made:

1) By using construction materials, concrete, rubber and steel, two types of 3D periodic structures are studied. For both cases, the frequency band gaps are found in low-frequency regions. Comparatively speaking, for the case with coated cube core, the frequency band gap is wider and lower than for the case with coated sphere core, as taking the side length of the core and the diameter of the sphere core with the same value.
2) Geometrical parameters and materials parameters of the unit cell play important role in the frequency band gap. The frequency band gap will be lowered with the increase of the side length of the unit cell and the thickness of the rubber coating. With the increasing the side length of the cube core, the band gap will be lower and wider. The softer coating material will give the lower and narrower band gap; the heavier core material will provide the lower and wider band gap.
3) Numerical simulations show the large vibration attenuation can be found in the band gap as vibration traveling through three units. The seismic isolation analysis shows that the proposed 3D periodic foundation has potential application in multi-dimensional structural vibration isolation.

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