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Effect Of Magnetic Field On Convection Of Radiating Gas in A Vertical Channel Through Porous Media

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Abstract - The free convective flow of radiating gas between two vertical thermally conducting and MHD walls in the presence of a uniform gravitational field through porous medium has been studied. Solutions for the velocity and temperature have been obtained for the optically thin limit when the wall temperature varies linearly with the vertical distance. Then we observed that the fluid velocity increases and the temperature difference between the walls and the fluid decreases with an increase in the radiation parameter. The effect of various parameter like the magnetic field (M)Rayleigh number Ra effect of porosity parameter (δ) and radiation parameter F has been discussed. The velocity profile and temperature field for fluid and thermal conductance walls for different parameter are discussed numerically and explained graphically.

Keywords - Radiation, Porous Medium, Heat Transfer Thermal Conductance and Magneto Hydrodynamic.

1. Introduction

In recent years, free convective flow of viscous fluids through porous medium has attracted the attention of a number of researchers in view of its wide application to geophysics, astrophysics, meteorology, aerodynamics, and boundary layer control and so on. In addition, convective flow through a porous medium has the application in the field of chemical engineering for filtration and purification processes. In petroleum technology, study the movement of natural gas oil and water through oil channels and in the field of agriculture engineering to study the underground resources, the channel flows through porous medium have numerous engineering and geophysical applications. However, these studies are confined to normal temperatures of the surrounding medium. If the temperature to the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effects of radiation and free convection. Magnetic effect also plays a very important role in all the fluid flow problems and it makes wide applications to many engineering fields if radiation effect taken together with it. Many times radiation with magnetic field affects and in many biomedical cases this happens. Channels are frequently used in various applications in designing ventilating and heating of buildings, cooling electronic components, drying several types of agriculture products grain and food and packed bed thermal storage. Convective flows in channels driven by temperature differences at bounding walls have been studied and reported extensively in literature. Radiative convective flows are frequently encountered in many scientific and environmental processes such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers and solar power technology. Several researchers have investigated convective flow in porous medium such as Nield and Bean [1], Sparrow and Cess [2], Burmeister [3], Bejan [4], Kaviany [5] and Vafai [6], Raptis [7,8] has studied the effect of radiation on free convection flow through a porous medium. The natural convection cooling of vertical rectangular channels in air considering radiation and wall conduction has been studied by Hall et al [9]. Al-Nimr and Haddad [10] have described the fully developed free convection in open-ended vertical channels partially filled with porous material. Thermal dispersion-radiation effects on non-Darcy natural convection in a fluid saturated porous medium have been investigated by Mohammadein and EL-Amin [11]. The effect of wall conductance on free convection between asymmetrically heated vertical plates has been studied by Kim et al. [12]. Greif et al. [13] have made an analysis on the laminar convection of a radiating gas in a vertical channel. Effect of wall conductance on convective magneto hydrodynamic channel flow has been investigated by Yu and Yang [14]. Gupta and Gupta [16] have studied the radiation effect on hydro magnetic convection in a vertical channel. Data and Jana [15] have studied the effect of wall conductance on hydro magnetic convection of a radiation gas in a vertical channel. Makinde and Mhone [17] investigated the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated

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porous medium and non-uniform wall temperatures. Narahari [18] has investigated the effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary. Mrinal Jana et al [21] have studied about Conduction of Radiating gas in presence of Porous medium in a vertical channel and for this they Considered the Viscous incompressible flow of the fluid.

In this paper, we have studied the Magneto hydrodynamic free convection flow of a radiating gas between two vertical thermally conducting walls embedded in porous medium. The governing equations are solved analytically. The effects of the permeability of the porous medium, effect of magnetic parameter, the influences of radiation parameter, thermal wall conductance on velocity filed and temperature distribution are investigated. On the basis of analysis their graphical representations are being made. It is observed that the fluid velocity equation shows $u_1(\eta)$ increases whereas the temperature distribution $\theta(\eta)$ decreases with an increase in the radiation parameter F. It is also observed that both the fluid velocity $u_1(\eta)$ and temperature $\theta(\eta)$ in the flow field increase with an increase in the porosity parameter δ. It is found that the fluid velocity decreases while the temperature increases with an increase in the thermal conductance Ψ. Further, it is found that radiation causes to decrease the rate of heat transport to the fluid thereby reducing the effect of natural convection. The rate of flow increases with an increase in either radiation parameter F or Rayleigh number.

2. Formulation Of The Problem And Its Solutions

Consider a fully developed flow of a viscous incompressible fluid flow in a vertical channel embedded in porous medium. The distance between the channel walls is 2L.. Employ a Cartesian coordinates system with z- axis vertically upwards along the direction of flow and y-axis perpendicular to it. The origin of the axes is such that the channel walls are at positions y=-L. and y = L (see Figure 1).

For the fully developed laminar flow in porous medium, the velocity and the temperature field have only a vertical component and all of the physical variables except temperature and pressure are functions of y. The temperature inside the fluid can be written as

$$T = T^*(y) + Nz \qquad \ldots(1)$$

where N is the vertical temperature gradient.

On the use of (1), the momentum and energy equations are simplified to the following form

$$v\frac{d^2u}{dy^2} - \frac{v}{k}u + g\beta(\theta^* + Nz) - \frac{\sigma B_0^2 u}{\ell} - \frac{\partial p}{\rho \partial z} = 0. \qquad \ldots(2)$$

$$-\frac{\partial p}{\rho \partial z} = 0 \qquad \ldots(3)$$

$$Nu = \alpha\frac{d^2\theta^*}{dy^2} - \frac{1\partial q_r}{\rho C_p \partial y} \qquad (4)$$

Where $\theta^* = T - T_w, v$ is the kinematic coefficient of fluid viscosity. g the acceleration due to gravity. k permeability of the porous medium and α the thermal conductivity.

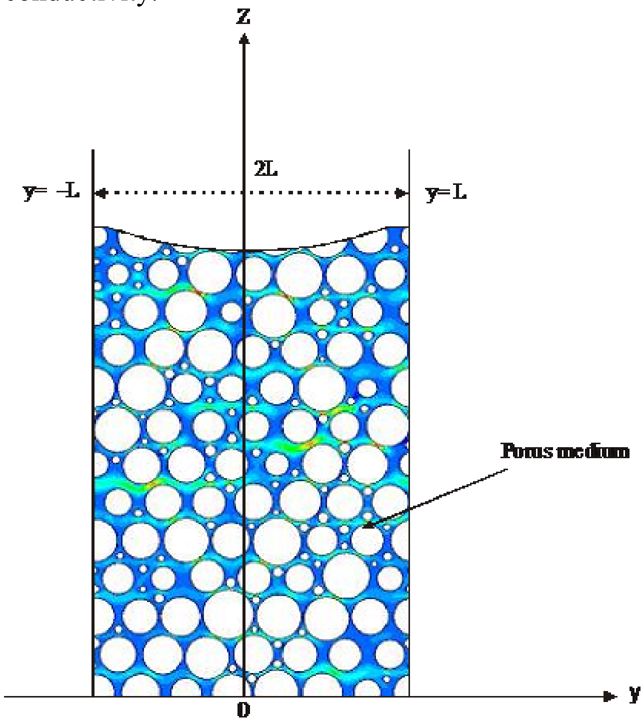

Fig. 1 – Geometry of the problem.

In the optically thin limit the fluid does not absorb its own emitted radiations. This means that there is no self-absorption but the fluid does absorb radiation emitted by the boundaries. Cogley et al [19] showed that in the optically thin limit for a non-grey gas near equilibrium the following relation holds.

$$\frac{\partial q_r}{\partial y} = 4\left(T - T_w\right)\int_0^\infty K_{\lambda_u}\left\{\frac{\partial e_{\lambda h}}{\partial T}\right\}_w d\lambda \qquad \ldots(5)$$

Where $K_{\lambda u}$ the absorption coefficient, $e_{\lambda h}$ is the Planck function and the subscript 'w' refers to values at the wall. Further simplifications can be made concerning the spectral properties of radiating gases ([20]) but are not necessary for our investigation

On the use of (5). Equation (4) becomes.

$$Nu = \alpha\frac{d^2\theta^*}{dy^2} - c\theta^*, \qquad \ldots(6)$$

where

$$c = \frac{4}{\rho c_p}\int_o^\infty k_{\lambda\alpha}\left\{\frac{\partial e_{\lambda h}}{\partial \mathrm{T}}\right\}_o d\lambda \qquad \ldots(7)$$

Subscript "0" indicates that all quantities have been

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evaluated at the entrance temperature T_{w0} which is the temperature of the wall at z=0.

Integrating Equation (3) we get

p = f (z) …(8)

On use of (8), Equation (2) becomes

$$\nu \frac{d^2u}{dy^2} + g\,\beta\theta^* - \frac{\nu}{k}u \frac{-\sigma B_0^2 u}{\rho} = -C_1 \quad \ldots(9)$$

where

$$C_1 = -\left[\frac{1}{\rho}\frac{\partial f}{\partial z} - g\,\beta N_Z\right] \quad \ldots(10)$$

Introducing the non-dimensional variables

$$\eta = \frac{y}{L}, u_1 = \frac{uL}{\alpha c_z}, \theta = -\frac{\theta^*}{NlC_z}, C_z = \frac{C_1L^3}{\nu\alpha} \quad \ldots(11)$$

$$y = \eta L$$

and on using (11), Equations (9) and (6) become

$$\frac{d^2u_1}{d\eta^2} - Ra\theta - \xi u_1 = -1$$

Where $\frac{1}{\delta} + M = \xi$ …(12)

$$\frac{d^2\theta}{d\eta^2} - F\theta = -u_1 \quad \ldots(13)$$

where $\delta = \frac{k}{L^2}$ is the porosity parameter, $Ra = \frac{g\beta NL^4}{\nu L}$ is the Rayleigh number, $F = \frac{cL^2}{\alpha}$ is the radiation parameter $M = \frac{\sigma\beta_0^2 L^2}{\rho\nu}$ is Magnetic parameter.

The dimensionless velocity and the temperature boundary conditions are

$$u_1 = 0 \; at \; \eta = \pm 1,$$

$$\frac{d\theta}{d\eta} + \frac{\theta}{\Psi} = 0 \;\; at \;\; \eta = \pm 1 \quad \ldots(14)$$

where Ψ is the thermal conductance ratio.

Eliminating μ_1 from (12) and (13), we obtain

$$\frac{d^4\theta}{d\eta^2} - \{\xi + F\}\frac{d^2\theta}{d\eta^2} + \{Ra + F\xi\}\,\theta = 1 \quad \ldots(15)$$

The solution of $\theta(\eta)$ satisfying the boundary conditions (14) is easily obtained. Achieving $\theta(\eta)$, one can determine $u_1(\eta)$ from (12) using the boundary condition (14).

The solutions for $\theta(\eta)$ and $u_1(\eta)$ subject to the boundary conditions (14) are

$$u_1(\eta) = a_4\left[F - a_2\left(F - m_2^2\right)\cosh m_2\eta\right. \quad \ldots(16)$$

$$\theta(\eta) = a_4\left[1 - a_2\cosh m_2\,\eta - a_3\{\cosh m_1\eta - a_1\cosh m_2\eta\}\right] \quad \ldots(17)$$

where

$$a_1 = \frac{\Psi m_1 \sinh m_2 + \cosh m_1}{\Psi m_2 \sinh m_2 + \cosh m_2}$$

$$a_2 = \frac{1}{\Psi m_2 \sinh m_2 + \cosh m_2}$$

$$a_3 = \frac{F - a_2\left(F - m_2^2\right)\cosh m_2}{\left(F - m_1^2\right)\cosh m_1 - a_1\left(F - m_2^2\right)\cosh m_2}$$

$$a_4 = \frac{1}{Ra + f\xi} \quad \ldots(18)$$

$$m_1^2 = \frac{1}{2}\left[\{\xi + F\} + \left\{(\xi - F)^2 - 4Ra\right\}^{1/2}\right]$$

$$m_2^2 = \frac{1}{2}\left[\{\xi + F\} - \left\{(\xi - F)^2 - 4Ra\right\}^{1/2}\right] \quad \ldots(19)$$

It is observed from the Equations (16) and (17) that the velocity and temperature depend on the parameters δ, F, Ra and Ψ.

Case-I: Constant wall temperature $(\psi = 0)$

The temperature distribution $\theta(\eta)$ and velocity $u_1(\eta)$ for constant wall temperature are given by

$$\theta_1(\eta) = \frac{1}{Ra + F\xi}\left[1 - \frac{m_2^2}{\left(m_2^2 - m_1^2\right)}\left(\frac{\cosh m_1\eta}{\cosh m_1}\right) + \frac{m_1^2}{\left(m_2^2 - m_1^2\right)}\left(\frac{\cosh m_1\eta}{\cosh m_2}\right)\right] \quad \ldots(20)$$

$$u_1(\eta) = \frac{1}{Ra + F\xi}\left[F - \frac{\left(F - m_1^2\right)m_2^2}{m_2^2 - m_1^2}\left(\frac{\cosh m_1\eta}{\cosh m_1}\right)\right.$$

$$\left. + \frac{\left(F - m_2^2\right)m_2^2}{\left(m_2^2 - m_1^2\right)}\left(\frac{\cosh m_2\eta}{\cosh m_2}\right)\right] \quad \ldots(21)$$

Where m_1 and m_2 are given by (19)

Case-II: Thermally insulated wall $(\psi = \infty)$

The temperature distribution $\theta(\eta)$ and velocity $u_1(\eta)$ for thermally insulated walls are given by

$$\theta_2(\eta) = \frac{1}{(Ra + F\xi)}\left[1 - \frac{F\left(m_2 \sin m_2 \cosh m_1\,\eta\right) - \left(m_1 \sin m_1 \cosh m_2\,\eta\right)}{m_2\left(F - m_2^2\right)\sinh m_2 \cosh m_1 - m_1\left(F - m_2^2\right)\sinh m_1 \cosh m_2}\right] \quad \ldots(22)$$

$$u_2(\eta) = \frac{F}{(Ra + F\xi)}\left[1 - \frac{\left(\begin{array}{l} m_2\left(F - m_1^2\right)\sinh m_2 \cosh m_1\eta \\ - m_1\left(F - m_2^2\right)\sinh m_1 \cosh m_2\eta \end{array}\right)}{\left(\begin{array}{l} m_2\left(F - m_1^2\right)\sinh m_2 \cosh m_1\eta \\ - m_1\left(F - m_2^2\right)\sinh m_1 \cosh m_2\eta \end{array}\right)}\right] \quad \ldots(23)$$

Where m_1 and m_2 are given in equation (19).

(i) The non-dimensional shear stress at the right wall $(\eta = 1)$ of the channel is given by

$$r = -a_4\left[a_3\left(F - m_1^2\right)m_1 \sinh m_1 + \left(a_2 - a_1a_3\right)\left(F - m_2^2\right)m_2 \sinh m_2\right] \quad \ldots(24)$$

(ii) The rate of heat transfer across the channel's wall is given as

$$\left[-\frac{d\theta}{dn}\right]_{n=1} = a_4\left[a_3m_1 \sinh m_1 + \left(a_2 - a_1a_3\right)m_2 \sinh m_2\right] \quad \ldots(25)$$

(ii) The non-dimensional flow rate is given by

$$W = \int_{-1}^{1} u_1(\eta)\,d\eta$$

$$= 2a_4\left[F - a_1\left(F - m_1^2\right)\frac{\sinh m_1}{m_1} - \left(a_2 - a_1a_3\right)\left(F - m_2^2\right)\frac{\sinh m_2}{m_2}\right] \quad \ldots(26)$$

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3. Results and Discussion

To study the effects of radiation, porosity of the porous medium and magnetic parameter on the velocity field u_1 and temperature distribution θ. We have presented the non-dimensional velocity u_1 and the temperature θ against η for various values of radiation parameter F, Rayleigh number Ra, Porosity parameter (δ), magnetic parameter M and the thermal conductance parameter Ψ.

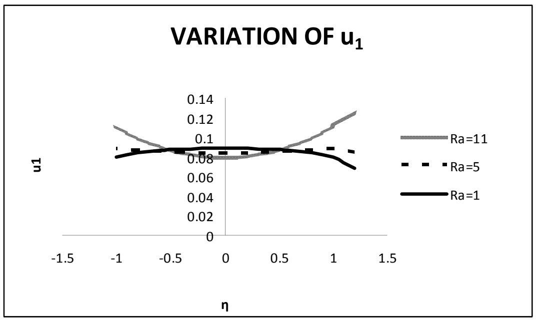

Fig. 2 – Figure is drawn in between u_1 and η for the different values of Rayleigh number Ra. This shows that as we increase Ra the fluid velocity u_1 decreases.

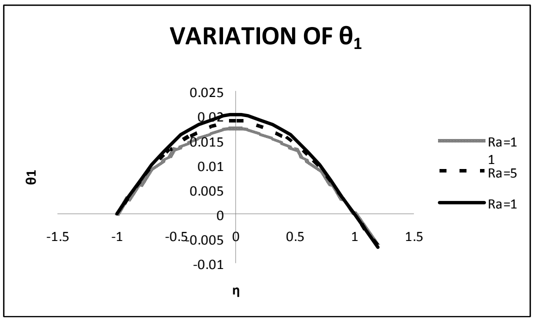

Fig. 3 – Figure is drawn in between θ_1 and η for the different values of Ra. The figure depicts that as we increase Ra, θ_1 increases.

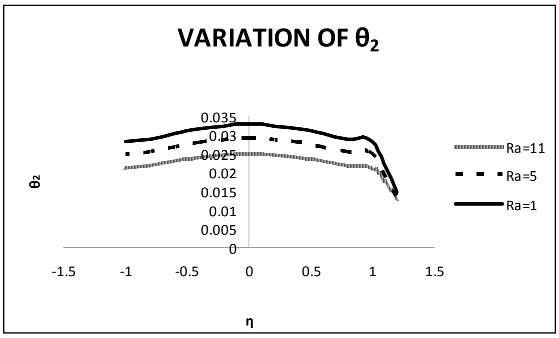

Fig. 4 – Figure is drawn in between u_2 and η for the different values of Ra. The figure shows that as we increase Ra, u_2 decreases.

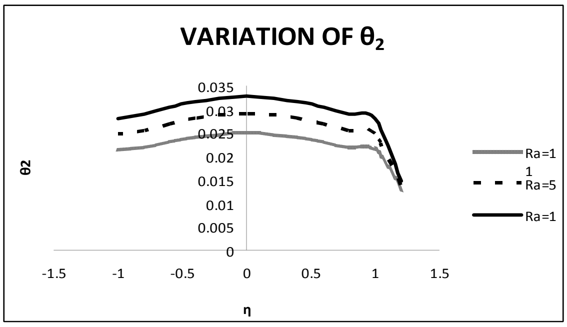

Fig. 5 – Figure is drawn in between θ_2 and η for the different values of Ra. The figure depicts that as we increase Ra, θ_2 decreases.

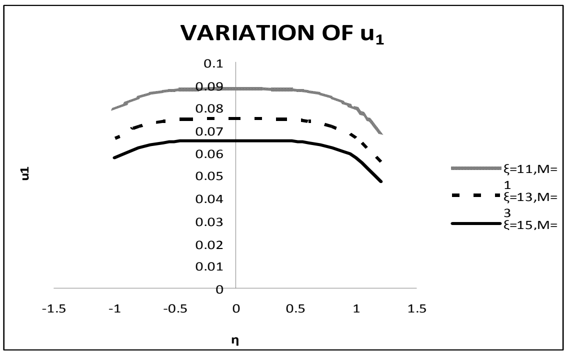

Fig. 6 – Figure is drawn in between u_1 and η for the different values of M figure shows that as we increase M, u_1 decreases.

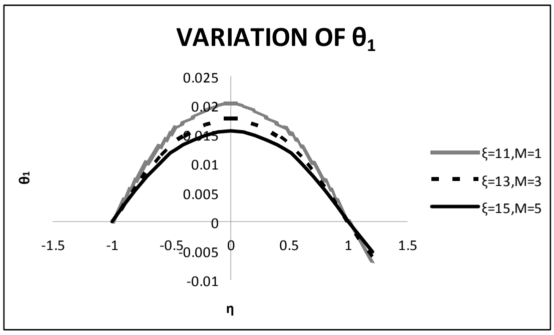

Fig. 7 – Figure is drawn in between θ_1 and η for the different values of M; figure depicts that as we increase M, θ_1 increases in the beginning but suddenly as we increase η with respect to M, it decreases.

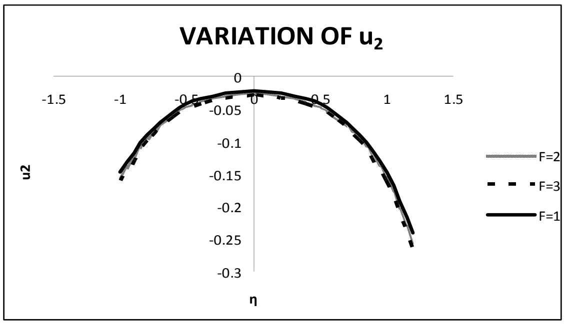

Fig. 8 – Figure is drawn between u_2 and η for the different values of M. The figure shows that as we increase M, u_2 increases.

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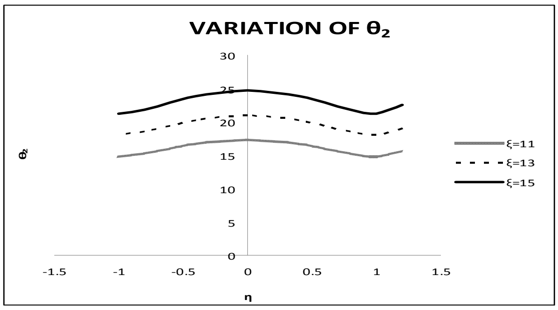

Fig. 9 – Figure is drawn between θ_2 and η for the different values of M figure depicts that as we increase M, θ_2 increases significantly.

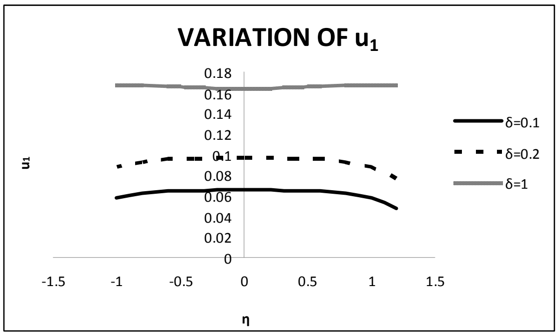

Fig. 10– Figure is drawn between u_1 and η for the different values of (δ) porous parameter. Figure shows that as we increase δ, u_1 increases significantly.

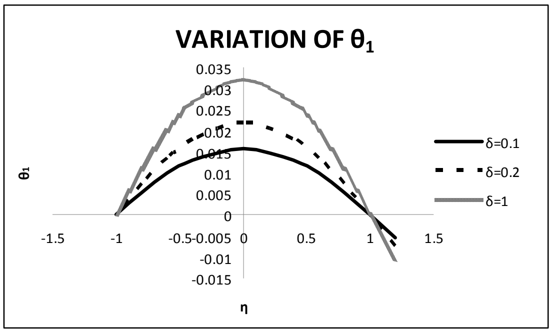

Fig. 11 – Figure is drawn between θ_1 and η for the different values of δ. As we increase δ, θ_1 decreases in the beginning but after reaching a certain value of η (η=1) it increases.

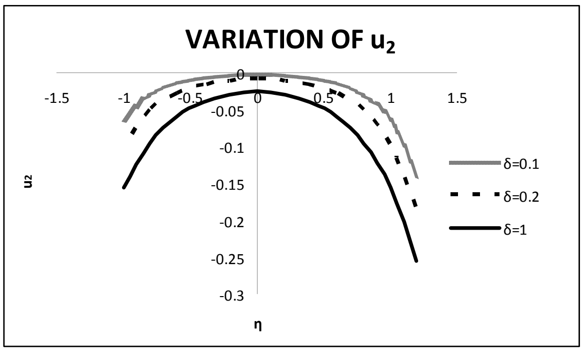

Fig. 12 – Figure is drawn between u_2 and η for the different values of δ. As we increase δ, u_2 decreases.

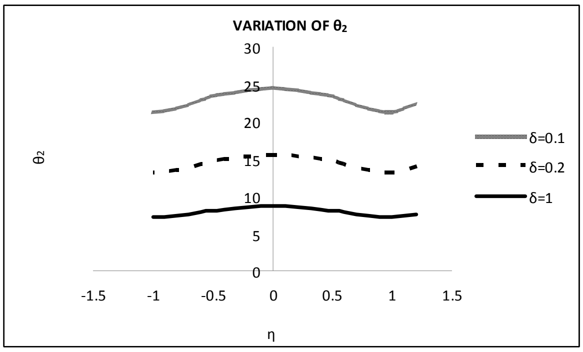

Fig. 13 – Figure is drawn between θ_2 and η for the different values of δ. As we increase δ, θ_2 effectively decreases.

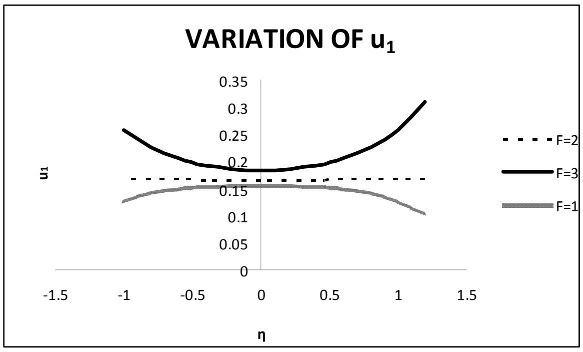

Fig. 14 – Figure is drawn between u_1 and η for the different values of F (Radiation Parameter) as we increase F, u_1 effectively increases.

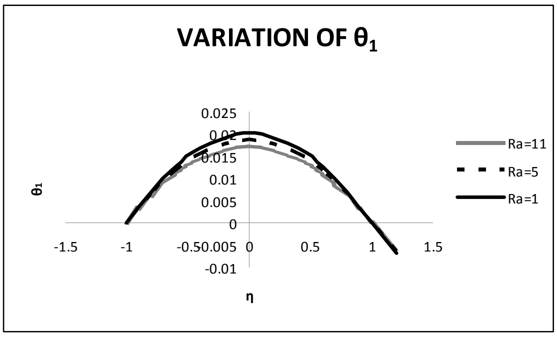

Fig. 15 – Figure is drawn between θ_1 and η for the different values of F. As we increase F, θ_1 in the beginning increases but after reaching a certain value η = 1, it decreases.

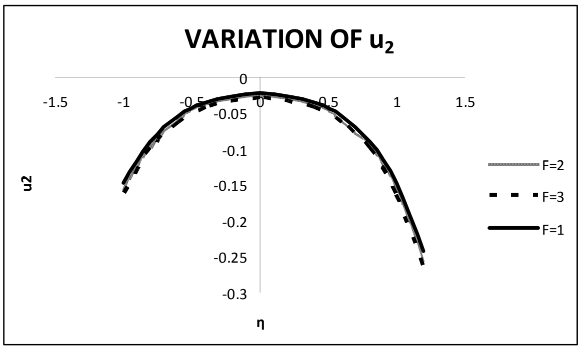

Fig. 16 – Figure is drawn between u_2 and η for the different values of F as we increase F, u_2 decreases.

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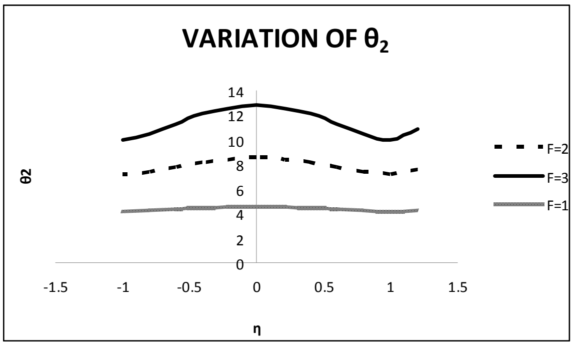

Fig. 17 – Figure is drawn between θ_2 and η for the different values of F as we increase F, θ_2 increases.

4. Conclusions

The fully developed free convection flow of a radiating gas in presence of magnetic field between two vertical thermally conducting walls embedded in porous medium has been studied. The effects of the porous parameter of the porous medium the influences of radiation parameter, and magnetic parameter, Rayleigh number on velocity and temperature fields are investigated and analyzed with the help of their graphical representations.

It is observed that the fluid velocity increases and the temperature distribution $u_1(\eta)$ decreases but $\theta_2(\eta)$ means temperature on increase in thermal conductance increases with an increase in the radiation parameter F. It is also observed that both the fluid velocity and temperature in the flow field increase with an increase in the porosity parameterδ. It is found that the fluid velocity decreases while the temperature increases with an increase in the thermal conductance of the wallsΨ. Further, it is found that radiation causes to decrease the rate of heat transfer to the fluid thereby reducing the effect of natural convection. The rate of flow decreases with an increase in Rayleigh number Ra and increases with increase in F.

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