

# Straightening Locus of a Curved Bimetallic Strip Subjected to Heating

Geoff Angel, George Haritos, Ian Campbell

**Abstract**—this work establishes a way of calculating the free end point position, of a pre-curved bi-metallic strip, that is subjected to uniform heating. The prediction of the endpoint of a bimetallic strip is required during the design phase of an electronic control circuit sensor switch that uses a sensing/activating unit containing a bimetallic strip. Bimetallic sensors are normally flat at ambient temperature, and at the required sensing temperature, the strip bends into a radius of curvature, which displaces the contact on the end of the strip, to make or break an electrical circuit. Although the normal, flat type of bimetallic sensor exists, this work concentrates on a pre-curved bimetallic sensor at ambient temperature. A curved bimetallic strip sensor provides a much larger sensing range and displacement at the free end of the strip, per degree of temperature change, than for a straight bimetallic strip. The greater sensing range is due to the arc length of the bimetallic strip being longer than the chord length of an equivalent straight bimetallic strip. The longer sensing range affords a greater flexibility in the positioning of the actual activation point. Pre-curved bimetallic test samples were subjected to heating whilst the motion of the free end point of the strip was recorded on a metal plate. As the heat applied to the samples was increased, many temperature points were recorded to generate approximate loci of points. The loci of test points compared well to theoretical curve generated by the derived formulae. Therefore the benefits of this work are that a pre-curved bimetallic strip offers a less critical sensing range, and the advantage that the mechanism can be designed to be much smaller and take less space in the product than for a comparable flat bimetallic strips sensor.

**Index Terms**— design, sensors, compact, bimetallic, thermal

## I. INTRODUCTION

The aim of this paper is to introduce a mathematical method of predicting the end point of a pre-curved bimetallic strip that is being uniformly heated. One end of the curved bimetallic strip is rigidly fixed against displacement and rotation, and the other end, is free to move. By the application of a uniformly distributed heat to the curved strip, the strip will straighten up.

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If a Cartesian coordinate system is adopted, a formula can be derived to describe the theoretical locus of the end point of the strip relative to an X and Y coordinate axis system. By using Timoshenko's Equation [1], for evaluating the bending of a bimetallic strip under heating conditions, in conjunction with the straightening formulae produced by this paper, it is possible to predict the radius of curvature and displacement of the end point of the free end, as a function of temperature. It will be shown by an actual bimetallic strip straightening test, how the locus of test points correlate to the theoretical path. This paper can be used at the design stage of a temperature controlled circuit, whereby it is necessary to know the exact position of the end of a curved bimetallic strip for a given particular temperature. With this paper it will also be possible to specify the geometric and material properties of a curved bimetallic strip necessary to achieve other critical design objectives in a temperature controlled circuit.

## II. THEORY

Timoshenko is used to evaluate the radius of curvature of a straight bimetallic strip. With the addition of a correction formula, it is still possible to use Timoshenko, to evaluate the straightening of a pre-curved bimetallic strip. With the radius of curvature found, and with other formulae derived in this work, it is possible to correlate the theoretical end point position of the free end of the strip, to the actual recorded data results from the test. Consider the curved bimetallic strip that is shown in Fig.2, it is rigidly fixed at one end and free to move at the other end. When uniformly heated, it will tend to straighten up if the material side of the strip with the higher coefficient of linear expansion  $\alpha_2$ , lies on the inside surface, see Fig.1. As the free end of the strip straightens up, it will adhere to a locus predetermined by the initial pre-curved "cold" radius of curvature and material properties and make-up of the bimetallic in question.

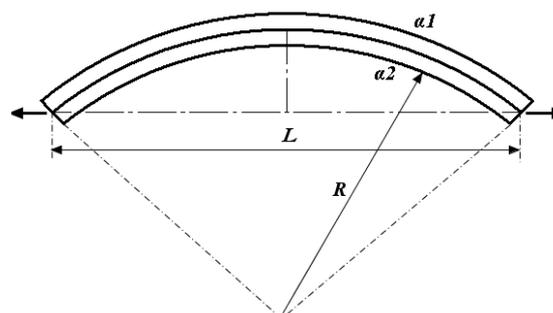


Fig.1 Curved bimetallic strip with  $\alpha_2$  is on the inside

### III. EVALUATION OF THE “HOT” RADIUS OF CURVATURE $R_h$

Application of Timoshenko curvature equation to obtain unloaded “hot” radius  $R_h$

#### Assumptions

- the pre-curved bimetallic strip is rigidly fixed at one end, and free to move at the other end.
- the strip is uniformly heated along the entire length of strip, and the strip remains truly circular.
- no external loads are applied during heating
- the material with the higher coefficient of linear thermal expansion  $\alpha_2$  is on the inside radius  $R$

From Timoshenko[1], the radius of curvature of a bimetallic strip is given by

$$\rho = \frac{t \cdot \left[ 3 \cdot (1+m)^2 + (1+m \cdot n) \cdot \left( m^2 + \frac{1}{m \cdot n} \right) \right]}{6 \cdot (\alpha_2 - \alpha_1) \cdot (T_H - T_C) \cdot (1+m)^2} \quad (1)$$

Where  $\rho$  is the radius of curvature as function of temperature from an ambient flat strip.

$t = t_1 + t_2$  total thickness of the strip,  $t_1, t_2$  being the material thicknesses.

$m = \frac{t_1}{t_2}$  ratio of thicknesses.

$n = \frac{E_1}{E_2}$  ratio of Young’s Modulus.

$T_H$  &  $T_C$  hot and cold temperatures states of the strip

$E_1, E_2$  are the linear Modulus of the two separate materials.  
 $\alpha_2$  &  $\alpha_1$  are the coefficients of linear thermal expansion for the two metals

Whereby  $\alpha_2$  is assumed to be numerically larger than  $\alpha_1$

The  $R_h$  correction equation evaluates the radius of curvature of a heated bimetallic strip from an initially pre-curved radius of curvature  $R_C$  by adding the reciprocals of both radii

$$\frac{1}{R_c} + \frac{1}{\rho}$$

Thus :

$$R_h = \frac{\rho R_c}{\rho - R_c} \quad (2)$$

With  $R_h$  established by the application of the Timoshenko formula, the corresponding “hot” chord length  $L_h$  can now be found. The general chord length of any arc is generally known to be given by:

$$L = 2R \sin\left(\frac{\theta}{2}\right) \quad (3)$$

Where:

$L$  is the chord length mm

$R$  is the radius of curvature mm

$A$  is the arc length (in radians) part of a true circle

$$\theta = \frac{A}{R} \text{ rad.}$$

And thus:

$$L_H = 2R_h \sin\left(\frac{A}{2R_h}\right) \quad (4)$$

The “hot” chord length of the straightened strip.

Evaluation of angle  $\theta$  as a function of hot radius of curvature

$R_h$  is by considering the geometry of the pre-curved bimetallic strip. From Fig.2, two Isosceles triangles exist,  $\Delta oab$  and  $\Delta odc$ . For both triangles, adding all the angles upto 180 degrees:  $R\gamma + \alpha = 180$  and  $2\beta + \omega = 180$ . The third relationship that can be found in Fig.2 is,  $\gamma - \beta = \theta$ .

By manipulation and substitution of these sub-formulae, it can be shown that

$$\theta = \frac{\omega}{2} - \frac{\alpha}{2} \quad (5)$$

with further substitution and manipulation this is equal to:

$$\theta = \frac{A_c}{2} \left( \frac{1}{R_c} - \frac{1}{R_h} \right) \quad (6)$$

Given that  $A_c = A_h$  the strip arc changes shape, not its length.

Where:

$A_c = A_h$  is the arc length of the curved bimetallic strip.

$R_c$  is the cold radius of curvature stated previously and initially known.

$R_h$  is the hot radius of curvature calculated by Timoshenko earlier.

Hence the “hot” endpoint position can be now calculated in terms of X, Y coordinate system, see Fig.3.

$$x = L_h \cos\left(\frac{A_c}{2} \left( \frac{1}{R_c} - \frac{1}{R_h} \right)\right) \quad (7)$$

$$y = L_h \sin\left(\frac{A_c}{2} \left( \frac{1}{R_c} - \frac{1}{R_h} \right)\right) \quad (8)$$

From (2) we have the radius of curvature  $R_h$  as a function of the temperature change from ambient, with this value entered into (7) and (8) the evaluation the  $x, y$  end point position of the bimetallic strip is possible. These formulae were used to generate the theoretical curves used later on in this paper.

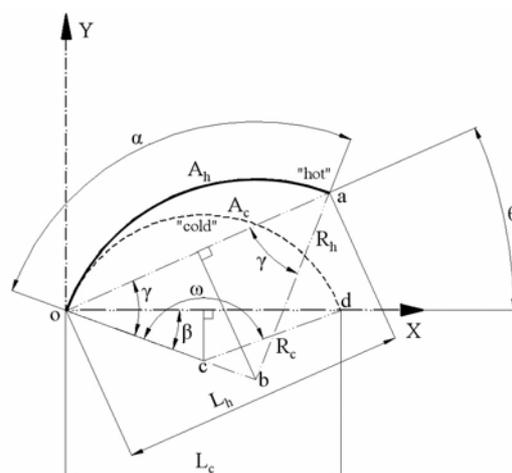


Fig.2 Curved bimetallic strip geometry

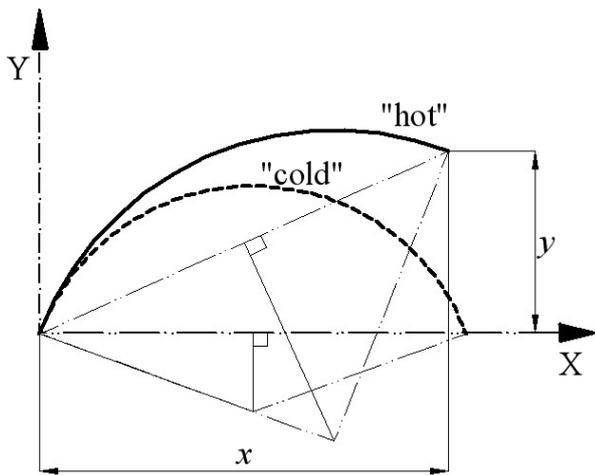


Fig.2 x & y position of heated strip

#### IV. TEST EQUIPMENT

Bimetallic strip used in test; Shivalik SBC-206-1 [2] which were initially 202mm long x 5mm wide x 0.4 mm thick straight bimetallic strip. Four bimetallic strip test samples were made by gently cold working the strips to form true arcs of a circle equal to D64mm, D80mm, D100mm, and D128mm. The bimetallic strips were formed with the material side with the highest coefficient of linear expansion on the inner surface. To ensure that the curved bimetallic strips conformed to a true arc during cold working, special formers were produced to check the diameter and roundness see Fig.4, the formers were held to +/- 0.25mm tolerances. The length of each test sample was cut back to equal half the circumference of the former, i.e. 100.53mm, 125.66mm, 157.07mm, 200.06mm long respectively, within a tolerance of +/- 0.25mm. Each test sample was subjected to heat treatment according to the Kanthal handbook [3], to 350 °C for a 2 hours before the actual testing took place, this was to normalize the strips from any work hardened induced stresses from the cold forming process.



Fig.4 Test samples checking for roundness & size

4-Curved bimetallic test samples, tagged as: D64, D80, D100, D128, Hanna HI 93530 K-Thermocouple Thermometer Digital: Thermocouple: position T1. TES1319 K-Type Thermometer 2 off: Thermocouple: position's T2 and T3. Solex, Digi-Thermo ST 4060 Digital Thermometer recording ambient temperature  $T_a$ . Each thermocouple was affixed to each test sample by a spring clip on the outside surface, and shielded by the body of the test sample from direct hot air flow.

The positioning pattern of the three thermocouples was the same for all test samples, see Fig. 6. Bosch 2.3kW GHG 660 LCD Professional ; variable flow hot air gun; adjustable heat settings in increments from 10 to 600 °C. Fan type nozzle for maximum flow spread along the test samples. 50°C the lowest temperature setting output of the gun. Hot air flow rate and position of heat gun fixed for all testing.

Heat flow was perpendicular to the Aluminium Base plate for a constant uniform heating environment. A - 5mm heat stabilising plate was placed underneath the Aluminium base plate. A heat stabilising shield was placed around the test pieces during testing.

A 1mm thick sheet Aluminium base plate was used for recording locus of points during the tests. Test sample holder clamped to the Aluminium base plate using a workshop "G" clamp. The test samples were clamped in the test sample holder to 1mm depth for each strip. The test samples were measured to be 1mm parallel to, and clear of the Aluminium base plate throughout all tests. See Fig.6. A black fine felt tip pen was used for recording data points on the Aluminium base plate, see Fig.5.

#### V. TEST METHOD

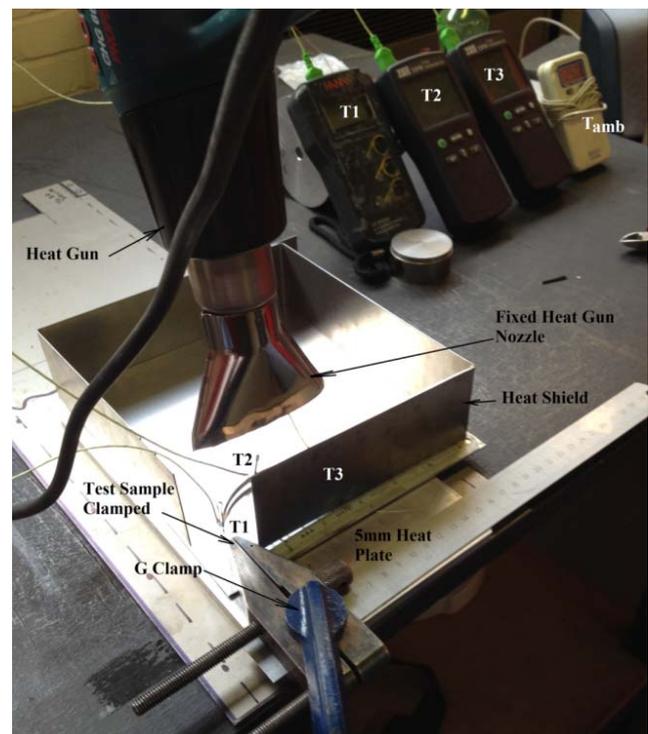


Fig.5 Test Setup

Each test sample was clamped parallel to the Aluminum base plate within the heat stabilized zone. One end of the bimetallic strip was rigidly fixed, the other end 1mm from the plate, was free to move, see Fig 6. Each test sample was subjected to uniform heating, and as the strip straightened up, the locus of the free end point was recorded on the Aluminum base plate using the felt tipped pen, and at each point, the corresponding thermocouple temperature was recorded, see Fig. 7. The heat from the gun was increased in steps of 20°C, and an identifiable locus of points was produced for each test sample, see Fig.7.

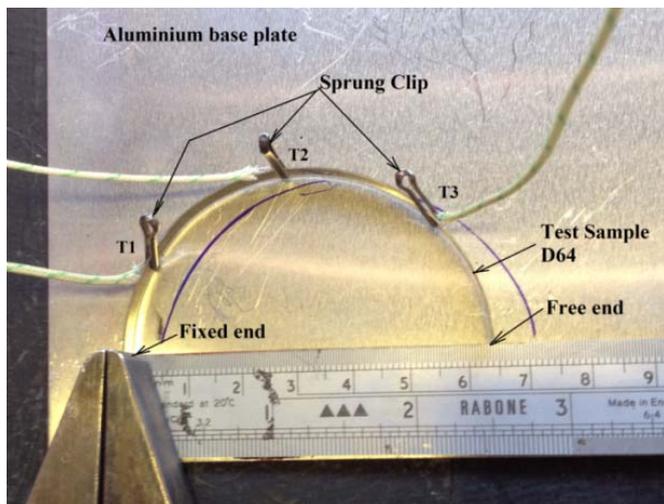


Fig.6 Test Sample setup

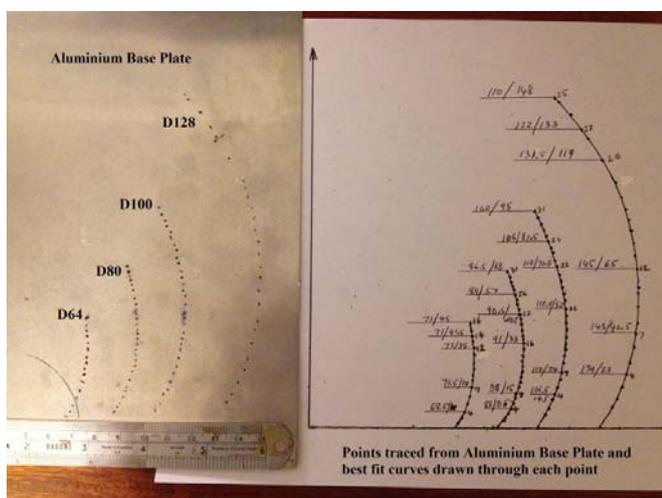


Fig.7 Full set of test data points transferred to paper and X, Y, distances measured

## VI. TEST RESULTS & DISCUSSION

The result of heating and plotting the loci of points are shown in Fig.8. The data points were plotted onto the surface of the Aluminum base plate. The range of points were plotted for a thermocouple temperature range of up to 210 °C for an ambient room temperature in the range of 21 to 24 °C, and sufficient data points were obtained to identify the locus of the free end of the strip for each test sample. For each test sample, five to seven data points corresponding to the specific thermocouple temperatures, were measured from the best fit curve in Fig.8 and recorded in Table's 2, 3, 4, & 5. see Appendix 3. Sample test points

were plotted against the calculated curves generated by the formulae presented in the theory section of this paper, and a good correlation was observed for the majority of the test points.

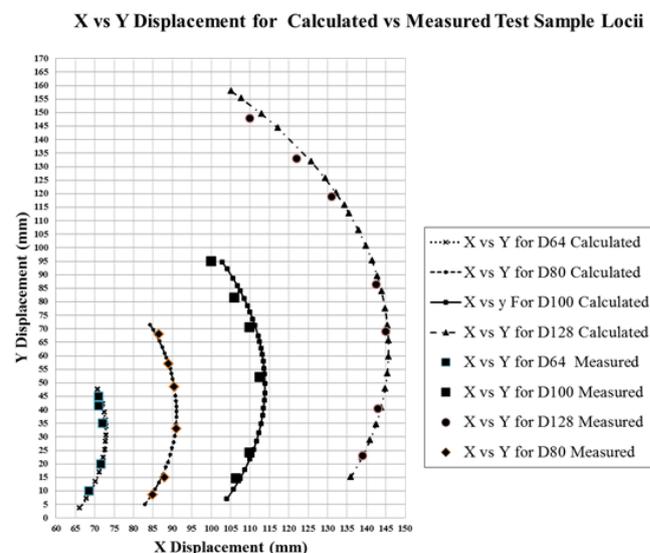


Fig.8 Comparison of theory to test data

Generation of the theoretical calculated data curves in Fig. 8, and in Tables 2, 3, 4, & 5 were computed using a Mathcad program. The properties used to calculate the theoretical values were as follows:

Thickness of each metal 0.2mm equal ; total thickness = 0.4mm, Young's Modulus of Steel 210 GN/m<sup>2</sup> , Young's Modulus of Invar 36 : 145 GN/m<sup>2</sup> , source.[4]

Coefficient of linear expansion for steel:  $20 \times 10^{-6}/K$ . [4]

Coefficient of linear expansion for Invar 36:  $1.85 \times 10^{-6}/K$ .

With the above values and the Timoshenko formula, coupled with the same thermocouple temperatures from the test samples, the theoretical data points were generated.

Despite the manual method of plotting and recording of the data points, a good correlation exists between the theoretically derived curves and the sample data points from the tests, as can be shown in Fig.8. The best correlation occurs on the smaller test samples, whereby the heat source was the closest to the test samples. The larger the test piece, the further the test sample moved from the fixed direct heat source and thus the scatter of the data points was the greatest. On sample D128, the largest deviation from the theoretical curve was recorded, this was due to the continuous movement of the free end of the bimetallic strip during heating, a phenomena known as hunting. The correlation results are shown in Table 1 with an overall average percentage error of the four test samples amounting to X% error **0.35** and the Y% error **5.2** the worst deviation in the D128 sample, Y axis which was **9.32%**, again due to the inaccuracy of recording caused by the hunting of the free end of the test sample. The test results were shown on the whole, to have a good correlation with the theory, and thus the equations in this paper can be used with a high confidence as a means of predicting the end point position of the free end of a curved bimetallic strip, when subjected to uniform heating, and unloaded from any external forces.

VII. CONCLUSION

This work provides a means of calculating the free end point position of a curved bimetallic strip subject to uniform heating. With the aid of a Microsoft Excel work sheet or other similar electronic worksheet, the major equations can be easily evaluated for any curved bimetallic strip to provide design options in any control circuit using a bimetallic element as the sensing unit. The low overall percentage correlation error between the test data and the theory, validates the formulae derived in this paper, and indicates that they can be applied with high degree of confidence to predict the movement of the end point of the strip due to heating.

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- [2] Shivalik S--. Bimetallic strip supplier. 2013.
- [3] Kanthal. Kanthal Thermostatic Bimetal Handbook. Box 502, SE-734 27 Hallstahammar, Sweden: Kanthal; 2008.
- [4] Matweb. Material Properties Data tables. 2013.

APPENDICES

APPENDIX 1 KEY TO TABLES

Point number recorded on Aluminum base plate

Hot air gun temperature (°C).

Thermocouple T1 recorded temperature (°C).

Thermocouple T2 recorded temperature (°C).

Thermocouple T3 recorded temperature (°C).

Tamb. Ambient surrounding recorded temperature (°C).

Average thermocouple temperature calculated (°C).

X distance calculated (mm).

Y distance calculated (mm).

Xm distance measured from best fit curve in millimeters.

Ym distance measured from best fit curve in millimeters.

Xdiff. is the difference between data & theory in (mm).

Ydiff is the difference between data & theory in (mm).

X percentage error for each test sample: X diff./64\*100 (e.g. for test sample D64).

Y percentage error for each test sample: Y diff./64\*100 (e.g. for test sample D64).

APPENDIX 2 PERCENTAGE ERROR SUMMARY TABLE

Table 1 Summary of overall percentage error

Test Sample	Ave. % Error	Ave. % Error
D64	1.15	3.03
D80	-0.36	6.42
D100	1.90	1.89
D128	-1.30	9.32
Overall Ave % Error	0.35	5.17

APPENDIX 3 DATA TABLES

Table 2 Test results for D64 test sample

Sample D64 Point	Recorded temperatures			Calculated			Measured			Correlation				
	Tg	T1	T2	T3	Ambient	T avg	X	Y	Xm	Ym	Xdiff	Ydiff	X% error	Y% error
1	50	39.1	37.2	39.8	23	38.70	66.06	3.52						
2	90	52.4	52.2	55.4	22.8	53.33	67.79	7.05						
3	130	63.9	63.7	68.2	23	65.27	68.99	9.96						
4	170	76.8	76.3	81.2	22.8	78.10	70.15	13.31	68.50	10.00	1.65	3.31	2.58	5.18
5	210	90.4	89.1	95.0	22.9	91.50	71.13	16.86						
6	250	101.3	98.8	104.4	22.9	101.50	71.73	19.59						
7	290	112.3	108.4	113.3	23	111.33	72.20	22.30	71.50	20.00	0.69	2.30	1.09	3.60
8	330	121.8	119.1	123.5	22.9	121.47	72.55	25.20						
9	370	133.4	129.6	132.8	22.9	131.93	72.77	28.21						
10	410	142.2	137.7	139.9	22.7	139.93	72.84	30.54						
11	450	153.1	150.1	149.2	22.9	150.80	72.80	33.72						
12	490	162.4	157.6	156.6	22.9	158.87	72.66	36.10	72.00	35.00	0.66	1.10	1.03	1.73
13	530	171.3	167.5	167.3	22.8	168.70	72.36	39.05						
14	570	183.4	177.6	175.2	22.9	178.73	71.93	42.00	71.00	41.50	0.93	0.50	1.45	0.79
15	610	190.6	185.1	184.0	22.9	186.57	71.49	44.33						
16	650	202.1	196.1	193.2	22.9	197.13	70.75	47.47	71.00	45.00	-0.25	2.47	-0.29	3.86
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
										average	0.74	1.94	1.15	3.03

Table 3 Test results for D80 test sample

Sample D80 Point	Recorded temperatures			Calculated			Measured			Correlation				
	Tg	T1	T2	T3	Ambient	T avg	X	Y	Xm	Ym	Xdiff	Ydiff	X% error	Y% error
1	50	39.1	37.6	39.4	24.4	38.70	82.91	5.03						
2	90	52.2	51.6	56.2	24.3	53.33	85.49	10.58						
3	110	58.8	56.9	63.8	24.3	59.83	86.49	13.13	85.00	8.50	1.49	4.63	1.87	5.79
4	150	73.1	70.1	78.3	24.3	73.83	88.33	18.83						
5	170	77.3	74.1	82.5	24.3	77.97	88.79	20.56	88.00	15.00	0.79	5.56	0.99	6.95
6	210	90.2	86.1	94.7	24.3	90.33	89.92	25.85						
7	330	122.8	116.1	123.0	24.1	120.63	91.06	39.50						
8	350	127.6	120.2	125.6	24.2	124.47	91.03	41.21	91.00	33.00	0.03	8.21	0.03	10.26
9	370	130.7	124.7	131.5	24.1	128.97	90.94	43.33						
10	450	152.8	141.8	145.3	24.2	146.63	90.08	51.47						
11	470	158.7	145.8	147.7	24.1	150.73	89.75	53.42	90.50	48.50	-0.75	4.92	-0.94	6.15
12	490	165.3	150.6	148.5	24.1	154.80	89.38	55.31						
13	530	174.2	158.5	157.0	24	163.23	88.46	59.26						
14	550	180.8	162.6	157.3	24.1	166.90	88.01	60.92	89.00	57.00	-0.99	3.92	-1.24	4.89
15	570	187.6	167.4	160.6	24	171.87	87.33	63.21						
17	650	209.1	187.0	174.8	24.1	190.30	84.21	71.60	86.50	68.00	-2.29	3.60	-2.87	4.50
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
										average	-0.64	5.24	-0.36	6.42

Table 4 Test results for D100 test sample

Sample Point	Recorded temperatures				Calculated				Measured			Correlation		
	Tg	T1	T2	T3	Ambient	T avg	X	Y	Xm	Ym	Xdiff	Ydiff	X% error	Y% error
1	50	36.4	35.6	37.0	23.6	36.3333	104.01	7.03						
2	90	48.0	47.1	52.1	23.5	49.0667	107.44	14.65						
3	110	53.0	51.8	58.4	23.5	54.4	108.67	17.96	106.50	14.50	2.17	3.46	2.17	3.46
4	150	65.8	62.2	69.8	23.5	65.9333	110.91	25.37						
5	170	70.1	66.8	74.3	23.5	70.4	111.61	28.33	110.00	24.00	1.61	4.33	1.61	4.33
6	210	82.1	76.1	82.3	23.5	80.1667	112.82	34.94						
7	310	108.2	96.7	98.7	23.3	101.2	113.82	49.85						
8	350	117.0	104.2	105.6	23.4	108.933	113.61	55.34	112.50	52.00	1.11	3.34	1.11	3.34
9	370	123.5	107.9	106.8	23.4	112.733	113.39	58.09						
10	450	142.5	120.3	113.8	23.4	125.533	112.07	67.37						
11	470	147.1	124.6	121.1	23.4	130.933	111.25	71.29	110.00	70.50	1.25	0.79	1.25	0.79
12	490	153.0	128.9	120.3	23.2	134.067	110.70	73.56						
13	530	162.9	133.4	126.4	23.3	140.9	109.30	78.57						
14	550	171.1	135.6	126.8	23.3	144.5	108.47	81.15	106.00	81.50	2.47	-0.35	2.47	-0.35
15	570	174.1	140.2	131.0	23.2	148.433	107.45	84.03						
16	590	180.3	142.3	131.4	23.3	151.333	106.69	86.02						
17	650	197.8	153.9	139.9	23.3	163.867	102.79	94.79	100.00	95.00	2.79	-0.21	2.79	-0.21
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
									average	1.90	1.89	1.89	1.90	1.89

Table 5 Test results for D128 test sample

Sample Point	Recorded temperatures				Calculated				Measured			Correlation		
	Tg	T1	T2	T3	Ambient	T avg	X	Y	Xm	Ym	Xdiff	Ydiff	X% error	Y% error
1	50	39.4	39.3	41.1	23.2	39.9333	135.94	14.94						
2	90	50.7	52.1	55.4	23.1	52.7333	140.93	28.72						
3	110	55.8	57.3	61.3	23.1	58.1333	142.46	34.51	139.00	23.00	3.46	11.51	2.70	8.99
4	150	68.6	69.4	72.4	23.2	70.1333	144.81	47.73						
5	170	73.6	74.9	76.0	23.1	74.8333	145.34	53.21	143.00	40.50	2.34	12.71	1.83	9.93
6	210	86.4	86.2	83.5	23.1	85.3667	145.66	65.46						
7	310	114.5	111.9	104.9	23.1	110.433	141.43	95.20						
8	350	125.6	121.3	112.8	23.1	119.9	137.98	106.35	142.50	86.50	-4.52	19.85	-3.53	15.51
9	370	130.5	128.2	117.1	23.1	125.267	135.57	112.59						
10	450													
11	470	144.3	135.9	129.8	23.1	136.667	129.37	125.54	131.00	119.00	-1.63	6.54	-1.27	5.11
12	490													
13	530													
14	550	180.1	149.5	132.4	23.1	154	117.24	144.14	122.00	133.00	-4.76	11.14	-3.72	8.70
15	570													
16	590	188.5	152.6	136.9	23.2	159.333	112.96	149.42						
17	650	204.3	158.4	141.4	23.1	168.033	105.16	157.87	110.00	148.00	-4.84	9.87	-3.78	7.71
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
									average	-1.66	11.94	-1.30	-1.30	9.32