A Sustainable Energy Harvesting Machine

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Abstract —this paper introduces a new type of machine that is intended to transform sustainable solar energy into electrical power via the attachment of an additional generator, for space applications. The energy transformation mechanism is via a set of radial pre- curved bimetallic blades arranged in a similar way to a gas turbine layout. The blades are cyclically heated and cooled thus producing a linear force that is translated into rotary output motion in the machine.

Index Terms—design, energy, conceptual, space, sustainable

I. INTRODUCTION

There are many patents for thermal motors, Low [1]or O'Hare[2], to mention just a few, that work on the principle of utilizing the work done by heating and cooling a bimetallic strip in some manner, but most are quite complex contraptions that are neither simple and practical, or proven. The thermal machine concept that this paper puts forward, is both extremely simple and elegant, and has been proven to operate and maintain a continuity of rotary output motion when exposed to a heating and cooling cycle. A series of pre-curved bimetallic blades are radially deployed within the motor to produce an axial thrust, the thrust and reaction within the motor creates rotational movement of the output shaft of the motor. In this short paper, the major steps of the mathematical model of the motor are provided. The basic Timoshenko[3] bi-metallic curvature equation is applied with minor adjustments, to enable the evaluation of the straightening out from an initially smaller radius of curvature, to a larger radius of curvature, as opposed to bending from flat. Figure 1, shows the basic pre-curved bimetallic blade geometry and its chord displacement axis. At ambient or "cold" temperatures, the pre-curved blade possesses a smaller radius of curvature. If mounted so that one end is freely pivoted, and the other end is free to move with a rotational degree of freedom, then the bending of the blade due to a uniform temperature change, is transformed into an axial extension of the free end, along the chord displacement axis.
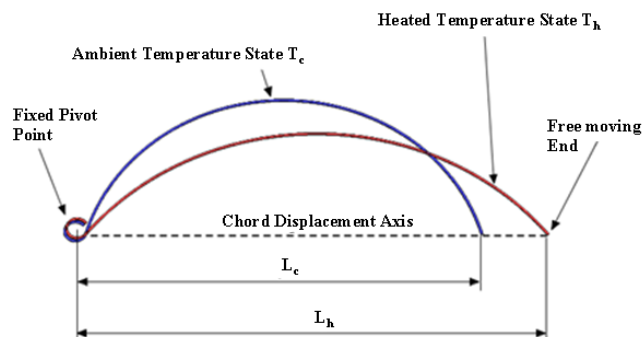
Fig. 1 Chord axial displacement, "straightening up" of a bi-metallic blade.

For the condition that the blade straightens up as in Figure 1, the material sides of the bimetal need to be as shown in Figure 2. A pre-curved bi-metallic blade behaves in the same way as a straight bi-metallic strip, that is, when uniformly heated it will tend to bend due to the differential expansion rates of the intimately bonded metals. The different expansion rates produce internal stresses, forces and couples that ultimately bend the strip. In a pre-curved bi-metallic blade, the material side with the higher coefficient of linear expansion "$\alpha 2$" must be on the inside face of the blade, for it to straighten out, see Figure 2. $\alpha 2$ and $\alpha 1$ denote the separate coefficients of linear expansion for the two metals making up the bimetallic blade.
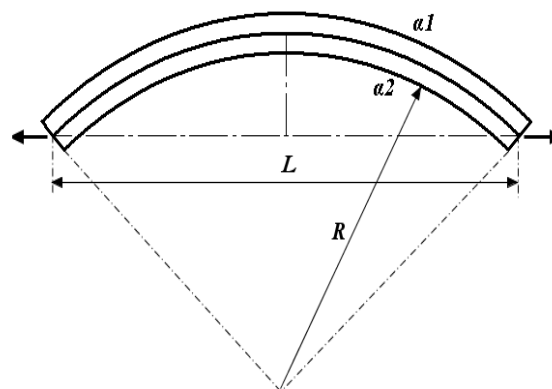

Fig.2. Bimetallic strip and the coefficients of linear expansions $\alpha 1, \alpha 2$.

II. THERMAL MACHINE CONCEPT DESCRIPTION

The machine concept that this paper proposes, consists of a series of radially orientated pre-curved bimetallic blades that are assembled between a rotating outer rotor, and a fixed central ratchet, see Figure 3. The machine put forward in this paper is entirely novel, and although there are many bimetallic powered engines patented, none of them are in the configuration proposed here.

III. MODE OF OPERATION

When an external heat source is applied locally to a curved bimetallic blade within the motor, the differential expansion rates of the two metals of the blade causes the blade to bend, when mounted as previously stated, the bending of the blade translates into a straightening action along the chord displacement axis, the extension of the straightening action, produces a reactionary thrust between the movable outer rotor and the fixed inner ratchet, that results in outer rotor movement. If the motor consists of many radial blades as shown in Figure 3, and providing that a heating and cooling cycle for each blade can be contrived, then continuous motion of the output rotor is possible, whilst the heating source is applied.
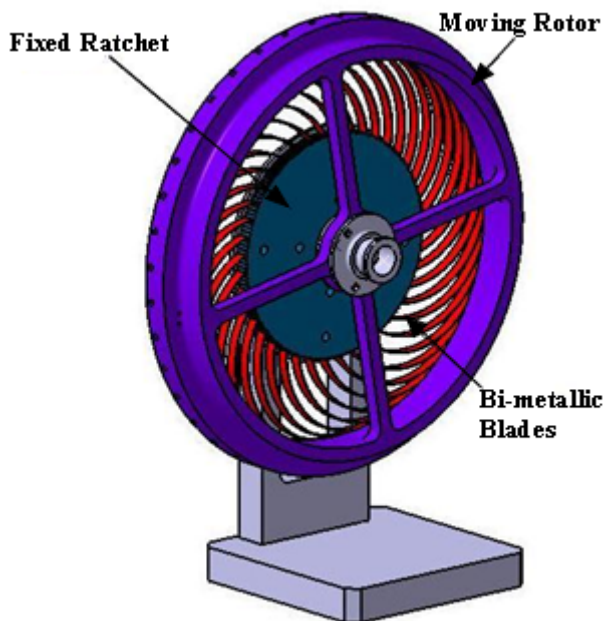

Fig.3 Thermal machine design concept.

IV. DESIRED DESIGN FEATURES

The following essential requirements or features govern and drive the ideology of the proposed thermal machine:
The machine had to be inherently simple & elegant in operation, i.e. only one rotating part.
The machine had to be modular in concept, i.e. facilitate multi-stage expansion.
The machine had to utilize existing proven technology.
The machine had to facilitate the ability to scale up or down, for different applications.
The machine had to be flexible as to its heating and cooling sources.
The machine had to be quiet and vibration less in operation.
The machine had to be designed for zero maintenance requirements.
The manufacture of the machine must be possible in low technology regions around the world.
The machine must not pollute or produce outgasses or exhausts fumes of any kind.
All the above criteria were met in the concept design put forth in this paper, the machine as shown in Fig. 3.

V. THERMAL MACHINE LAYOUT

Figure 3, shows just one of the possible layout configurations of this type of thermal machine. If the main components that make up the machine as proposed are: a rotor, a series of radial blades, and a stator, then the possible layout permutations and configurations are as follows:
A) Inner stator fixed, outer ratchet moving, here the blades are stationary.
B) Outer stator fixed, inner ratchet moving, here the blades are stationary
C) Inner ratchet fixed, outer rotor moving, here the blades move with the outer rotor
D) Inner rotor moving, outer ratchet fixed, here the blades are moving with inner rotor. The chosen layout concept as shown in Figure 3, was C), that is inner ratchet fixed, outer rotor moving.
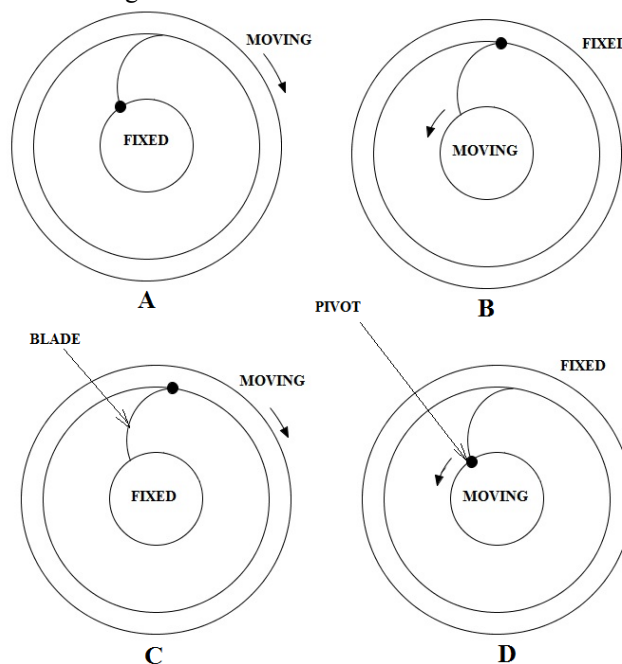

Fig. 4 Thermal machine layout choices.

For Prototype 1, as designed in Figure 3, and as realized in Figure 5, concept layout "C" was chosen, that is, the inner stationary ratchet was fixed, and the outer rim on which the blades were all pivoted, was the output moving rim. The rationale for picking configuration C or D is that it is easier to have a fixed heating and cooling zone, than to have a rotating heating and cooling mechanism, which itself means more components and contradicts the design ideal of a simple, single moving element design.

Fig.5 Prototype 1 with 20 radial "push" blades

In layout configurations A & B of Figure 4, the position of the blades are fixed, for these two concepts to work, the heating and cooling mechanism would need to travel around the machine. In concepts C&D, the blades are pivoted on the moving output shaft, thus the heating and cooling areas can be fixed, such that the blades automatically self - drive into and out of the fixed heating and cooling zones, as the rotor moves around.
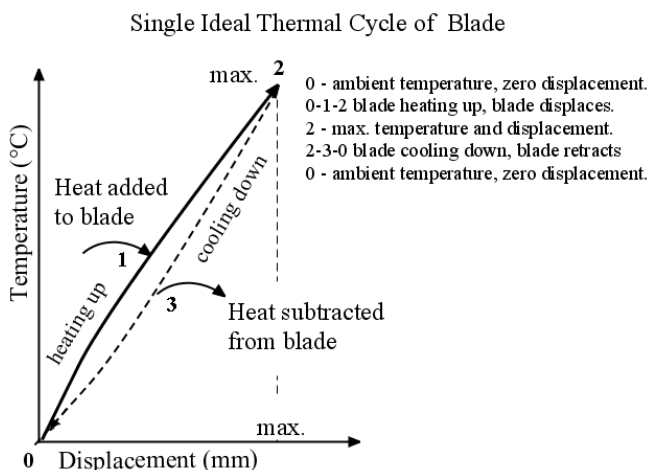
Single Ideal Thermal Cycle of Blade

Fig.6 Ideal thermal cycle of blade

An ideal heating and cooling cycle of a single blade is shown in Fig.6. The cycle shown is reduced to the basic events of heating up the bimetallic and cooling it down again. The heat transfer mechanism assumes uniform heating and cooling along the entire length of blade, and the rate of heat transfer gain is the same for heating up, as it for heat transfer loss in cooling down. To achieve a repeatable cycle time, the heat up and cool down time must also be the same. The mechanism to obtain continous rotational motion in the machine is achieved by the overlap cycle. The overlap cycle
ensures continuity of motion by overlapping the individual thermal cycles of all the blades in the machine, see Fig. 7.
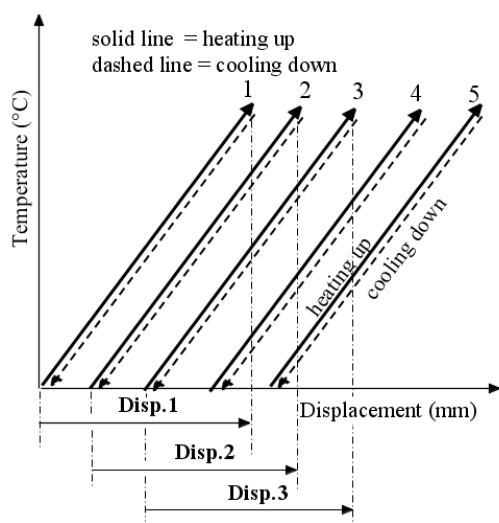

Fig.7 Theoretical overlap cycle of 4 blades

To evaluate the number of blades, and thus overlap cycles required to maintain the continuity of motion in a bimetallic powered machine, is a complex and involved calculation, because it is a function of many variables that are described later in this paper. Figure 7 depicts a simple 5-blade overlap cycle with blades number 1 to 5. According to the overlap cycle, as blade 1 heats up, it moves the rotor, that in turn brings blade 2 into the heating zone. As blade 2 heats up, it too adds displacment to the rotor bringing blade 3 into the heating zone, and so forth. At point 1, blade 1 has reached maximum temperature and displacement and now becomes an inactive blade, that is being driven by the rest of the active blades, into the cooling zone, for it's cycle to repeat. Figure 5, shows a single layer machine and Figure 8 depicts the recent development thermal machine, a 6-stack, multi-layer 54 blade prototype. The 6-stack prototype is a research project that is currently being developed at the University of Hertfordshire Sustainable Energy Technologies Centre.[4]

Fig.8 6-stack thermal machine

VI. THEORETICAL PRINCIPLES

The evaluation of the force that can be derived by the application of uniform heat to an individual pre-curved bimetallic blade, can be calculated with the aid of two separate important formulae. The first formula is by Timoshenko[3], who provides the radius of curvature of a bimetallic strip as a function of temperature. The second step in evaluating the thrust generated by this change of curvature, when mounted as previously stated, is by the application of the Castigliano's[4] minimum energy theorem to find the axial force and displacement in beams. Due to the nature of this paper only the major steps of the mathematical model of the thermal motor are included. The full derivation of the Castigliano[4]formula contains further work on the geometry of arcs, and that section of the derivation has been omitted for the sake of brevity.

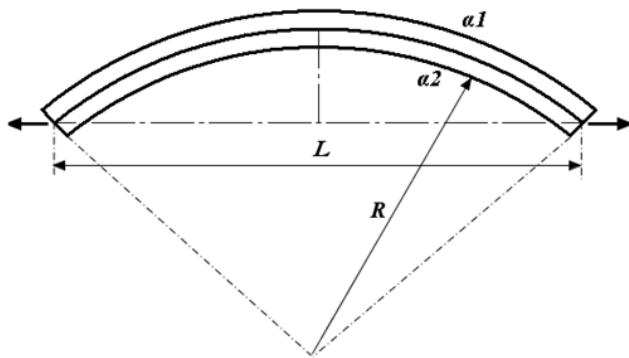## VII. APPLICATION OF TIMOSHENKO'S CURVATURE EQUATION

Fig.9 $\alpha 2 \& \alpha 1$ for a strip of a material with $\alpha 2$ is on the inside.

Assumptions
Pre-curved bimetallic strip is simply supported at each end with a rotational degree of freedom.
Strip is uniformly heated along the entire length of strip, and the strip remains truly circular.
No external loads are applied during heating
The material with the higher coefficient of linear thermal expansion $\alpha 2$ is on the inside radius R.
All equations and formulae are in S.I. units, i.e. N, mm, ^{8}C
Thus by Timoshenko [3]

$$\rho = \frac{t \cdot \left[3 \cdot (1+m)^2 + (1+m \cdot n) \cdot (m^2 + \frac{1}{m \cdot n}) \right]}{6 \cdot (\alpha_2 - \alpha_1) \cdot (T_H - T_C) \cdot (1+m)^2} \quad (1)$$

Where ρ is the radius of curvature as a function of temperature from an ambient flat strip.
$t = t_1 + t_2$: total thickness of the strip, t_1, t_2 being the material thicknesses
$m = \dfrac{t_1}{t_2}$: ratio of thicknesses

$n = \dfrac{E_1}{E_2}$: ratio of Young's Modulus

$T_H \& T_C$: hot and cold temperatures states of the strip
E_1, E_2: are the linear Modulus of the two separate materials $\alpha 2 \& \alpha 1$: are the coefficients of linear thermal expansion for the two metals whereby $\alpha 2$ is assumed to be numerically larger than $\alpha 1$.
The R_h correction equation evaluates the radius of curvature of a heated bimetallic strip from an initially pre-curved radius of curvature R_c by adding the reciprocals of both radii $\dfrac{1}{R_c} + \dfrac{1}{\rho}$.

With the R_h established by the application of the Timoshenko formula, the corresponding "hot" chord length L_h can now be found.
The general chord length of any arc is generally known to be given by:

$$L = 2R \sin(\frac{A}{2R}) \quad (2)$$

L : is the chord length
R : is the radius of curvature
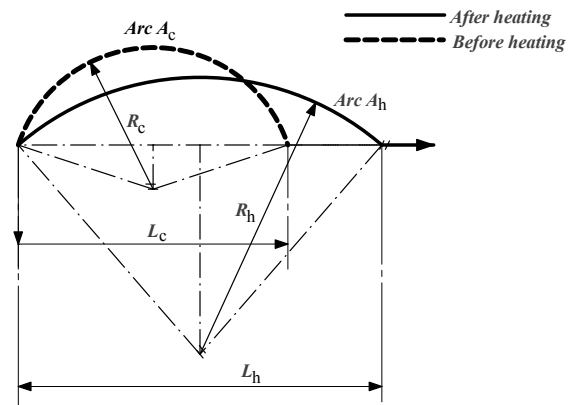A : is the arc length (in radians) part of a true circle

Fig.10 Ambient and heated bimetallic blade radii.

Thus
$$L_H = 2R_h \sin(\frac{A}{2R_H}) \quad (3)$$

and
$$L_c = 2R_c \sin(\frac{A_b}{2R_c}) \quad (4)$$

Whereby A_b is the curved arc length of the bimetallic strip
Hence, using Timoshenko's radius of curvature formula with the chord formulae, it is possible to evaluate the hot length of bimetallic strip, L_h. Given that L_c can be easily evaluated if the pre-curved radius of curvature R_c is known, then the unloaded displacement x, from L_c to L_h can be evaluated by simple subtraction.

Thus;
$$x = L_h - L_c \quad (5)$$

Given $A_b \& R_c$, and ρ from Timoshenko that gives R_h.

VIII. DISPLACEMENT AND FORCE

Application of the Castigliano [4],formula, for evaluating both the force and chord displacement of the pre-curved blade when considered to be a beam, after integration is given by :

$$\delta = \frac{4FR_c^3}{EI} \left[\frac{\omega}{2} - \frac{3\sin(\omega)}{4} + \frac{\omega\cos(\omega)}{4} \right] \quad (6).$$

The total axial deflection of the curved bimetallic strip when subjected to an axial force F
By re-arranging (6) in terms of F, the force developed by a curved bimetallic strip subjected to heating is obtained as a function of the extension x provided previously in (5).
Thus:
$$F = \frac{EIx}{4R_c^3 \left[\frac{\omega}{2} + \frac{1}{4}(\omega\cos(\omega) - 2\sin(\omega)) \right]} \quad (7)$$

Force developed by the bimetallic strip by heating alone.
Where: $x = L_h - L_c$ was evaluated by Timoshenko previously, and values ω, E, I and R have all been stated previously.

δ : is the total deflection due to load F, the direction of F

E: is the averaged modulus of metals in the bimetallic strip

F: an externally applied axial load /or Force by the strip

I: is the Moment of Inertia of section of strip

R_c : is the radius of curvature of the bimetallic strip at

ambient temperature.

A : is the total arc length of the bimetallic strip

$\omega = \dfrac{A_b}{R_c}$: total angle of the entire arc A_b of L_c

The full derivation of the formulae is outside the scope of this short paper, but can be accessed by contacting the author.

Thus for any externally applied static load F, the net loaded displacement x for any temperature differential Δt, experienced by the strip is given by:

$$x = L_h - \delta - L_c \qquad (8)$$

For any application of the above formulae, it is normal to know initial values of $R_c, A_b, E_a, I, \alpha_2 \,\& \, \alpha_1, T_h, T_c$, from which all other values can be calculated.

IX. MACHINE GEOMETRY

For a thermal machine layout type "C" of Figure 4, the general blade geometry within the thermal motor is shown in Figure 11.
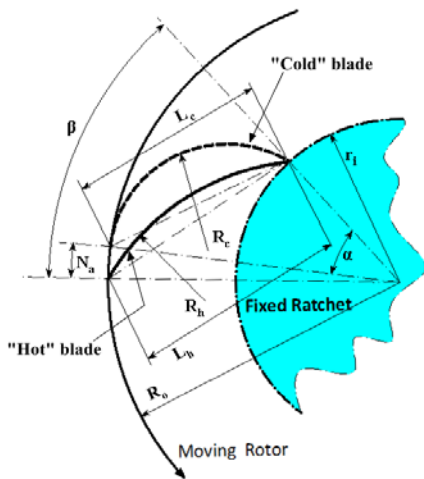

Fig.11 showing a single bimetallic blade in two states of heating

For an ideal machine with no external load, upon heating a single blade which is pivoted on the outer moving rim at one end, and is engaged into a ratchet tooth of the fixed stator, will change its chord length and displace as (2). This linear displacement, or straightening up of the blade, causes an angular displacement of the outer moving rim as a reaction through the blade occurs. It can be shown that the outer rim displaces through an angle given by the following formula using the Cosine rule:

$$N_a = a\cos\left[\frac{R_0^2 + r_t^2}{2R_o^2 r_t}\right] - a\cos\left[\frac{R_o^2 + r_t^2 - L_c^2}{2R_o^2 r_t}\right] \qquad (9)$$

Thus the angle of rotation due to one blade expanding during its heating cycle is calculated by (9). Angle N_a is a function of L_h which is driven by the hot radius of

curvature R_h, itself, a function of the Timoshenko formula (1). The force F developed by a single bimetallic blade is now translated into a tangential force vector by (13), see the force vector geometry in Figure 12. From the geometry in Figure 12, angle α is equal to:

$$\alpha = \frac{R_o + r_i^2 - L_c}{2R_0^2 r_i} \qquad (10)$$

$$x = \frac{r_t^2 - L_c^2 - R_0^2}{-2R_0 L_c} \qquad (11)$$

$$\mu = \frac{\pi}{2} - x \qquad (12)$$

$$F_t = F \cdot \cos(\mu) \qquad (13)$$

Thus from the geometry formulae just shown, it is possible to evaluate the tangential force component as a result of the thrust generated by a single bimetallic blade, onto the moving output shaft. Figure 12, shows the geometry of a single bimetallic blade within the motor. It can be seen that as the angle of **μ** decreases, the greater the tangential force vector F_t becomes. It should be noted that angle μ is dependent on the relative sizes of the radii R_0 and r_i, and R_0, r_i control the blade geometry, that is, the length of the bimetallic blade A_b and its ambient radius of curvature R_c. The optimum configuration for performance is thus dependent on many geometric variables, and these variables are dependent upon the overall size of machine itself, which is a function of the required application.
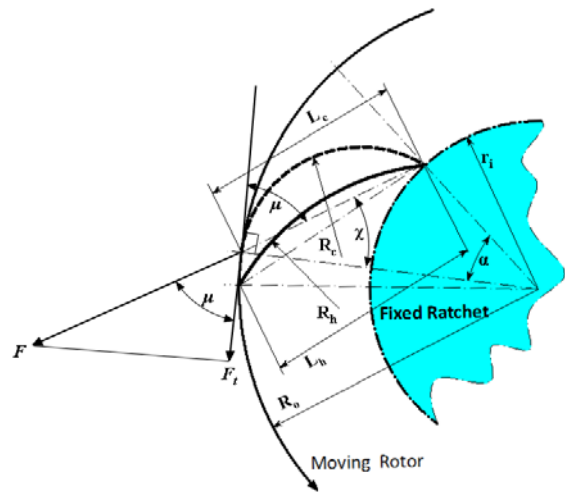

Fig.12 Force F geometry as a function of angle μ

The force vector F_t of a single blade times the distance R_0, provides the output torque of the machine.

Thus $\qquad\qquad T_{out} = F_t R_0 \qquad (14)$

Output torque of a single blade as a function of a temperature difference.

X. DISCUSSIONS

The formulae presented here are the major stages of the mathematical model of the thermal machine, however to evaluate the machine performance in depth, more

information is required which is beyond the scope of this short paper. The formulae presented here are for the evaluation of a single bimetallic blade in ideal conditions, assuming perfect heat transfer to and from the blade during the heating and cooling cycle. For a full mathematical model, the following detailed information is required: mass, size, geometry of the thermal machine components, available potential temperature differences, and required external driving loads. With this information, a heating and cooling cycle can be calculated which includes the number, size, geometry of the blades, and the required overlap cycle.

The overall efficiency of the machine is defined by the classical definition; energy in, divided by energy out, minus internal frictional losses. In a machine of this type, the overall efficiency is acknowledged to be very low; this is due to the very low frequency of the heating and cooling cycle. However, whilst evaluating the efficiency, it should be noted that this device does not consume valuable, diminishing fossil fuel based products, and thus for certain applications, the "waste" of the input energy, or efficiency, is not a problem, the question is, can the device provide sufficient, reliable non-polluting power to satisfy the requirements of the application?, whereby the input energy to the machine, comes from a renewable energy source such as solar power.

The answer to this question, is highly dependent on the size and detail design of the machine and the materials used. From the early test data of the first Prototype1 Fig.(5), the rotational speed of the outside moving rim was approximately 1 degree per sec, this was for 20 blades in a single stack or layer machine, with only one or two blades active any time. The size of Prototype 1 was φ203mm and the blades were 80mm long with a radius of curvature R_c = 40mm.

Another important aspect of the machine, is that could be designed to capture waste energy such as hot exhaust gases that would otherwise be lost to the surroundings. The overall efficiency of the machine is highly dependent on the heat transfer mechanism, which is a function of the design of the machine, including materials chosen and the geometry of the blades. The size of the machine also has a direct bearing on the efficiency of the device, in that, the larger the machine can be, the greater the energy harvesting scale factor.

XI. CONCLUSION

The thermal machine concept proposed, is therefore a very simple, elegant machine that provides many advantages for those special applications, that require the attributes that this type of sustainable device offers. Although space applications are the ideal environment for this device because of its inherent simplicity, reliability and lack of maintenance, it could equally be used in remote terrestrial desert region applications whereby a "set and forget" power source is required. The concept can also be used to recapture fossil fuel based waste exhaust energies that would otherwise be lost to the surroundings. Moreover, because the device is versatile, in that it only requires a temperature differential to work, it could be designed to be powered by solar radiation, hot and cold gases, liquids, or geothermal energy. A typical terrestrial application would be to power a water pump in the remote hot desert.

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