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Kinematics of a Spatial RCCC Parallel Manipulator

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***Abstract* — A spatial RCCC (R – revolute kinematic pair, C – cylindrical kinematic pair) parallel manipulator is considered in this paper. Structural scheme of this manipulator has constant and variable parameters. Constant parameters characterize the geometry of links, and variable parameters characterize the relative position of the elements of kinematic pairs. In this paper the direct kinematics of a spatial RCCC parallel manipulator is solved.**

***Index Terms* — parallel manipulator, cylindrical and revolute kinematic pair, binary link, direct kinematics.**

I. INTRODUCTION

Parallel manipulators are characterized by high stiffness, high speed, low moving inertia, and large payload capacity and play a very important role in numerous applications (robotic machining, aircraft simulators, pointing devices). [1]-[5]. In this paper a spatial RCCC parallel manipulator with one degree of freedom, i.e. an autooperator, is considered. An autooperator or a fixed – sequence manipulator works on one hard-coded program and it cannot readjust in change of technological operation. However an autooperator is a reliable device having a simple control system. Therefore it is advisable to use an autooperator in automatic machinery instead of manipulator with many degrees of freedom.

In [6] a structural synthesis of a spatial RCCC parallel manipulator (Fig. 1) is carried out and the constant and variable parameters are defined. This manipulator is formed by connection of the link *BP* of the RC manipulator *ABP* having three degrees of freedom with a frame by binary CC link *CD* having two negative degrees of freedom. 0Degree of freedom of a spatial kinematic chain is defined by formula [7]

$$W = 6n - \sum_{k=1}^{5} kp_k - \delta \, , \qquad (1)$$

where n – number of mobile links, k – class of kinematic pair

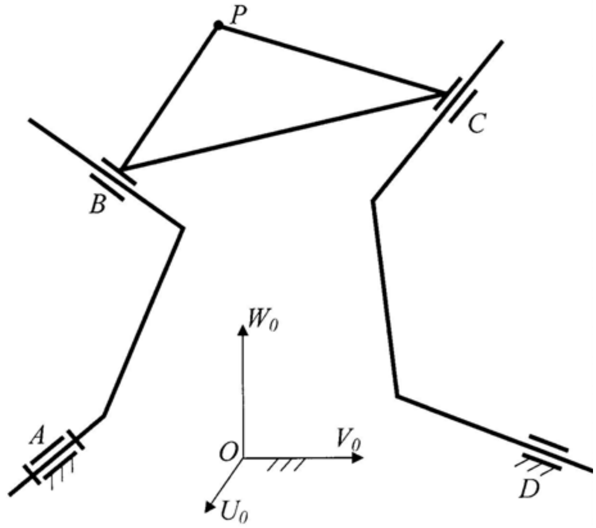

Fig. 1. A spatial RCCC parallel manipulator.

(joint), p_k - number of kinematic pair k-th class, δ - number of local mobility. Class of kinematic pair is determined by the number of restrictions on relative movement of its elements. For the spatial RCCC parallel manipulator: n=3, p_4 =3 (cylindrical kinematic pairs *B*, *C*, and *D*), p_5 =1 (revolute kinematic pair *A*), $\delta = 0$. Hence, $W = 6 \cdot 3 - 1 \cdot 5 - 3 \cdot 4 = 1$.

To define the geometry of the parallel manipulator the Cartesian coordinate systems *UWV* and *XYZ* are used which are fixed with elements of kinematic pairs. The axis *W* and *Z* of the coordinate systems *UVW* and *XYZ* are directed along the axis of rotation or translation of the kinematic pairs elements, and the axis *U* and *X* are directed along the shortest distance the axis *W* and *Z*. The axis *V* and *Y* supplement the coordinate systems *UVW* and *XYZ*. The transformation matrix between the coordinate systems *UVW* and *XYZ* is made up [4]. The constant and variable parameters of the parallel manipulator structural scheme are the elements of this matrix. The transformation matrix T_{jk} between the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ that are fixed on the ends of the binary link has a view

$$\mathbf{T}_{jk} = \mathbf{T}_{jk}(a_{jk}, b_{jk}, c_{jk}, \alpha_{jk}, \beta_{jk}, \gamma_{jk}) =$$

$$= \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix}, \qquad (2)$$

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where $t_{11} = 1$, $t_{12} = t_{13} = t_{14} = 0$,

$t_{21} = a_{jk} \cdot \cos\gamma_{jk} + b_{jk} \cdot \sin\gamma_{jk} \cdot \sin\alpha_{jk}$,

$t_{22} = \cos\gamma_{jk} \cdot \cos\beta_{jk} - \sin\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \sin\beta_{jk}$,

$t_{23} = -\cos\gamma_{jk} \cdot \sin\beta_{jk} - \sin\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \cos\beta_{jk}$,

$t_{24} = \sin\gamma_{jk} \cdot \sin\alpha_{jk}$,

$t_{31} = a_{jk} \cdot \sin\gamma_{jk} - b_{jk} \cdot \cos\gamma_{jk} \cdot \sin\alpha_{jk}$,

$t_{32} = \sin\gamma_{jk} \cdot \cos\beta_{jk} + \cos\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \sin\beta_{jk}$,

$t_{33} = \cos\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \cos\beta_{jk} - \sin\gamma_{jk} \cdot \sin\beta_{jk}$,

$t_{34} = -\cos\gamma_{jk} \cdot \sin\alpha_{jk}$, $t_{41} = c_{jk} + b_{jk} \cdot \cos\alpha_{jk}$,

$t_{42} = \sin\alpha_{jk} \cdot \sin\beta_{jk}$, $t_{43} = \sin\alpha_{jk} \cdot \cos\beta_{jk}$, $t_{44} = \cos\alpha_{jk}$.

The following six parameters define the relative positions of the two coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$: a_{jk} - a distance from axis W_j to axis Z_k which is measured along the direction of t_{jk} ; t_{jk} – a common perpendicular between axes W_j and Z_k ; α_{jk} - an angle between positive directions of axes W_j and Z_k which is measured counter clockwise relatively to positive direction of t_{jk} ; b_{jk} - a distance from direction of t_{jk} to direction of the axis X_k which is measured along positive direction of an axis Z_k ; β_{jk} - an angle between positive directions of t_{jk} and axis X_k which is measured counter clockwise relatively to positive direction of axis Z_k ; c_{jk} - a distance from direction of an axis U_j to direction of t_{jk} which is measured along positive direction of an axis W_j; γ_{jk} - an angle between positive directions of axis U_j and t_{jk} which is measured counter clockwise relatively to positive direction of an axis W_j.

A link with two kinematic pairs is called a binary link. Constant parameters characterize the geometry of links, and variable parameters characterize the relative positions of the kinematic pairs elements. Structural scheme of the spatial RCCC parallel manipulator with the chosen Cartesian coordinate systems and the constant and variable parameters are shown in Fig. 2, where $U_0V_0W_0$ is a absolute coordinate system, θ_A - a generalized coordinate, i.e. a variable parameter of the active revolute kinematic pair *A*.

II. Direct kinematics of a spatial RCCC parallel manipulator

In direct kinematics of the spatial RCCC parallel manipulator the coordinates of the point *P* in the absolute coordinate system $U_0V_0W_0$ are defined by the given generalized coordinate and constant parameters by the following expression

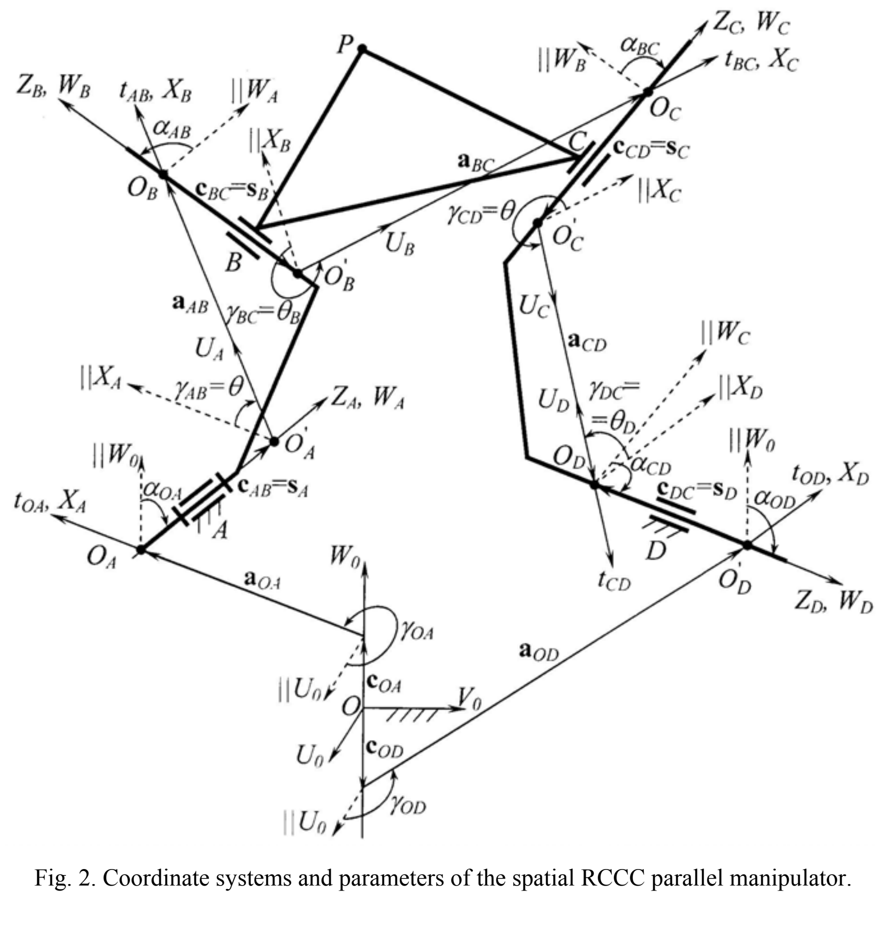

Fig. 2. Coordinate systems and parameters of the spatial RCCC parallel manipulator.

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$$\begin{bmatrix}1\\U_P\\V_P\\W_P\end{bmatrix}=\mathbf{T}_{OA}\left(a_{OA},\alpha_{OA},0,0,c_{OA},\gamma_{OA}\right)\times$$

$$\times\mathbf{T}_{AB}\left(a_{AB},\alpha_{AB},0,0,c_{AB},\gamma_{AB}\right)\times$$

$$\times\mathbf{T}_{BC}\left(a_{BC},\alpha_{BC},0,0,c_{BC},\gamma_{BC}\right)\cdot\begin{bmatrix}1\\{}^Cx_P\\{}^Cy_P\\{}^Cz_P\end{bmatrix}, \quad (3)$$

where ${}^Cx_P, {}^Cy_P, {}^Cz_P$, - the coordinates of the point P with respect to the coordinate system $X_CY_CZ_C$.

To determine the unknown parameters $c_{BC}=s_B$ and $\gamma_{BC}=\theta_B$ of the passive cylindrical kinematic pair B in (3) it is necessary to solve the problem of position analysis of the spatial CCC dyad BCD. For this purpose we connect the points O_B and O'_D by the vector $\mathbf{l}_{O_BO'_D}=\mathbf{r}_{O'_D}-\mathbf{r}_{O_D}$, where module and coordinates of the unit vector $\mathbf{e}_{O_BO'_D}$ are defined by the expressions

$$l_{O_BO'_D}=\left[(\mathbf{r}_{O'_D}-\mathbf{r}_{O_B})^2\right]^{\frac{1}{2}}, \quad (4)$$

$$\left.\begin{aligned}e_{O_BO'_D,U}&=\cos(U_0,\hat{\mathbf{l}}_{O_BO'_D})=(U_{O'_D}-U_{O_B})/l_{O_BO'_D}\\e_{O_BO'_D,V}&=\cos(V_0,\hat{\mathbf{l}}_{O_BO'_D})=(V_{O'_D}-V_{O_B})/l_{O_BO'_D}\\e_{O_BO'_D,W}&=\cos(W_0,\hat{\mathbf{l}}_{O_BO'_D})=(W_{O'_D}-W_{O_B})/l_{O_BO'_D}\end{aligned}\right\}. \quad (5)$$

Then we form the vector equation of closed loop $O_BO'_BO_CO'_CO_DO'_DO_B$

$$-s_B\mathbf{e}_B+a_{BC}\mathbf{e}_{BC}-s_C\mathbf{e}_C+a_{CD}\mathbf{e}_{CD}+$$
$$+s_D\mathbf{e}_D-l_{O_BO'_D}\mathbf{e}_{O_BO'_D}=0, \quad (6)$$

where $\mathbf{e}_B, \mathbf{e}_C$ and $\mathbf{e}_D$ - the unit vectors of the axes O_BZ_B, O_CZ_C and O_DZ_D with respect to the coordinate system $U_0V_0W_0$, $\mathbf{e}_{BC}=a_{BC}{}^{-1}\cdot\mathbf{a}_{BC}$, $\mathbf{e}_{CD}=a_{CD}{}^{-1}\cdot\mathbf{a}_{CD}$.

Considering that

$$\mathbf{e}_{BC}=\sin^{-1}\alpha_{BC}\cdot(\mathbf{e}_B\times\mathbf{e}_C),$$
$$\mathbf{e}_{CD}=\sin^{-1}\alpha_{CD}\cdot(\mathbf{e}_C\times\mathbf{e}_D),$$

we write (6) in the following form

$$-s_B\mathbf{e}_B+a'_{BC}(\mathbf{e}_B\times\mathbf{e}_C)-s_C\mathbf{e}_C+$$
$$+a'_{CD}(\mathbf{e}_C\times\mathbf{e}_D)+s_D\mathbf{e}_D-l_{O_BO'_D}\mathbf{e}_{O_BO'_D}=0, \quad (7)$$

where $a'_{BC}=a_{BC}\cdot\sin^{-1}\alpha_{BC}$, $a'_{CD}=a_{CD}\cdot\sin^{-1}\alpha_{CD}$.

Radius vectors $\mathbf{r}_{O_B}$ and $\mathbf{r}_{O'_D}$ of the points O_B and O'_D, and the unit vectors $\mathbf{e}_B$ and $\mathbf{e}_D$ with respect to the coordinate system $U_0V_0W_0$ in (4) and (7) are defined by the equations, respectively

$$\begin{bmatrix}1\\\mathbf{r}_{O_B}\end{bmatrix}=\mathbf{T}_{OA}\mathbf{T}_{AB}\begin{bmatrix}1\\{}^B\mathbf{r}_{O_B}\end{bmatrix}=\mathbf{T}_{OA}(a_{OA},0,c_{OA},\alpha_{OA},0,\gamma_{OA})\times$$

$$\times\mathbf{T}_{AB}(a_{AB},0,c_{AB},\alpha_{AB},0,\gamma_{AB})\cdot[1,\ 0,\ 0,\ 0]^T=$$

$$=\begin{bmatrix}1\\\cos\gamma_{OA}(a_{OA}+a_{AB}\cos\gamma_{AB})-\\-a_{AB}\sin\gamma_{OA}\cos\alpha_{OA}\sin\gamma_{AB}+c_{AB}\sin\gamma_{OA}\sin\alpha_{OA}\\\sin\gamma_{OA}(a_{OA}+a_{AB}\cos\gamma_{AB}+\\+a_{AB}\cos\gamma_{OA}\cos\alpha_{OA}\sin\gamma_{AB}-c_{AB}\cos\gamma_{OA}\sin\alpha_{OA}\\c_{OA}+a_{AB}\sin\alpha_{OA}\sin\gamma_{AB}+c_{AB}\cos\alpha_{OA}\end{bmatrix}, \quad (8)$$

$$\begin{bmatrix}1\\\mathbf{r}_{O'_D}\end{bmatrix}=\mathbf{T}_{OD}\cdot\begin{bmatrix}1\\{}^D\mathbf{r}_{O'_D}\end{bmatrix}=\mathbf{T}_{OD}\cdot\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}1\\a_{OD}\cos\gamma_{OD}\\a_{OD}\sin\gamma_{OD}\\c_{OD}\end{bmatrix}, \quad (9)$$

and

$$\begin{bmatrix}0\\\mathbf{e}_B\end{bmatrix}=\mathbf{T}_{OA}\mathbf{T}_{AB}\begin{bmatrix}0\\{}^B\mathbf{e}_B\end{bmatrix}=\mathbf{T}_{OA}(a_{OA},0,c_{OA},\alpha_{OA},0,\gamma_{OA})\times$$

$$\times\mathbf{T}_{AB}(a_{AB},0,c_{AB},\alpha_{AB},0,\gamma_{AB})\cdot[0,\ 0,\ 0,\ 1]^T=$$

$$=\begin{bmatrix}0\\(\cos\gamma_{OA}\sin\gamma_{AB}+\sin\gamma_{OA}\cos\alpha_{OA}\cos\gamma_{AB})\sin\alpha_{AB}+\\+\sin\gamma_{OA}\sin\alpha_{OA}\cos\alpha_{AB}\\(\sin\gamma_{OA}\sin\gamma_{AB}-\cos\gamma_{OA}\cos\alpha_{OA}\cos\gamma_{AB})\sin\alpha_{AB}-\\-\cos\gamma_{OA}\sin\alpha_{OA}\cos\alpha_{AB}\\-\sin\alpha_{OA}\cos\gamma_{AB}\sin\alpha_{AB}+\cos\alpha_{OA}\cos\alpha_{AB}\end{bmatrix}, \quad (10)$$

$$\begin{bmatrix}0\\\mathbf{e}_D\end{bmatrix}=\mathbf{T}_{OD}\begin{bmatrix}0\\{}^D\mathbf{e}_D\end{bmatrix}=\mathbf{T}_{OD}(a_{OD},0,c_{OD},\alpha_{OD},0,\gamma_{OD})\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=$$

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$$= \begin{bmatrix} 0 \\ \sin\gamma_{OD}\cdot\sin\alpha_{OD} \\ -\cos\gamma_{OD}\cdot\sin\alpha_{OD} \\ \cos\alpha_{OD} \end{bmatrix}. \tag{11}$$

To determine the unit vector $\mathbf{e}_C$ in (7) we write the following system of equations

$$\left.\begin{aligned} &\mathbf{e}_B\cdot\mathbf{e}_C = \cos\alpha_{BC} \\ &\mathbf{e}_C\cdot\mathbf{e}_D = \cos\alpha_{CD} \\ &\mathbf{e}_C^2 = 1 \end{aligned}\right\}, \tag{12}$$

where α_{BC} - the given angle between the directions of cylindrical kinematic pairs *B* and *C* elements of the binary link 2, i.e. between the unit vectors $\mathbf{e}_B$ and $\mathbf{e}_C$; α_{CD} - the given angle between the directions of cylindrical kinematic pairs *C* and *D* elements of the binary link 3.

We write the system (12) in the following expanded form

$$\left.\begin{aligned} &e_{BU}e_{CU} + e_{BV}e_{CV} + e_{BW}e_{CW} = \cos\alpha_{BC} \\ &e_{DU}e_{CU} + e_{DV}e_{CV} + e_{DW}e_{CW} = \cos\alpha_{CD} \\ &e_{CU}^2 + e_{CV}^2 + e_{CW}^2 = 1 \end{aligned}\right\}. \tag{13}$$

System of equations (13) contains two linear equations and one quadratic equation for the unknown projections e_{CU}, e_{CV}, e_{CW} of the unit vector $\mathbf{e}_C$ on the axis of the absolute coordinate system $U_0V_0W_0$. This system of equations has two solutions. Need to find a solution of the systems (12) and (13), and to solve a problem of choosing the right solution.

The first two equations of (13) are presented in a view

$$\left.\begin{aligned} &e_{BV}e_{CV} + e_{BW}e_{CW} = \cos\alpha_{BC} - e_{BU}e_{CU} \\ &e_{DV}e_{CV} + e_{DW}e_{CW} = \cos\alpha_{CD} - e_{DU}e_{CU} \end{aligned}\right\}. \tag{14}$$

Solving (14) for the unknown parameters e_{CV} and e_{CW} we have

$$e_{CV} = \frac{1}{A_U}\left(A_V e_{CU} + D_W\right),\ e_{CW} = \frac{1}{A_U}\left(A_W e_{CU} - D_V\right), \tag{15}$$

where

$$\begin{aligned} &A_U = e_{BV}e_{DW} - e_{BW}e_{DV}, \\ &A_V = e_{BW}e_{DU} - e_{BU}e_{DW}, \\ &A_W = e_{BU}e_{DV} - e_{BV}e_{DU}, \\ &D_V = e_{DV}\cos\alpha_{BC} - e_{BV}\cos\alpha_{CD}, \\ &D_W = e_{BW}\cos\alpha_{BC} - e_{BW}\cos\alpha_{CD}. \end{aligned}$$

Note, that the vectors

$$\mathbf{a} = [A_U, A_V, A_W]^T = \mathbf{e}_B \times \mathbf{e}_D,$$
$$\mathbf{d} = [D_U, D_V, D_W]^T = \mathbf{a}\times\mathbf{e}_C$$

determine the directions of the common perpendiculars to the vectors $\mathbf{e}_B$, $\mathbf{e}_D$ and $\mathbf{a}$, $\mathbf{e}_C$ respectively.

Substituting (15) in the third equation of the system (13) we have a quadratic equation for the unknown parameter e_{CU}

$$A^2 e_{CU}^2 + 2B_U e_{CU} + C_U = 0, \tag{16}$$

where

$$\begin{aligned} &A^2 = A_U^2 + A_V^2 + A_W^2, \\ &B_U = A_V D_W - A_W D_V, \\ &C_U = D_V^2 + D_W^2 - A_U^2. \end{aligned}$$

Then the solution of (16) has a view

$$e_{CU} = \frac{1}{A^2}\left(-B_U \pm R_U\right), \tag{17}$$

where $R_U = \sqrt{{B_U}^2 - A^2 C_U}$, and the signs $\pm$ correspond to the two different solutions of the direct kinematics of the parallel manipulator.

After determining the unit vectors $\mathbf{e}_B$, $\mathbf{e}_C$, $\mathbf{e}_D$ the vector equation (7) in projections on the axis of the absolute coordinate system $U_0V_0W_0$ is presented as a system of linear equations for the unknown lengths of motions of the cylindrical pairs

$$\mathbf{F}\cdot\mathbf{s} = \mathbf{g}, \tag{18}$$

where

$$\mathbf{F} = \begin{bmatrix} -e_{BU} & -e_{CU} & e_{DU} \\ -e_{BV} & -e_{CV} & e_{DV} \\ -e_{BW} & -e_{CW} & e_{DW} \end{bmatrix},\quad \mathbf{s} = \begin{bmatrix} s_B \\ s_C \\ s_D \end{bmatrix},$$

$$\mathbf{g} = \begin{bmatrix} l_{O_BO'_D} e_{O_BO'_D,U} - a'_{BC}(e_{BV}\cdot e_{CW} - e_{BW}\cdot e_{CV}) - \\ - a'_{CD}(e_{CV}\cdot e_{DW} - e_{CW}\cdot e_{DV}) \\ \\ l_{O_BO'_D} e_{O_BO'_D,V} - a'_{BC}(e_{BW}\cdot e_{CU} - e_{BU}\cdot e_{CW}) - \\ - a'_{CD}(e_{CW}\cdot e_{DU} - e_{CU}\cdot e_{DW}) \\ \\ l_{O_BO'_D} e_{O_BO'_D,W} - a'_{BC}(e_{BU}\cdot e_{CV} - e_{BV}\cdot e_{CU}) - \\ - a'_{CD}(e_{CU}\cdot e_{DV} - e_{CV}\cdot e_{DU}) \end{bmatrix}.$$

Suppose that the vectors $\mathbf{e}_B$, $\mathbf{e}_C$ and $\mathbf{e}_D$ are linearly

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independent, i.e., matrix **F** is not singular. Then the matrix **F** is invertible and the parameters s_B, s_D, s_C can be determined from the system (18) by equation

$$\mathbf{s} = \mathbf{F}^{-1} \cdot \mathbf{g} . \tag{19}$$

To determine the angular $\theta_B = \gamma_{BC}$ of the cylindrical pair B we consider the coordinate system $O_B X_B Y_B Z_B$ (Fig. 3). We transfer the unit vectors $\mathbf{e}_{AB}$ and $\mathbf{e}_{BC}$ to this coordinate system, where

$$\mathbf{e}_{AB} = a_{AB}^{-1} \cdot \mathbf{a}_{AB} = \sin^{-1} \alpha_{AB} (\mathbf{e}_A \times \mathbf{e}_B) ,$$

$$\begin{bmatrix} 0 \\ \mathbf{e}_A \end{bmatrix} = \mathbf{T}_{OA} \cdot \begin{bmatrix} 0 \\ {}^A\mathbf{e}_A \end{bmatrix} =$$

$$= \mathbf{T}_{OA}(a_{OA}, 0, c_{OA}, \alpha_{OA}, 0, \gamma_{OA}) \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\gamma_{OA} \cdot \sin\alpha_{OA} \\ -\cos\gamma_{OA} \cdot \sin\alpha_{OA} \\ \cos\alpha_{OA} \end{bmatrix} .$$

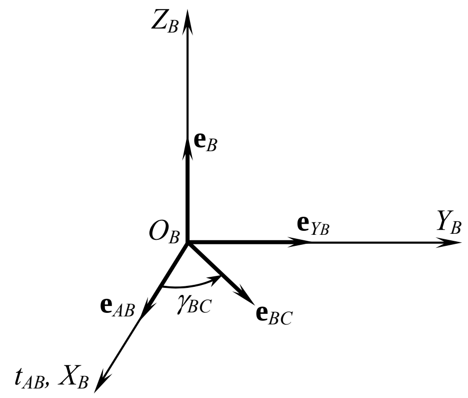

Fig. 3. Coordinate system $O_B X_B Y_B Z_B$.

The axis $O_B Y_B$ is directed according to the rule of the right Cartesian coordinate system, and its unit vector $\mathbf{e}_{Y_B}$ is defined as $\mathbf{e}_{Y_B} = \mathbf{e}_B \times \mathbf{e}_{AB}$. Then the unknown angle θ_B is determined by the following system

$$\left. \begin{aligned} \cos\theta_B &= (\mathbf{e}_{AB} \cdot \mathbf{e}_{BC}) = \frac{(\mathbf{e}_A \times \mathbf{e}_B) \cdot (\mathbf{e}_B \times \mathbf{e}_C)}{\sin\alpha_{AB} \sin\alpha_{BC}} \\ \sin\theta_B &= (\mathbf{e}_{BC} \cdot \mathbf{e}_{Y_B}) = \frac{(\mathbf{e}_B \times \mathbf{e}_C) \cdot [\mathbf{e}_B \times (\mathbf{e}_A \times \mathbf{e}_B)]}{\sin\alpha_{BC} \sin\alpha_{AB}} \end{aligned} \right\} . \tag{20}$$

Similary it can be shown that the angles $\theta_C = \gamma_{CD}$ and $\theta_D = \gamma_{DC}$ of the cylindrical pairs are determined by the following systems, respectively

$$\left. \begin{aligned} \cos\theta_C &= (\mathbf{e}_{BC} \cdot \mathbf{e}_{CD}) = \frac{(\mathbf{e}_B \times \mathbf{e}_C) \cdot (\mathbf{e}_C \times \mathbf{e}_D)}{\sin\alpha_{BC} \sin\alpha_{CD}} \\ \sin\theta_C &= (\mathbf{e}_{CD} \cdot \mathbf{e}_{Y_C}) = \frac{(\mathbf{e}_C \times \mathbf{e}_D) \cdot [\mathbf{e}_C \times (\mathbf{e}_B \times \mathbf{e}_C)]}{\sin\alpha_{CD} \sin\alpha_{BC}} \end{aligned} \right\} \tag{21}$$

and

$$\left. \begin{aligned} \cos\theta_D &= (\mathbf{e}_{OD} \cdot \mathbf{e}_{DC}) = \frac{(\mathbf{e}_O \times \mathbf{e}_D) \cdot (\mathbf{e}_D \times \mathbf{e}_C)}{\sin\alpha_{OD} \sin\alpha_{CD}} \\ \sin\theta_D &= (\mathbf{e}_{DC} \cdot \mathbf{e}_{Y_D}) = \frac{(\mathbf{e}_D \times \mathbf{e}_C) \cdot [\mathbf{e}_D \times (\mathbf{e}_O \times \mathbf{e}_D)]}{\sin\alpha_{CD} \sin\alpha_{OD}} \end{aligned} \right\} . \tag{22}$$

Here $\mathbf{e}_{Y_C}$ and $\mathbf{e}_{Y_D}$ - unit vectors of the axis $O_C Y_C$ and $O_{D'} Y_D$ respectively, and $\mathbf{e}_O = [0,\ 0,\ 1]^T$ - unit vector of the axis OW_O.

III. Conclusion

A direct kinematics of a spatial RCCC parallel manipulator is discussed in this paper. Variable parameters of passive kinematic pairs and position of working point are determined by given generalized coordinate and constant parameters. An angle between elements of active revolute kinematic pair is a generalized coordinate. Rotation and translation motions of three passive cylindrical kinematic pairs are unknown variable parameters. Constant parameters characterize geometry of binary links. Direct kinematics of the parallel manipulator is solved on the basis of position analysis of spatial dyad with cylindrical joints. Analysis shows that two sets of direct kinematic solutions exist for the RCCC parallel manipulator.

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