

# Fast Frontal View Gait Authentication Based on the Statistical Registration and Human Gait Modeling

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**Abstract**—We study the problem of analyzing and classifying frontal view gait video data. In this study, we suppose that frontal view gait data as a mixing of scale changing, human movements and speed changing parameters. We estimate these parameters using the statistical registration and modeling on a video data. To demonstrate the effectiveness of our method, we conducted experiment, assessing the proposed method for frontal view human gait authentication. We apply K-nearest-neighbor classifier, using the estimated parameters, to perform the human gait authentication, and present results from an experiment involving 120 subjects. As a result, our method shows high recognition rate and low calculation cost.

**Index Terms**—Frontal view gait authentication, gait analysis, human gait modeling, statistical registration.

## I. INTRODUCTION

WE study the problem of analyzing and classifying frontal view gait video data. A study on the human gait analysis is very important in the fields of the health/sports management, medical and biometrics research.

Gait analysis is mainly based on motion capture system and video data. The motion capture system can give the precise measurements of trajectories of moving objects, but it requires the laboratory environments and we cannot be used this system in the field study. On the other hand, the video camera is handy to observe the gait motion in the field study.

From the standpoint of health/medical research area. Gage [1] proposed brain paralysis gait analysis using gait video data. Kadaba *et al.* [2] discussed importance of lower limb in the human gait using gait video data too. Many gait analysis have recently analyzing using video analysis software (e.g. Dartfish, Contemplas, Silicon Coach). For example, Borel *et al.* [3] and Grunt *et al.* [4] proposed infantile paralysis gait analysis using lateral view gait video data.

On the other hand, from the standpoint of statistics, Olshen *et al.* [5] proposed the bootstrap estimation for confidence intervals of the functional data with application to the gait cycle data observed by the motion capture system.

In recent years, gait analysis is very important in the biometrics research area too. From a practical perspective, the method based on the video data is very important authentication technique, where data does not require the subject to help this system. For instance, face recognition is one of the famous authentication techniques using video

data. However, it needs strong learning. Gait authentication, on the other hand, is a viable alternative (e.g. Soriano *et al.* [6]; Barnich & Droogenbroeck [7]).

Soriano *et al.* [6] proposed the gait authentication method based on the data matching techniques using the dynamic time warping of human silhouettes. Barnich & Droogenbroeck [7] proposed histogram based gait authentication method and they apply to the frontal view gait authentication.

However, most studies have not focused on frontal view gait analysis, because such data has many restrictions on analysis based on the filming conditions. Frontal view gait authentication case is certainly exists. For instance, in corridor like structure, it is difficult to apply the lateral view gait authentication [8].

The video data filmed from the frontal view is difficult to analyze, because the subject getting close in to the camera, and data includes the scale-changing parameters [7], [8]. To cope with this, Okusa *et al.* [9] and Okusa & Kamakura [10] proposed a registration for scales of moving object using the method of nonlinear least squares, but Okusa *et al.* [9] and Okusa & Kamakura [10] did not focus on the human leg swing.

Okusa & Kamakura [11], [12] focus on the gait analysis using arm and leg swing model with estimated parameters and application to the human gait authentication and normal/abnormal gait analysis. However, their models have many of parameters, and it raise calculation cost and instability of parameter estimation.

In this study, we redesign the frontal view human gait model for the gait authentication. We suppose that frontal view gait data as a mixing of scale changing, human movements and speed changing parameter. We validate the effectiveness of each parameters, and modify the previous model for the gait authentication.

To demonstrate the effectiveness of our method, we conducted experiment, assessing the proposed method for frontal view human gait authentication. We apply K-nearest-neighbor classifier, using the estimated parameters, to perform the human gait authentication, and present results from an experiment involving 120 subjects. As a result, our method shows high recognition rate and low calculation cost.

## II. FRONTAL VIEW GAIT DATA

In this section, we describe an overview of frontal view gait data. Many of gait analysis using lateral view gait data, because lateral view gait is easy to detect the human gait features. However, in a corridor like structure, the subject is approaching a camera. Such case is difficult observe lateral view gait.

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In a lateral view gait, at least two cycles or four steps are needed. For more robust estimation of the period of walking, about 8m is recommended. To capture this movement, the camera distance required is about 9m. Practically, having such a wide space is difficult. On the other hand, frontal view gait video is easy to observe 8m (or more) gait steps [8].

Figure 1 is an example of frontal view gait data recorded by Figure 2 situation. Figure 1 illustrates difficulty of frontal view gait analysis. Even if subject do the same motion with the same timing, frontal view gait data includes scale changing components. Figure 3 shows subject's width time-series behavior of frontal view gait data. This figure illustrates frontal view gait data contains many of time-series components.



Fig. 1. Frontal view gait data

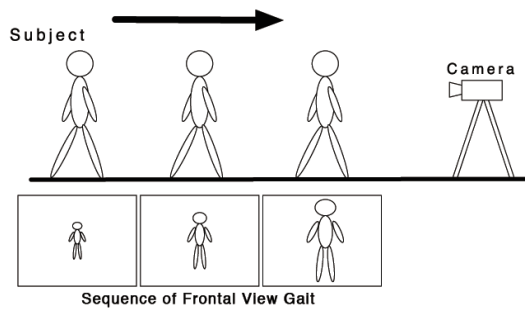


Fig. 2. Filming situation of frontal view gait data

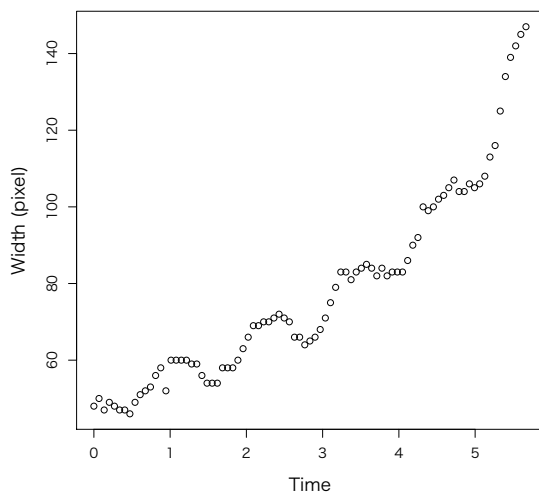


Fig. 3. Time-series behavior of frontal view subject width

### III. MODELING OF FRONTAL VIEW GAIT DATA

#### A. Preprocessing

The raw video data is difficult to observe subject width and height time-series behavior, because data contains background. We separate subject from background using inter-frame subtraction method (Eq. 1).

$$\Delta^{(T)} = |I^{(T+1)} - I^{(T)}|, T = 1, \dots, (n - 1),$$

$$\Delta^{(T)}(p, q) = \begin{cases} 1 & (\Delta^{(T)}(p, q) > 0) \\ 0 & (\text{Otherwise}). \end{cases} \quad (1)$$

Here,  $\Delta^{(T)}$  is an inter-frame subtraction image,  $I^{(T)}$  is grey scaled video data image at frame  $T$ ,  $(p, q)$  is the pixel coordinate.

a) *Subject Width/Height Calculation:* Inter-frame subtraction method can separate the subject and background. However, it is difficult to measure the time-series behavior of the subject width and height. In this section, we describe the subject width and height calculation method using inter-frame subtraction data.

Let us suppose that inter-frame subtraction image is binary matrix. We can measure the subject height and width by integration calculation of row and column at each frame. In this study, we focus on the human gait arm and leg swing of the frontal view gait. We assume that subject width and height time-series behavior consist of the arm and leg swing behavior.

#### B. Relationship between camera and subject

Figure 4 shows a relationship between camera and subject. From figure 4, Width and height modeling has same structure. In this section, we describe the subject's width modeling. We can assume simple camera structure. We consider the virtual screen exists between observation point and subject, and we define  $x_i$  as subject width on the virtual screen at  $i$ -th frame ( $i = 1, \dots, n$ ).

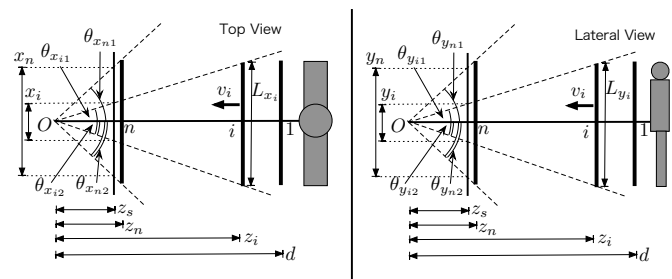


Fig. 4. Relationship between camera and subject

Here we define  $z_i, z_j$  as distance between observation point and subject at  $i$ -th,  $j$ -th frame,  $z_s$  as distance between observation point and virtual screen,  $\theta_{x_{i1}}, \theta_{x_{i2}}$  as subject angle of view from observation point at  $i$ -th frame,  $d$  as distance between observation point and 1st frame,  $v_i$  as subject speed at  $i$ -th frame. Okusa *et al.* [9] defined the subject length  $L$  was constant. We assume that  $L$  has the time-series behavior and we define  $L_i$  is the subject length at  $i$ -th frame.

$x_i$  at  $i$ -th frame depends on  $\theta_{x_{i1}}, \theta_{x_{i2}}$  as shown in Figure 4.

$$x_i = z_s(\tan \theta_{x_{i1}} + \tan \theta_{x_{i2}}). \quad (2)$$

Similarly, the subject length at  $i$ -th frame is

$$L_{x_i} = z_i(\tan \theta_{x_{i1}} + \tan \theta_{x_{i2}}). \quad (3)$$

From Eq.(2), Eq.(3), ratio between  $x_n$  and  $x_i$  is

$$\frac{x_n}{x_i} = \frac{L_{x_n} z_i}{L_{x_i} z_n} \quad (4)$$

Frame interval is equally-spaced (15 fps). Okusa *et al.* [9] assumed the average speed is constant. We can assume that average speed from  $i$ -th frame is  $(n - i) = (z_i - z_n)/\bar{v}$ , therefore  $z_i$  is  $z_i = z_n + \bar{v}(n - i)$ . We substitute  $z_i$  to Eq.(4)

$$x_i = \frac{M_{x_i} \gamma}{\gamma + (n - i)} x_n + \epsilon_i, \quad (5)$$

where  $\gamma$  is  $z_n/\bar{v}$ ,  $M_{x_i}$  is  $L_{x_i}/L_{x_n}$ ,  $\epsilon_i$  is noise. From Eq.(5), predicted value  $\hat{x}_i^{(n)}$  is registration from  $i$ -th frame's scale to  $n$ -th frame's scale

$$\hat{x}_i^{(n)} = \frac{\gamma + (n - i)}{M_{x_i} \gamma} x_i. \quad (6)$$

Similarly, we can define subject height as

$$y_i = \frac{M_{y_i} \gamma}{\gamma + (n - i)} y_n + \epsilon_i, \quad (7)$$

where  $M_{y_i}$  is  $L_{y_i}/L_{y_n}$ .

Next, we discuss the scale changing, human movement, and speed changing parameter estimation model.

### C. Scale changing parameter estimation

From Eq.(5), scale parameter is  $\gamma$ . Solve Eq.(5) for  $\gamma$  shows

$$\gamma = \frac{x_i(n - i)}{x_i - M_{x_i} x_n}. \quad (8)$$

Here  $\gamma$  is the ungaugable parameter, and we estimate it using nonlinear least squares method

$$S(\gamma, M_{x_i}) = \sum_{i=1}^n \left\{ x_i - \frac{M_{x_i} \gamma}{\gamma + (n - i)} x_n \right\}^2. \quad (9)$$

### D. Human movement parameter estimation

$M_{x_i}$  and  $M_{y_i}$  are movement model of the subject. If the subject is the rigid body, movement model  $M_{x_i}$  and  $M_{y_i}$  are constant. Meanwhile, human gait is not a constant.  $M_{x_i}$  and  $M_{y_i}$  needs the movement model because the subject body is moving wildly.

b) *Human gait modeling: arm swing:* Collins *et al.* [13] has reported that arm swing is an very important role in the gait motion. We consider the human gait model based on Collins *et al.* [13] model (see Figure 5).

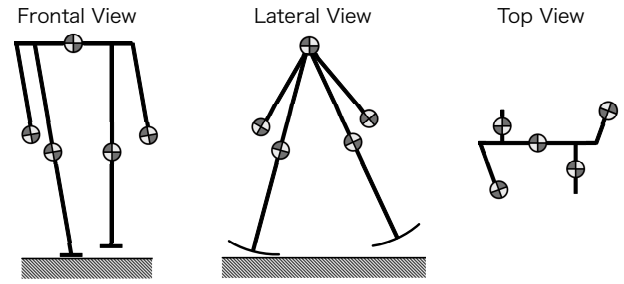


Fig. 5. Gait model

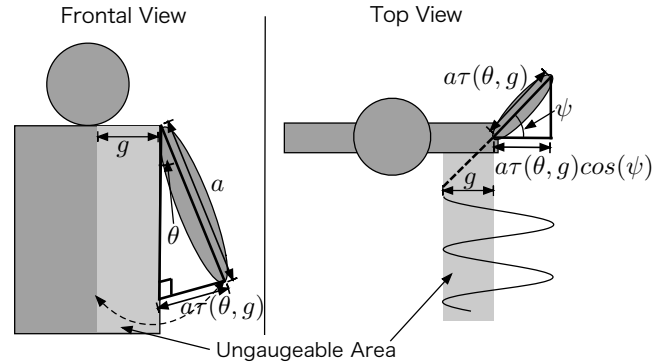


Fig. 6. Arm swing model

It seems reasonable to think that arm swing is single pendulum. Collins *et al.* [13] model assumed that arm swing is move to anteroposterior direction. Our model, on the other hand, can assume that arm swing move to an oblique direction (Figure 6).

Figure 6's model has an ungaugable area. Our method's width/height calculation is based on integration calculation of row and column at each frame. If the arm move to inside the body area, arm length is ungaugable. Arm swing model is

$$x_i = \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i)}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, n)} + s \right) \gamma}{\gamma + (n - i)} x_n + \epsilon_i$$

$$W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i) = P_1 \tau(fi + Q_1, g_1) + P_2 \tau(fi + Q_2, g_2)$$

$$\tau(\theta, g) = \begin{cases} \sin(\theta) + g & (\sin(\theta) + g > 0) \\ 0 & (\text{Otherwise}) \end{cases} \quad (10)$$

where  $P_1 = a_1 \cos(\psi)$  and  $P_2 = a_2 \cos(\psi)$ .  $P_1 \tau(fi + Q_1, g_1)$  and  $P_2 \tau(fi + Q_2, g_2)$  are right and left arm model respectively. From Eq.(10), we estimate each gait parameter using nonlinear least squares method.

$$S(\gamma, P_1, P_2, Q_1, Q_2, g_1, g_2, f, s) = \sum_{i=1}^n \left\{ x_i - \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i)}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, n)} + s \right) \gamma}{\gamma + (n - i)} x_n \right\}^2 \quad (11)$$

Here,  $f$  is gait cycle frequency,  $s$  is adjustment parameter,  $P_1, P_2$  are arm swing amplitude parameters,  $Q_1, Q_2$  are arm phase parameters, and  $g_1, g_2$  are ungaugable area parameters.

c) *Human gait modeling: leg swing*: Leg swing modeling is simpler than arm swing model because leg swing model does not have a ungaugable area. Okusa *et al.* [9] and Okusa & Kamakura [10] does not consider the leg swing. It seems reasonable to think like arm swing that leg swing is single pendulum (Figure 7).

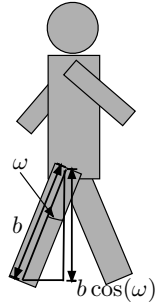


Fig. 7. Leg swing model

Leg swing model is

$$y_i = \frac{\left( \frac{H(b_1, Q_3, f, i)}{H(b_1, Q_3, f, n)} + s \right) \gamma}{\gamma + (n - i)} y_n + \epsilon_i$$

$$H(b_1, Q_3, f, i) = b_1 \cos(fi + Q_3). \quad (12)$$

Here  $b_1$  is leg swing amplitude parameter, and  $Q_3$  is leg phase parameter.

#### E. Speed changing parameter estimation

Frontal view video data is difficult to see the subject's speed. If our gait model is correct, observed value  $x_i$  and  $y_i$  is same as the fitted value of gait model at point  $\ell_i$ . Previous model's  $\ell_i$  assumes equally spaced ( $\ell_i = i = 1, \dots, n$ ). We estimate  $\ell_{x_i}$  and  $\ell_{y_i}$  value for minimize the observed value and model fitted value at  $\ell_i$ . We can define estimated value  $\ell_{x_i}$  and  $\ell_{y_i}$  as a virtual space coordinate at i-th frame (Figure 8).

Eq.5, Eq.7 with the coordinate estimation shows

$$x_i = \frac{M_{x_i} \gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$

$$y_i = \frac{M_{y_i} \gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i. \quad (13)$$

Here,  $\ell_{x_i}, \dots, \ell_{x_n}$  and  $\ell_{y_i}, \dots, \ell_{y_n}$  are virtual space coordinate parameters of width and height respectively. From Eq.13, arm swing and leg swing model with the coordinate estimation shows Eq.14, Eq.15.

$$x_i = \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_i})}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_n})} + s \right) \gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$

$$W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_i}) =$$

$$P_1 \tau(f \ell_{x_i} + Q_1, g_1) + P_2 \tau(f \ell_{x_i} + Q_2, g_2)$$

$$\tau(\theta, g) = \begin{cases} \sin(\theta) + g & (\sin(\theta) + g > 0) \\ 0 & (\text{Otherwise}). \end{cases} \quad (14)$$

$$y_i = \frac{\left( \frac{H(b_1, Q_3, f, \ell_{y_i})}{H(b_1, Q_3, f, \ell_{y_n})} + s \right) \gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i$$

$$H(b_1, Q_3, f, \ell_{y_i}) = b_1 \cos(f \ell_{y_i} + Q_3). \quad (15)$$

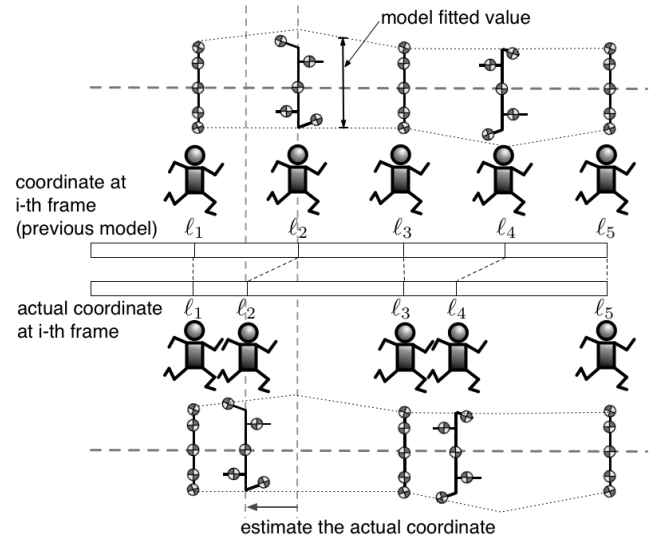


Fig. 8. Virtual space coordinate estimation

We suppose that virtual space coordinate of subject is  $\hat{\ell}_i = (\hat{\ell}_{x_i} + \hat{\ell}_{y_i})/2$ . Then, we can assume that subjects speed is 1st order difference of  $\hat{\ell}_i$ , and acceleration is 2nd order difference of  $\hat{\ell}_i$ .

#### IV. EFFECTIVENESS OF EACH PARAMETERS

In this section, we discuss the effectiveness of each parameters. From the standpoint of security, we think authentication method needs high accuracy, low calculation cost.

Eq.14 and Eq.15 models has many of parameters, we need to estimate  $n + 11$  parameters. It raise calculation cost and instability of parameter estimation. Okusa & Kamakura [11] discussed parameter estimation algorithm and its application to the human authentication. However, Okusa & Kamakura [11] algorithm can not simultaneous processing for multiple subjects.

To cope with this, we confirm the most affected parameters for authentication. We choose randomly selected 30 subject from 120 subject and calculate the interclass stability index  $C$  for each estimated parameters. Parameter  $k$ 's interclass stability  $C_k$  calculation is

$$C_k = \sum_{p \neq q} |\Theta_{k,p} - \Theta_{k,q}|. \quad (16)$$

This index means most minimum interclass stability index parameter is most effective parameters for gait authentication. Where,  $\Theta$  is the set of the estimated parameters from Eq.14 and Eq.15 models, and  $p, q$  are learning and test data's subject ID, respectively. Note, that length of the virtual space coordinate parameters  $\ell_i$  are not equal at each subjects, because it depends on moving distance and speed. Therefore, we set  $\bar{\ell} = \frac{1}{n} \sum \ell_i$  as a representative value of  $\ell_i$ .

We ascending sorting  $C_k$  and we choose the smaller  $C_k$  value parameters until over 90% authentication rate. From this process, finally we can choose parameters  $P, b_1, \gamma, f$ .

#### A. Modified gait model

From the validation results of effectiveness of each parameters, most effective parameters for the gait authentication are  $P, b_1, \gamma, f$ . Accordingly, we modify the Okusa & Kamakura [11] model for estimate these parameters.

TABLE I  
RSS, AIC, cAIC, CALCULATION TIME VALUE OF WIDTH AND HEIGHT DATA

Model	Method	Subject ID									
		A	B	C	D	E	F	G	H	I	J
Okusa & Kamakura [11] model	RSS	170.46	67.02	122.28	382.93	193.88	302.20	125.73	55.34	1.17	0.68
	AIC	626.95	558.86	581.82	676.03	663.96	657.80	577.59	510.50	236.67	190.94
	cAIC	-4.48	-80.53	-25.72	60.52	0.68	42.29	-22.00	-81.13	-410.69	-440.49
	Calc. Time	7.766	6.830	8.231	8.007	6.092	7.090	6.929	8.426	7.462	7.581
Proposed model	RSS	354.50	192.40	229.01	859.82	619.89	580.83	134.61	175.97	1.08	2.83
	AIC	354.79	309.24	311.51	416.31	414.43	386.11	268.71	286.10	-107.87	-26.86
	cAIC	355.33	309.77	312.07	416.87	414.94	386.66	269.28	286.68	-107.35	-26.32
	Calc. Time	0.986	0.669	0.793	0.782	1.093	0.865	0.626	0.912	0.535	0.814
Number of frames:		79	80	76	77	83	77	75	74	81	79

Calc. Time: Calculation Time [sec]

Here modified width model is

$$x_i = \frac{(W(P, Q_1, f, i) + s_1)\gamma}{\gamma + (n-i)} x_n + \epsilon_i$$

$$W(P, Q_1, f, i) = P \sin(fi + Q_1). \quad (17)$$

Where  $P$  is amplitude of arm swing,  $Q_1$  is phase of human gait,  $f$  is gait cycle frequency and  $s$  is adjustment parameter.

Similarly, modified height model is

$$y_i = \frac{(H(b_1, Q_1, f, i) + s_2)\gamma}{\gamma + (n-i)} y_n + \epsilon_i$$

$$H(b_1, Q_1, f) = b_1 \sin(2fi + Q_1). \quad (18)$$

Differences points between Okusa & Kamakura [11] model and our modified model are two points. Firstly, we reduce the model parameters from the validation results of effectiveness of each parameters. Secondly, we standardize the parameters between leg swing and arm swing model. These measures have efficacy for calculation cost and parameter estimation stability. Our modified models are easy and stable to estimate  $P, b_1, \gamma, f$  parameters.

In next session, we validate the effectiveness of our model.

## V. EXPERIMENTS AND RESULTS

### A. Gait parameter estimation

To validate the effectiveness of our modified model, we compare Eq.17 and Eq.18 model with Eq.14 and Eq.15 model by Residual Sum of Squares (RSS), Akaike Information Criterion (AIC) [14] and Consistent Akaike's Information Criterion (cAIC) [15] value. We took movie of 10 subjects walking video data from frontal view (10 steps, Male, average height: 176.4cm, sd: 3.07cm) and apply to our proposed method.

Figure 9 is plot of the subject width(pixel) time-series behavior. Here, continuous line represent fitted value of Eq.18. From Figure 9, proposed model is good fitting for frontal view gait data.

Table I is RSS, AIC, cAIC, Calculation time value of previous model (Eq.14 and Eq.15 model) and proposed model (Eq.17 and Eq.18 model). In Table I, most minimal RSS and cAIC models are previous model (Eq.14, Eq.15 model). Meanwhile, most minimal AIC and calculation time model are proposed model (Eq.17, Eq.18 model).

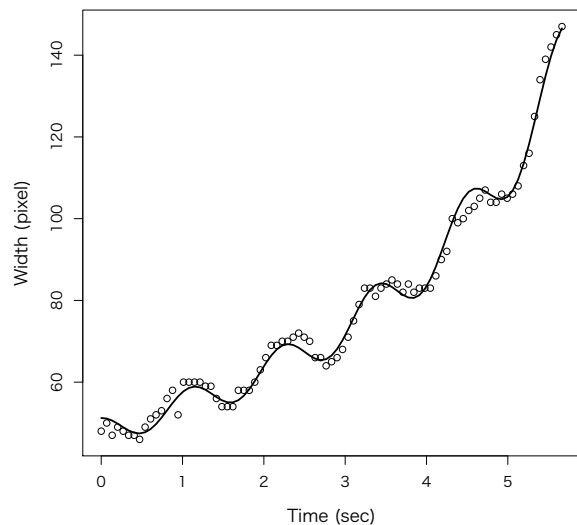


Fig. 9. Fitted Value of subject's width

In this research, we focus on the calculation cost and authentication accuracy. From Table I, our proposed models calculation cost is about 90% faster than previous model. From the standpoint of calculation cost, we think our proposed model is very high performance.

Naturally, low calculation cost not means high authentication accuracy. In next section, we validate the authentication accuracy between previous model with proposed model.

### B. Gait authentication

In this section, we discuss the human gait authentication. In this paper, we apply K-NN classifier (K=1), using the all estimated parameters, to perform the gait authentication, and present results from an experiment involving 120 subjects (10 steps, Male: 96 (average height: 173.24cm, sd: 5.64cm), Female: 24 (average height: 156.25cm, sd: 3.96cm)). To evaluate our estimated parameters, we apply these parameters to leave-one-out cross-validation test.

We compared proposed method with Okusa & Kamakura [11] method. Okusa & Kamakura [11] method have high performance than Soriano *et al.* [6] method and Barnich & Droogenbroeck [7] method.

Figure 10 is plot of the gait cycle vs. authentication rate. "Gait cycle" means the gait steps used in the authentication (one gait cycle is two steps). Here dashed line is Okusa &

Kamakura [11] method's authentication rate, continuous line is proposed method's authentication rate. Figure 10 shows our method has the better performance compared to Okusa & Kamakura [11].

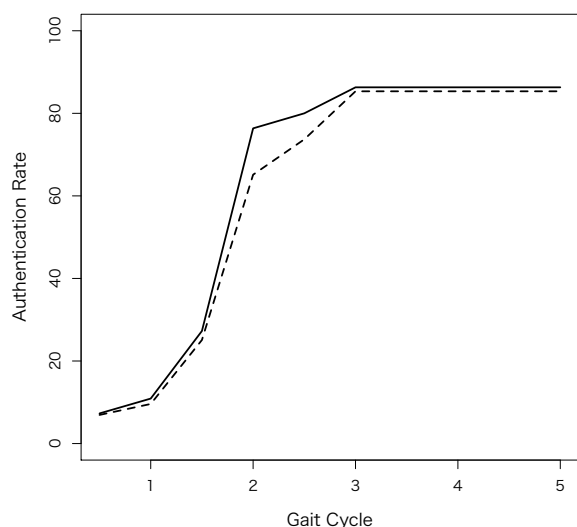


Fig. 10. Gait cycle vs. authentication rate

Table II is authentication rate (10 steps case) and average calculation time of proposed method and Okusa & Kamakura [11] method. In table II, our method shows high performance and low calculation cost. It is probable that caused by proposed method choose most effective parameters and reject the negative effective parameters.

TABLE II  
GAIT AUTHENTICATION RATE(%) AND  
AVERAGE CALCULATION TIME (SEC)

	Authentication rate(%)	Calc. time
Proposed method	86.3	0.93
Okusa & Kamakura [11]	85.3	9.42

## VI. CONCLUSION

In this article, we proposed the human gait model for the frontal view human gait authentication. Our model is able to estimate stably human gait feature quantity by low calculation cost. Our model is 90% faster than our previous model. Moreover, estimated parameters have high accuracy for the human gait authentication than previous method.

In next phase, we need to compare our method with more another gait authentication method. Additionally, we need to implement the gait authentication system based on the proposed method and demonstrate it.

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