# Cost-Efficiency Method for Production Scheduling

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*Abstract* – This paper seeks to determine a dynamic Pareto-optimal method for various problems of production scheduling, based on simultaneous application of two criteria: relative manufacturing cost criterion and average orders utility criterion. In this method the concept of production intensity as a dynamic production process parameter is used. The method is applicable both for "make-to-order" and "make-to-stock" manufacturing strategies. The software used allows scheduling for middle quantity of jobs. The result of software application is the set of non-dominant versions proposed to a user for making a final choice.

*Index Terms* – **Parallel machines, Pareto-optimality, production intensity, scheduling, single machine.**

## I. INTRODUCTION

All possible quality criteria of the first SCOR model level relate to one of the four categories: customer service, economical indices, demand satisfaction flexibility, product development. The criteria of the last category are not usually considered in production planning, and demand satisfaction flexibility is ensured by strategy selection – "make-to-stock" or "make-to-order". Therefore production planning quality mainly depends on the customer service level and production cost. The high customer service level (efficiency) may only be achieved through timely order completion. However, prompt order completion contradicts high level of utilization and increases expenses. This trend is known as 'dilemma of operation planning' [1].

In the last years some researches were have been conducted in order to solve the 'dilemma of operation planning', which studied multicriteria scheduling. Solution of this problem to a considerable extent depends on the chosen criteria of schedule quality. The main criteria used were overall production time or makespan $C_{max}$, the highest tardiness $T_{max}$, average tardiness $\overline{T}$, etc. A typical example of such method was demonstrated in the article by [2], where for a single machine three criteria were used: the criterion of "First In, First Out" or FIFO; the criterion of relative setup time SSU+ and the critical ratio criterion CR. The SSU+ criterion is the ratio between setup time $s_i$ and total job duration $\sum p_i$ for a single job type.

Due to high complexity of this problem, appropriate researches mainly apply various heuristic methods. Some of these methods are aimed at finding Pareto optimal solutions for two selected criteria. In some instances a set of criteria was reduced to a single generalized criterion by means of their summation with a different weight for each criterion

For instance, criteria of total tardiness $\sum T_i$, total cycle duration $\sum F_i$ and summary machine load $K$ are often used for a single machine.

For parallel machines this problem was considered as a problem with a single criterion created by linear combination of initial criteria [3]. The ant colony system algorithm was applied for a flow shop [4] with respect to both makespan and total flowtime as optimization criteria.

The above criteria are indirect criteria and do not reflect SCOR model requirements directly. Therefore some recent articles [5] were dedicated to scheduling by direct application of the cost criterion and the timely service criterion. For this purpose the mentioned paper describes a rather complicated system for sequential planning.

Reduction of costs in scheduling may be achieved through technological grouping of jobs, which provides for low setup time needed to shift from a job to a job within the group. As it was shown in the article by [6], the criterion of relative setup expenses $U$ and the criterion of average orders utility $\overline{V}$ may be considered for group scheduling. This paper demonstrates possibility to apply these criteria for certain scheduling problems.

The remainder of this paper is organized as follows. In Section 2 the problem is formulated, the function of direct expenses and the function of current orders utility are determined. Section 3 is dedicated to group scheduling for a single machine. In Section 4 group scheduling for parallel machine is considered. Section 5 is dedicated to jobshop production scheduling. Section 6 contains some concluding remarks.

## II. UTILITY FUNCTIONS IN SCHEDULING

Assume we are drawing up a plan at $t = 0$, and certain job due dates have already expired, i.e. some $d_i \leq 0$. Accordingly, it is necessary to use the dynamic customer service criterion, which is valid in the range of due dates $-\infty < d_i < \infty$.

The customer service level may be assessed by the current order utility function *V*. From the manufacturer's point of view, the order value increases proportionately to work amount $p_i$, since staff engagement increases. Besides, the more is the time reserve for completing an order, the more attractive is the order, since there is an opportunity to prepare for order execution. Eventually the order time reserve is decreasing, and the order value is diminishing. After all, if the due date has expired, the order value becomes negative.

The manufacturer's attitude to the order changes in time and the appropriate function is named *production intensity* [7] :

$$H_i = \frac{w_i p_i}{G} \frac{1}{(d_i - t)/\alpha G + 1} \qquad \text{at} \quad d_i - t \geq 0 \quad \text{and}$$

$$H_i = \frac{w_i p_i}{G}\left(\frac{t - d_i}{\alpha G} + 1\right) \quad \text{at} \quad d_i - t \leq 0, \qquad (1)$$
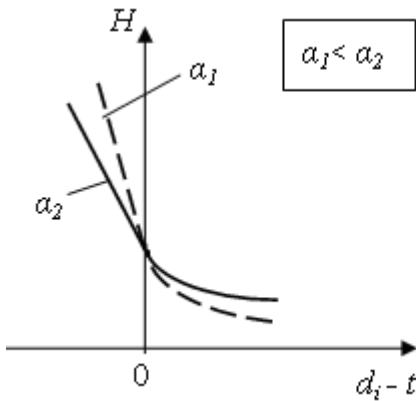


Fig. 1.   Production intensity diagrams

where $p_i$ – processing time of job $i$, $w_i$ – priority weight coefficient of job $i$, $G$ – plan bucket duration, $\alpha$ – "psychological coefficient".
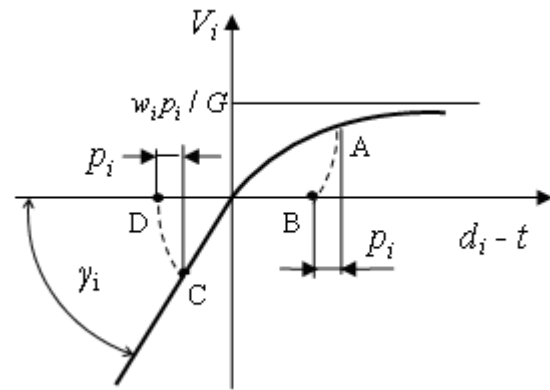


Fig. 2.   Current order utility function

The curves in Fig. 1 differ in the psychological coefficient value. The psychological coefficient determines the degree of placidity, when the time reserve is more than zero, and the degree of nervousness, when the time reserve is less than zero. The higher is the $\alpha$ coefficient, the more placid is the attitude to delays and the lower is the intensity.

Due to its additivity property, the production intensity may be used to describe status of planning objects and to assess various production environments: to compare intensity at various facilities (work centers, shops, departments). The production intensity concept may be used for determination of the current order utility function $V$ (Fig. 2). Assume that the current utility for an order $i$ is

$$V_i = \frac{w_i p_i}{G} - H_i. \qquad (2)$$

The curve in Fig. 2 for the positive value $d_i - t \geq 0$ tends to the horizontal asymptote

$$V_i = w_i p_i / G. \qquad (3)$$

In the negative part $d_i - t \leq 0$ the curve turns into the straight line with

$$tg\gamma_i = \frac{w_i p_i}{\alpha G^2}. \qquad (4)$$

Now let us consider the nature of utility function change in the period from job execution start until its completion. Let the point A in the positive field correspond to the start of the job $i$ with the processing time $p_i$. During job execution the remaining $p_i$ decreases in a linear manner. However, since production intensity increases in time, the current order utility $V_i$ would decrease in time ($d_i - t$) during $p_i$ in a non-linear mode as depicted by a dashed line AB. Similarly, if job execution starts at the point C in the negative field in Fig.

2, the current order utility $V_i$ would increase along a dashed line CD.

Assume that the order quantity on the planning horizon equals $N$. Then their total utility $V$ amounts to the sum of all order utilities, because orders, as a rule, are independent:

$$V = \sum_{i=1}^{N} V_i = \frac{1}{G}\sum_{i=1}^{N} w_i p_i - \sum_{i=1}^{N} H_i. \qquad (5)$$

The value of the function $V$ changes in time since the time reserve until the moment of scheduled completion also changes. Besides, some orders are get completed, and new orders appear in time.

Let us assume that a certain job that corresponds to the node of the scheduling versions tree at the level $l$ is completed at the moment of time $C_l$. Let us also assume that the job $k$ starts at the moment $t_k$, which is more than or equal to $C_l$. Then the average utility of the entire set of jobs $J$ from starting until completion of the job $k$ in the node at the level $l+1$ equals

$$\overline{V}_{l+1,k} = \frac{1}{t_k + p_k}\int_0^{t_k + p_k} V dt = \frac{1}{t_k + p_k}\left(\overline{V}_l \times C_l + \int_{C_l}^{t_k + p_k} V_k dt\right)$$

$$. \qquad (6)$$

In the formula (6) the value $\overline{V}_l$ equals to the average utility of the entire set of jobs on the planning horizon from the start moment $t = 0$ until completion $C_l$ of the last scheduled job. For example, at the moment $t = 0$ the machine is available, the quantity of completed jobs is $l = 0$, and $C_0 = 0$ accordingly. The value of $t_k$ depends on the moment $C_l$ or the job $k$ arrival moment. The integral in (6) in accordance with (2) equals

$$\int_{C_l}^{t_k+p_k} V_k dt = \frac{1}{G} \int_{C_l}^{t_k+p_k} \sum_{i \in J-I_l} w_i p_i dt - \int_{C_l}^{t_k+p_k} \sum_{i \in J-I_l} H_i dt ,$$

(7)

where $I_l$ – the set of jobs that would be completed in accordance with scheduling before the moment $t_k$.

Possible versions of using the formula (7) for a single machine and rules to compute integrals it contains are described in [8].

The function of negative expenses utility (loss function) for a single machine may be used as the first criterion in the dilemma "cost / efficiency". If the sequence number of planning job is $n$

$$U = \frac{1}{c}[c_s \sum_{l=0}^{n} s_l + c_r \sum_{l=0}^{n} (t_{kl} - C_l)] ,$$

(8)

where $c_s$ – cost of setup time unit, $c_r$ – cost of idle time unit, $s_l$ – setup time on level $l$.

Therefore, the scheduling optimization for a single machine may be achieved with the highest possible value of average orders utility $\overline{V}$ and the least possible value of expenses $U$ on the planning horizon $h$. In this process, $\overline{V}$ has to be computed according to the formula (6), $U$ - according to the formula (8) at every level of the possible solution tree.

## III. GROUP SCHEDULING FOR SINGLE MACHINE

Assume there is a set of jobs that arrive to a machine for processing in any sequence. These jobs refer to any of $S$ various types. The setup time $s_{fq}$ depends on sequence of transition from a type $f$ to a type $q$; when a job is available, the machine may also be idle, this results in additional costs. In accordance with the well-known three-part scheduling classification, the considered problem is:

$$1 | r_i, d_i, s_{fq} | U, \overline{V} .$$

(9)

There are two target functions in the formula (9), and they may both be improved only within certain limits. The Pareto compromise curve serves as such limit, because in its points the criterion $U$ improvement (diminution) means the criterion $\overline{V}$ deterioration (diminution). For solving the problem (9) it makes sense to apply the method, based on the MO-Greedy approach [9], which requires building a tree to search for non-dominated solutions. Using the expressions (1, 2), (6, 7) and formulas in [8], we can calculate the criterion $\overline{V}$ value in every tree node. The criterion $U$ value may be computed by the formula (8) in every node as well.

Now it is possible to formulate the MO-Greedy algorithm for solving the problem.

Step 1. *(Initial computation of utility functions)*

Let us suppose that the level number is $l = 0$; the machine is available, so $C_l = 0$; the initial expense function value is $U_0 = 0$; the initial orders utility function $V_0$ may be computed by the formula (5).

Step 2. *(Utility function computation at next levels)*

For each job $k$ that arrived before the moment $C_l$ and has not yet been completed, values $U_{l+1,k}$ and $\overline{V}_{l+1,k}$ are computed using the formulas (8) and (6).

Step 3. *(Determination of dominated tree nodes)*

If the level $l+1 < N$, then for domination on the level $l+1$ of the tree node $j$ with a job $i$ over the tree node $r$ with a job $k$ it is sufficient to comply with the following inequations

$$U_{l+1,j} \leq U_{l+1,r}, \overline{V}_{l+1,j} \geq \overline{V}_{l+1,r} \text{ and } g_i < g_k ,$$

(10)

besides, the first or the second inequation is strong.

Otherwise: on the last level $l+1 = N$ domination is possible, if

$$U_{l+1,j} \leq U_{l+1,r}, \overline{V}_{l+1,j} \geq \overline{V}_{l+1,r} .$$

(11)

Step 4. *(Transition to the next level or stopping)*

Level number increment $l = l+1$.

If the level is $l = N$, then STOP.

Otherwise, go to Step 2.

The inequation (10) applies the necessary start moment $g_i$, which is determined as

$$g_i = d_i - p_i.$$

(12)

Let us consider the following example assuming it is possible to perform three types of jobs on a single machine, where the setup time is sequence-depending. In Table I there is a list of five jobs to be scheduled within the 7-hours horizon. The norms of setup time $s_{fq}$ change from 0.1 to 0.3.

Table I        Job characteristics

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Product type | 1 | 2 | 1 | 3 | 1 |
| Processing time $p_i$, hours | 1 | 2 | 1 | 2 | 1 |
| Due date $d_i$ | -1 | 2 | 3 | 3 | 6 |
| Time of job arrival $r_i$ | -4 | 0 | 1 | 1 | 2 |
| Necessary start time $g_i$ | -1 | 1 | 3 | 2 | 6 |

|   | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 2 |  | Non-dominated versions on horizon 8 |  |  |  |
| 3 |  | Version 1: 1, 2, 3, 4, 5 |  |  |  |
| 4 |  | Version 2: 1,3, 2, 4, 5 |  |  |  |
| 5 |  | Version 3: 2, 1,3, 4, 5 |  |  |  |
| 6 |  | Version 4: 2, 1,3,5, 4 |  |  |  |
| 7 |  |  |  |  |  |

Fig. 3.  Computation results for data in Table I

As it appears from Table I, three jobs of the type 1 shall be scheduled, so grouping is expedient. To search for non-dominated solutions on the basis of the criteria $U$ and $\overline{V}$, the program using the VBA language for MS Excel has been designed. Numeral solution results recorded in an MS Excel sheet are given in Fig. 3. The jobs in every version of the solution were automatically grouped by job types. For instance, in the versions 2 and 3 jobs 1 and 3 are grouped as related to the type 1; in the version 4 jobs 1, 3 and 5 are grouped.
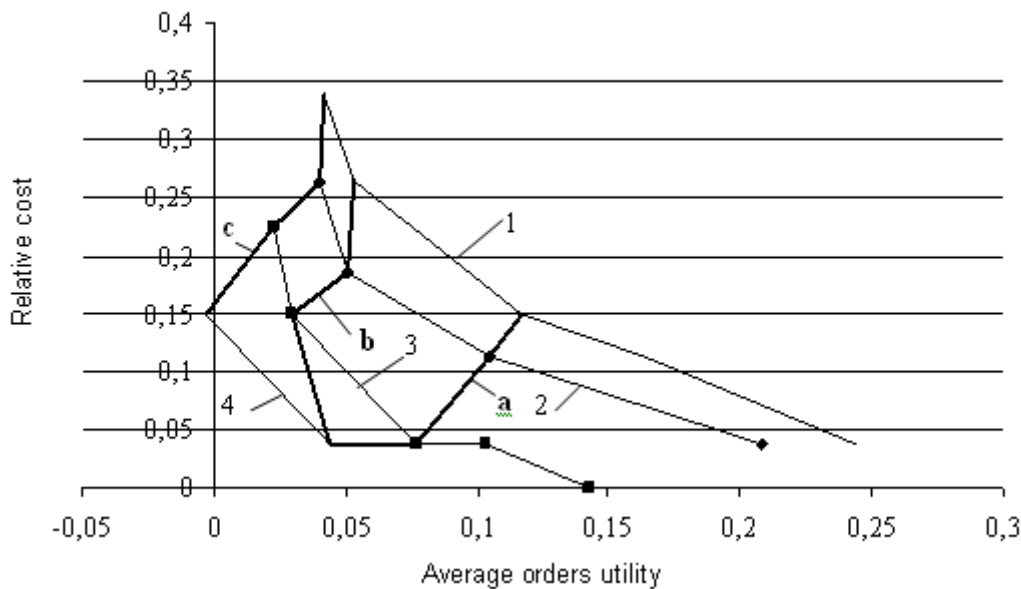
Fig. 4.   Diagrams of  scheduling versions $1 \div 4$  utility;
**a** - horizon 6;  **b**– horizon 7;  **c**– horizon 8.

Every  sequence  version  $1 \div 4$  corresponds  to  a trajectory on the plane of the average order utility criterion $\overline{V}$ and the relative cost criterion $U$ , as shown in  Fig. 4. By joining the points for different versions that correspond to the same horizon, it is possible to depict the set                 of Pareto-optimal solutions. In Fig. 4 such compromise curves **a, b, c** for horizons 6, 7, 8 are drawn accordingly.

Analysis of the diagrams in Fig. 4 makes it possible to make some conclusions about properties of solutions on the $U$ $\overline{V}$ plane.  The average orders utility of $\overline{V}$ decreases at every subsequent horizon. This is quite natural, since the time reserve for order completion decreases. At the same time the relative cost $U$  increases.  Sparseness of version points in the plane increases together with the horizon.

IV.   GROUP   SCHEDULING   FOR   PARALLEL MACHINES           AND           "MAKE-TO-STOCK" MANUFACTURING STRATEGY

The  average  job  $i$  processing  time        for  parallel unrelated machines equals to

$$\overline{p}_i = \frac{1}{m}\sum_{j=1}^{m} p_{ij} \, , \qquad (13)$$

where  $p_{ij}$ – processing time of  a job $i$   on a machine  $j$ .

When  scheduling  for  the  "make-to-stock"  strategy orders of customers are not considered.  At the same time it does not mean that a planner may neglect them.  When the "make-to-stock" strategy is used, it is necessary to provide (if applicable) the sufficient stock of every manufacturing product during the whole manufacturing time. Such stock has  to  provide  for  both  satisfaction        of  expected  average demand and minor demand deviations of average values. For  the  last  purpose  it  is  necessary  to  provide  a  certain safety stock.

For  parallel  machines  the  recurrent  formula  (6)  may  be used  without  changes,  if      completion  of  a  job  on  the previous level $l$  happens before  job completion  on the

subsequent  $l+1$  level.  Otherwise,  instead  of  the  formula (6) the following formula is used

$$\overline{V}_{l+1,k} = \overline{V}_l + \frac{1}{C_{il}} \int_{C_{jq}}^{t_k + p_k} V_k\,dt \;\; , \qquad (14)$$

where $i$ – the machine number  in the tree node, from which a new branch grows;  $j$– the machine number, for which a  new job $J_k$   in a new branch is meant; $C_{il}$ - the end  of  machine  $i$   operation  on  the  level $l$ ; $C_{jq}$ - the completion time of  the last job, which was planned on the level  $q$  on the machine  $j$ .  The algorithm for this problem is similar to the algorithm for the single machine problem. The difference consists in the necessity for node branching   for every parallel machine.

Let us consider the task of scheduling for   six parallel unrelated machines and        products of six various types. Assume  that  manufacturing  of  any  product  on  every machine  is  only   possible  in  a  volume  larger  than  a  so  called "technical lot". This lot may equal to the machine volume (for  process  manufacturing),  the  package  quantity  for discrete manufacturing, the transit norm (for instance, by weight),  etc.  Duration  of  technical  lot  manufacturing correlates  with  technical  parameters  of  the  machine  and depends on the product type.

During     task  execution  the   stocks  and  backlogs  of every  product  are  changing.   Apparently,  the  schedule  is good,  when  average  stocks  are  not  high  and  there  is  no backlog.  Fig. 5a presents three    stock dynamics processes most  typical  for  scheduling  that  describe  processes  for products P2, Q1 and R1.  Process dynamic of products P1 and P3 to a considerable extent is similar to the process of the product Q1, the process of the product Q2 is similar to process of the product P2.
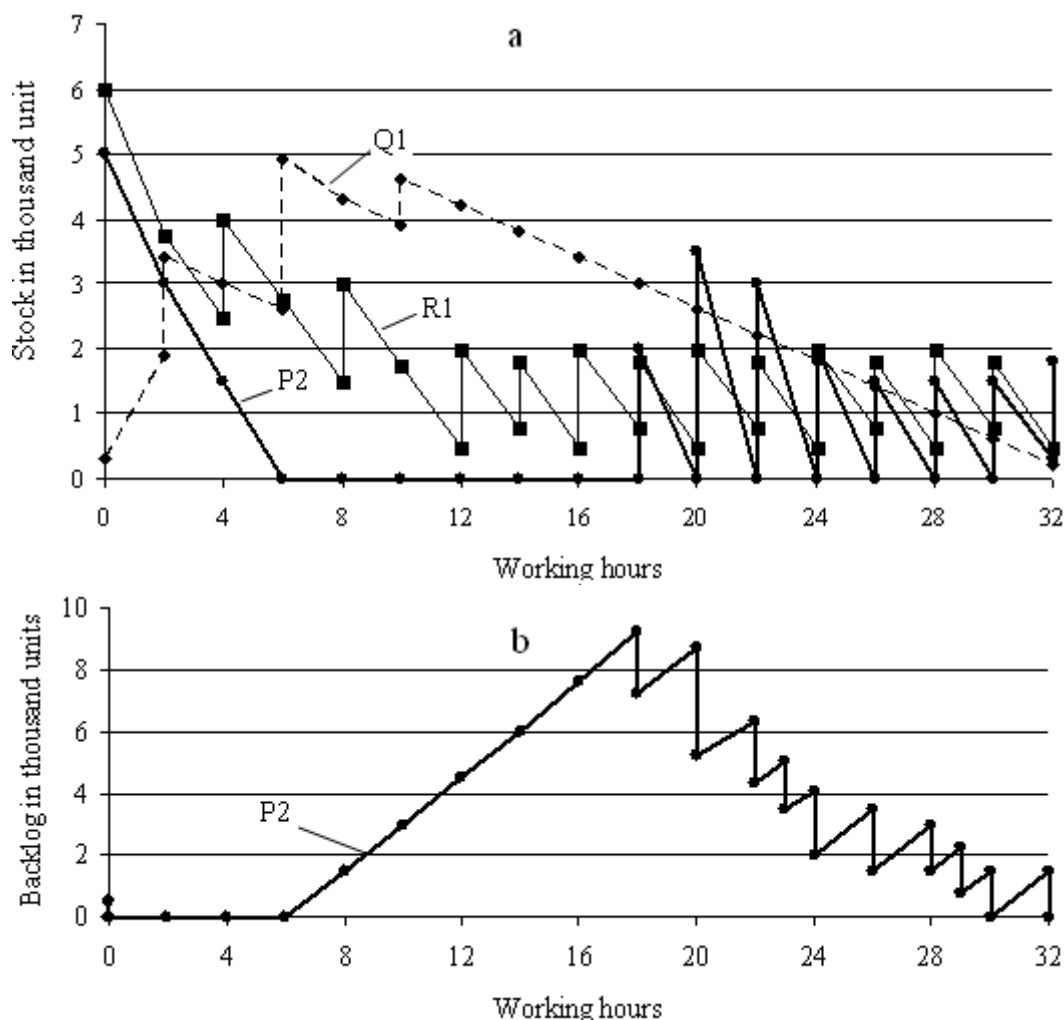
Fig. 5.   Dynamics of stocks and backlog during the task execution

It follows from the diagrams in Fig. 5a   that the most even is the process of manufacturing of the product R1, and there was no backlog of this product. Note that for the product R1 the highest safety stock is provided, which substantially caused such scheduling result. For the product  Q1  the plan provides  for manufacturing of  high  stock  in  the initial stage  of  the  scheduling   interval  and  then gradual consumption  from this stock to the safety stock level. In this case there is no backlog as well.

The situation   with the    product   P2 is much more complicated. According     to  the  schedule,  the  initial considerable  stock  should  be spent  in  six hours;  then the backlog  of  this  product   arises  and  grows  as  shown in Fig. 5b. It is only possible to reduce    the backlog of this product   to zero at the end of the planning interval.

## V.   GROUP   SCHEDULING   FOR   JOBSHOP MANUFACTURING

This  problem  may  be  considered  as  scheduling  for several  groups  of  parallel  machines  of  various  purposes. In this case every job consists of a set of operations, and every operation  has  to  be  executed  on  a  machine  with  a corresponding  purpose.  Let us assume that a set of jobs for manufacturing may be divided into groups of several types,

and     operation  setup  norms      $s_{ij}$   depend  on  the corresponding machine group  $j$  and job kind $i$ .

For example, let us consider scheduling for 20 jobs, each job contains from 3 to 5 operations to be executed in any given sequence. Assume there are 6 types of the job and the quantity of the machine group is 5.   Table II specifies machine properties   at the start of scheduling.

As it follows from Table II, there are 2 machines in the group  1,  the  group  2  includes  3  machines,  there  are  2 machines in the group 4 and each other group includes   one machine.  Besides,  the  machine  2  in  the  group  1  and  the machine 4 in the group 2 are excluded from planning.

Table II   Machine properties   at  scheduling start

| Machine number | Number of machine group | Start point | Settings at start | Release moment |
|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 0 |
| 2 | 1 | 0 | 1 | 3 |
| 3 | 2 | 1 | 4 | 2 |
| 4 | 2 | 0 | 2 | 0 |
| 5 | 2 | 1 | 2 | 4 |
| 6 | 3 | 1 | 1 | 0 |
| 7 | 4 | 1 | 4 | 5 |
| 8 | 4 | 1 | 3 | 0 |
| 9 | 5 | 1 | 5 | 0 |

|   | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Machine 1: 2/1, (8/1, 6/1), 5/2, (16/1, 10/1), (3/2, 12/1), 13/1, 14/1, 18/1, 20/1, (11/2, 15/2) | | | | | | |
| 2 | Machine 3: 3/1, 5/1, (7/1, 15/1, 11/1, 4/4), 17/2, (18/3, 12/4), (7/3, 15/4), 20/5 | | | | | | |
| 3 | Machine 5: 9/1, (8/2, 6/2), 10/3, 20/3, 19/2, 13/3 | | | | | | |
| 4 | Machine 6: 1/4, 4/2, (2/2, 9/2), (5/3, 17/1, 17/3), (3/4, 12/2, 18/2), | | | | | | |
| 5 |       (10/2, 16/2), 7/2, (19/1, 8/3), (13/2, 20/4), 10/5, 15/5 | | | | | | |
| 6 | Machine 7: (3/3, 12/3) | | | | | | |
| 7 | Machine 8: 4/3, 6/3, (5/4, 17/4), 20/2, (15/3, 11/4), (8/4, 19/3) | | | | | | |
| 8 | Machine 9: (2/3, 9/3), 6/4, (5/5, 14/2), 3/5, 17/5, 11/3, (18/4, 12/5), (10/4, 16/3), 8/5 | | | | | | |

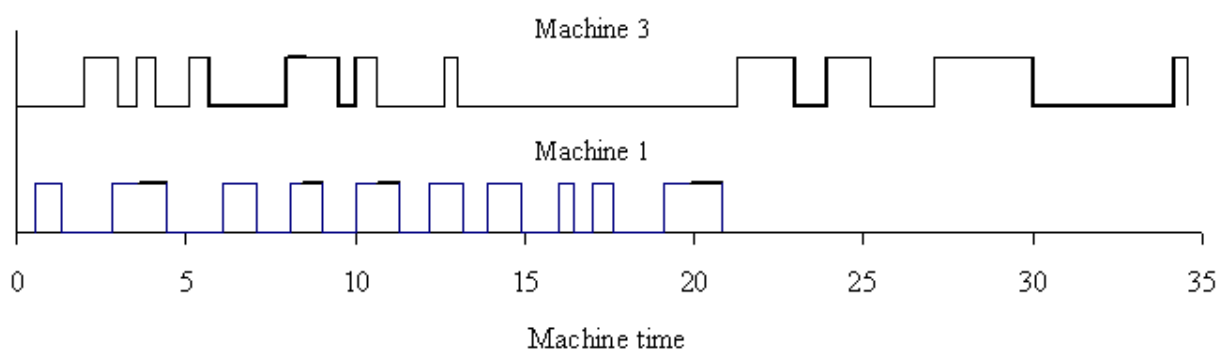Fig. 6.   Scheduling results for one of non-dominated versions



Fig. 7. Gantt diagrams for two machines

Applying of the scheduling method described above produces two non-dominated versions of a schedule and one of them is given in Fig. 6 as a record in the MS Excel sheet. Numbers in the sequence for every machine correspond to the number of a job and (in a fraction) the operation number of this job to be completed on this machine. Numbers in brackets form a group of jobs of identical type that do not require setup.

In Fig. 7 the Gantt diagrams for the machines 1 and 3 are depicted. Rectangles in the diagrams correspond to working operations, gaps stand for idle time. Thick lines correspond to operations without setups as their job type is the same as the previous one.

## VI. SUMMARY AND CONCLUSION

We have studied the dynamic method to solve the "operation planning dilemma" for a single machine, parallel machines and jobshop manufacturing. The relative expenses $U$ criterion and the average orders utility $\bar{V}$ criterion are used to define the correlation "cost/efficiency" on the planning horizon. The average orders utility value is determined, depending on the production intensity $H_i$ of every order, which changes in time. To design a schedule, a set of Pareto-optimal solutions shall be calculated on the planning horizon, and the final decision shall be made by the user.

The described method has considerable advantages compared to others due to an option of scheduling jobs with negative due dates. Besides, the method allows analyzing values of   relative setup cost and the average orders utility in order to choose the most acceptable schedule version.

For a single machine this method may by applied both for series-batch and parallel-batch fulfillment of various jobs. Quantity of possible jobs in the last case is determined by the machine work size and size of batches to be processed. Computations show that the           suggested method of scheduling for a single machine is suitable in the large parameter interval.

If   a flowshop manufacturing is the case, it may be considered as a particular case of jobshop manufacturing with uniform operation sequence for corresponding machine groups.  Therefore the suggested method may be applied for flowshop manufacturing both for "make-to-order" and "make-to-stock"  manufacturing strategies.  In real practice various additional constraints may be necessary for scheduling. For example, often it is needed to take into account the current device wear and tear, limited storage possibilities, general shipping terms, etc. In our opinion, it is reasonable that a user and the primary developer work out a customized program for every particular case.

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