

# Manufacture of 3D Möbius-Listing Models with a 3D Printer

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**Abstract**— The challenge addressed is to make real solid models (which can be felt and seen in 3D) of Möbius-Listing prototypes proposed by mathematical equations. The mathematical objects were generated by Wolfram Mathematica 8 and Autodesk 3ds-Max. Both programs provided versions in stereolithography format (STL). The objects were first printed with a 3D printer, with the chiral objects in a different color to distinguish them or for comparison purposes. In order to prepare future prototypes for milling in a 5-axis milling machine, we used Edgcam CAM software for their design and manufacture.

**Index Terms**— Elliptical Möbius-Listing, Squared and rectangular section half twisted torus, Squared and rectangular section Möbius-Listing, Chirality

## I. INTRODUCTION

There are two main methods used to obtain elliptical, squared, and rectangular Möbius-Listing prototypes. The first method uses Mathematica, which has to use the parametric representation of the Möbius-Listing formula created with the aid of the ParametricPlot3D function of the Wolfram Mathematica 8 kernel. In this method, the Mathematica kernel can be used to generate a 3d figure and saved as a stereolithography format (STL). The second method uses Autodesk 3ds Max. In this procedure, the strategy is to make a symmetric planar figure, then extruded it into a long cylinder, subsequently twisted by a half turn in either the clockwise or counterclockwise direction to obtain both chiral forms [1,2]. Finally the entire twisted cylinder is bent by  $2\pi$  to close the figure in 3D. The prototypes were saved in STL format. As before, Edgcam CAM software is used for its design and manufacture to prepare the piece for eventual cutting by a milling machine [3,4,5].

The prototypes were manufactured by both methods and printed using the Uprint Plus 3D printer. The rectangular section, half-twist ring torus is modeled as a parallelepiped of length  $L_z$ , with width  $L_y$  and thickness  $L_z$ . The whole parallelepiped is twisted half a turn on an axis

parallel to its length  $L_x$  and bent around a circle along its length  $L_x$  to make the half-twisted, rectangular-section ring torus. If one of the initial dimensions (thickness  $L_z$ ) is collapsed to a limit of zero thickness, then it results in a Möbius strip (in fact the Möbius strip is not a physically obtainable object, because a zero thickness prototype cannot be made).

A more realistic model of the Möbius strip must include a thickness and is better described by our model. Then, the proposed name of "parallelogram-section half-twisted ring torus with almost no thickness" is appropriate. It has the topological property of one side being preserved; the border or perimeter is then another one-sided surface perpendicular to the former, which is negligible when the aspect ratio approaches zero. Another topological property of having only one surface preserved occurs if it is accepted that the edge is negligible and that a wave over the surface has to go around twice to close properly. Moreover, the chiral property of the Möbius strip is also preserved, i.e., "two enantiomers." The non-orientable property of the Möbius strip is not present in the parallelogram-section, half-twisted ring torus, except at the limit when the parallelogram section is reduced to a slab without any thickness. In other words, the non-orientable property of the Möbius strip is intimately related to the zero thickness

An important observation is that the elliptical section of the Möbius-Listing figure is interesting because it is a figure which is symmetric and does not have an edge; however, it preserves chirality and other topological properties. It is easy to deduce from the inspection of a paper analog the parametric equations of a Möbius strip edge or perimeter:

$$x = a \cos \theta + \mu \cos \varphi / 2 \cos \theta \quad (1)$$

$$y = b \sin \theta + \mu \cos \varphi / 2 \sin \theta \quad (2)$$

$$z = \quad \quad \quad + \mu \sin \varphi / 2 \quad (3)$$

where  $a = b = \mu$  for a circular symmetry torus.  $\mu$  is half of the width of the Möbius strip with  $\mu < r < (a^2 + b^2)^{1/2}$ , and  $\varphi = \theta$  if the zero is chosen properly.  $\theta$  varies from 0 to  $4\pi$  to return to the same point; in other words "We go around it twice, so that shading comes out right." [3,4].

## Overview of Rapid Prototyping

The term rapid prototyping (RP) refers to a class of technologies that can automatically construct physical models from Computer-Aided Design (CAD) data. These "3D printers" allow designers to quickly create tangible prototypes of their designs instead of just 2D pictures. Such models have numerous uses [6].

They make excellent visual aids for communicating ideas with coworkers or customers. In addition, prototypes can be used for design testing. For example, an aerospace engineer might mount a model airfoil in a wind tunnel to measure lift

Manuscript received March 18, 2013.

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and drag forces. Designers have always used prototypes; RP allows them to be made faster and less expensively.

In addition to prototypes, RP techniques can be used to make tooling (referred to as rapid tooling) and even production-quality parts (rapid manufacturing). For small production runs and complicated objects, rapid prototyping is often the best manufacturing process available. Of course, "rapid" is a relative term. Most prototypes require from 3–72 hours to be made, depending on the size and complexity of the object. This may seem slow, but it is much faster than the weeks or months required to make a prototype by traditional methods such as machining. These dramatic time savings allow manufacturers to bring products to market faster and less expensively. In 1994, Pratt & Whitney achieved "an order of magnitude [cost] reduction [and] time savings of 70 to 90 percent" by incorporating rapid prototyping into their investment casting process.

## II. THE BASIC PROCESS

Although several rapid prototyping techniques exist, all employ the same basic six-step process. The steps are:

1. Generate a mathematical model Mathematica program
2. Create a CAD model of the design
3. Convert the CAD model to STL format
4. Slice the STL file into thin cross-sectional layers
5. Construct the model one layer atop another
6. Clean and finish the model

**Generate mathematical model with Mathematica:** Mathematica has to use the parametric representation of the formula of Möbius-Listing perimeter, and with the aid of the ParametricPlot3D function in the Mathematica 8 kernel, the STL (stereolithography format) can be generated (STL being an acronym derived from stereolithography) [7].

**CAD Model Creation:** The object to be built is modeled using a Computer-Aided Design (CAD) software package. Solid modelers, such as Pro/ENGINEER, tend to represent 3D objects more accurately than wire-frame modelers such as AutoCAD, and therefore they yield better results. The designer can use a pre-existing CAD file or can create one expressly for prototyping purposes. This process is identical for all of the RP build techniques.

**Conversion to STL Format:** The various CAD packages use a number of different algorithms to represent solid objects. To establish consistency, the STL format has been adopted as a standard in the rapid prototyping industry. The next step, therefore, is to convert the CAD file into STL format. This format represents a 3D surface as an assembly of planar triangles, "like the facets of a cut jewel." The file contains the coordinates of the vertices and the direction of the outward normal of each triangle. Because STL files use planar elements, they do not exactly represent curved surfaces. Increasing the number of triangles improves the approximation, but at the cost of a larger file size. Large, complicated files require more time to pre-process and build, so the designer must balance accuracy with manageability to produce a useful STL file. Since the STL

format is universal, this process is identical for all RP build techniques.

**Slice the STL File:** In the fourth step, a pre-processing program prepares the STL file for the build. Several programs are available, and most allow the user to adjust the size, location, and orientation of the model. Build orientation is important for several reasons. First, the properties of rapid prototypes vary from one coordinate direction to another. For example, prototypes are usually weaker and less accurate in the z- (vertical) direction than in the x-y plane. In addition, the orientation of a part partially determines the amount of time required to build the model. Placing the shortest dimension in the z-direction reduces the number of layers, thereby shortening the build time. The pre-processing software slices the STL model into a number of layers ranging from 0.01 mm to 0.7 mm thick depending on the build technique. The program may also generate an auxiliary structure to support the model during the build. These supports are useful for delicate features such as overhangs, internal cavities, and thin-walled sections. Each RP machine manufacturer supplies their own proprietary pre-processing software.

**Layer by Layer Construction:** The fifth step is the actual construction of the part. Using one of several techniques, the RP machines build one layer at a time from polymers, paper, or powdered metal. Most machines are fairly autonomous, needing little human intervention.

**Clean and Finish:** The final step is post-processing. This involves removing the prototype from the machine and detaching any supports. Some photosensitive materials need to be fully cured before use. Prototypes may also require minor cleaning and surface treatment. Sanding, sealing, and/or painting the model will improve its appearance and durability.

**Fused Deposition Modeling (FDM):** In this technique, filaments of heated thermoplastic are extruded from a tip that moves in the x-y plane. Like a baker decorating a cake, the controlled extrusion head deposits very thin beads of material onto the build platform to form the initial layer. The platform is maintained at a lower temperature so that the thermoplastic quickly hardens. After the platform lowers, the extrusion head deposits a second layer upon the first. Supports are built along the way and fastened to the part either with a second, weaker material or with a perforated junction.

Stratasys, of Eden Prairie, MN makes a variety of FDM machines ranging from fast concept modelers to slower, high-precision machines. Materials include acrylonitrile butadiene styrene (ABS; standard and medical grade), elastomer (96 durometer), polycarbonate, polyphenolsulfone, and investment casting wax.

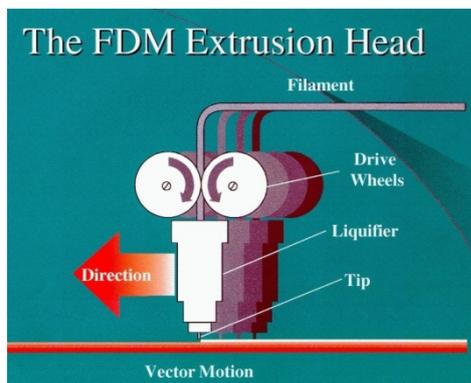


Fig. 1. Schematic diagram of fused deposition modeling.



Fig. 2. Rapid prototype machine, exterior and interior.



Fig. 3. Material bag used for printing.

### III. RESULTS

After the shapes were 3D printed, the figures were compared and photographed. The figures are provided here and mainly show the topological properties discussed above, i.e., chirality with one of the sides collapsed to the Möbius perimeter. The non-orientable property is just observed in the thickness limit. Also, the  $C_2$  symmetry is self-evident.

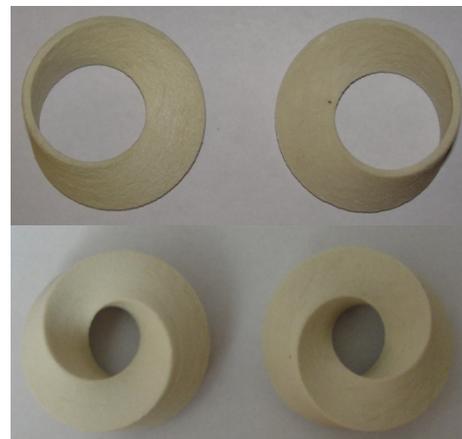


Fig. 4. Twisted torus from squared section to Möbius-Listing quadrangular section.

### IV. CONCLUSION

From the manufacture of these models one can deduce that it is possible to mill machine models for other experimental purposes, including for superconducting experiments and to measure the magnetic field around a magnetized Möbius-Listing model and electrical field of charged Möbius-Listing charge with electrons to measure the electric field around the Möbius-Listing model.

The present work was made as preparation for milling using a 5-axis milling machine.

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