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Photoacoustic Tomography with Diffusion Approximation

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***Abstract*—A framework is presented for identifying projection information in photoacoustic measurements under a diffusion approximation for tomographic image reconstruction. We propose to solve the inverse problem with respect to a thermoelastic wave transport. By means of a numerical simulation, we examine how our diffusion model relates to the signal registration and deduce an adjustment on small animal image reconstruction.**

***Index Terms*—inverse problem, diffusion approximation, photoacoustic imaging.**

I. Introduction

THERE is considerable interest on biomedical image reconstruction with photoacoustic data due to its non-invasive and non-ionizing testing utilization opportunities. Photoacoustic tomography is a hybrid image modality referring to electromagnetic (EM) energy absorption and sound generation within the object of interest. Ideally, acoustic detectors, positioned on the perimeter of the object, register projection information of the EM energy absorption map. In case of an acoustically homogeneous media the underlying transport is commonly modeled in terms of a plane wave propagation [1]. This model approach facilitates the solution of the inverse problem tremendously. One possible solution is obtained by applying the inverse (spherical) Radon transform to the measured data, and the source gets backprojected. In a recent study [2], the author works on a wave transport model incorporating an attenuation approximation in such a way that the Radon inversion is still a reliable solution. This efficient strategy no longer uses the raw detector measurement, but treated data, which approximates projection information under the consideration of attenuation.

The thermoelastic expansion, intrinsic to the photoacoustic effect, generally constitutes the inhomogeneous part of the wave equation. In this article we make the conjecture, that the transport may correspond to the thermoelastic wave equation, inspired by heat waves [3], and as a consequence we present a novel approach on photoacoustic image reconstruction with a diffusion approximation. Then, we analyze the model estimation in accordance to the findings in [2] and we apply the results on the photoacoustic section image reconstructions.

This paper is organized as follows: In section II, we present a conceptual model linking of thermoelastic wave transport to photoacoustic image reconstruction in terms of an inverse problem. As the underpinning issue of this work, we propose a model extension for photoacoustic methods with a diffusion approximation. Along with the suggested wave equation, we present in section III the common backprojection approach applied on a phantom, mimicking the zebra-fish case study of [4], referring to both plane wave and thermoelastic wave model. Furthermore, we outline a strategy for a numerical inversion with diffusion. In section IV, we deduce an adjustment of the zebra-fish section image reconstruction and express the visual results. Finally, we outline our conclusion in section V and reveal implications for biomedical image reconstruction.

II. The extended photoacoustic inverse Problem

In this section, we present a conceptual model linking of photoacoustic methods to tomographic image reconstruction in its mathematical formulation as an inverse problem. Thereby, we deduce a new photoacoustic transport approximation inspired by heat waves [3], that permits general interpretations in the broader domain of diffraction tomography.

The photoacoustic transport can be described in terms of a system of partial differential equations by modeling a specific transport acting on the pressure distribution function $p : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}_+$ over space and time. In case that the source, or rather the initial distribution $f(\mathbf{x}) = p(\mathbf{x}, 0)$ is a weakly scattering or small object, the occurring phenomena allow an approximation of the transport in free space expressed by a linear integral equation of second order with initial (functional and temporal gradient) conditions, $\forall \mathbf{x} \in \mathbb{R}^n$ and $\forall t \in \mathbb{R}$,

$$\mathcal{L}p(\mathbf{x}, t) = 0 \quad (1)$$

$$p(\mathbf{x}, 0) = f(\mathbf{x}) \quad (2)$$

$$\partial_t p(\mathbf{x}, 0) = 0. \quad (3)$$

The homogeneous linear differential equation (1) reveals an acoustic homogeneous media under stress confinement. By the principle of Duhamel the system (1-3) has an equivalent inhomogeneous version [5],

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$$\begin{aligned} \mathcal{L}p(\mathbf{x},t) &= f(\mathbf{x})\partial_t\delta(t), &(4)\\ \partial_t p(\mathbf{x},0) &= 0, &(5)\\ p(\mathbf{x},t_-) &= 0, \quad \forall t_- < 0, &(6)\end{aligned}$$

where δ represents the temporal delta function related to the illumination. In photoacoustic tomography it is assumed that for any source f there exists a trace g of the forward problem that corresponds to what sensors register at the boundary of the observed region Ω over a fixed time interval $t \in [0, T]$,

$$p(\mathbf{y}, t) = g(t), \quad \mathbf{y} \in \partial\Omega. \tag{7}$$

Photoacoustic imaging in a homogeneous and non-attenuating environment is usually based on a plane wave transport according to [1], and is modeled by the linear d'Alembert operator $\Box$ (including the Laplace operator ∇^2) and a constant wave speed c,

$$\mathcal{L}_0 p_0(\mathbf{x},t) := \Box p_0(\mathbf{x},t) = (\partial_t^2 - c^2\nabla^2)p_0(\mathbf{x},t). \tag{8}$$

The associated inverse problem can be expressed in terms of the (spherical) Radon transform

$$\mathcal{R}f = g, \tag{9}$$

where the actual integrating geometry varies with the detector shape. The inverse operator is known [6], hence the problem can be solved numerically.

"Nowadays, there is a trend to incorporate more and more modelling into photoacoustics" [7]. For miscellaneous applications of photoacoustic methods on non-homogeneous media we propose to consider the thermoelastic wave equation:

$$\mathcal{L}_d p_d(\mathbf{x},t) := \Box p_d(\mathbf{x},t) + d(t) * \partial_t p_d(\mathbf{x},t), \tag{10}$$

introducing the weight function d for the gradient transport. This diffusion approach should not be ignored when heat propagation is considerable in the experimental analysis. A feasible approach to estimate the appropriate weights is done by optimizing

$$\{d : [0,T] \to \mathbb{C} \mid \min_{t\in[0,T]} \|d(t)\partial_t g(t) + \partial_t^2 g(t)\|_2\}. \tag{11}$$

From a point of view of the model description (4-6) we can compare both waveform expressions in the Fourier space,

$$\begin{aligned} (c^2\nabla^2 + \omega^2)\hat{p}_0(\mathbf{x},\omega) &= \frac{i\omega}{\sqrt{2\pi}} f(\mathbf{x}), &(12)\\ (c^2\nabla^2 + \omega^2 - i\omega\hat{d}(\omega))\hat{p}_d(\mathbf{x},\omega) &= \frac{i\omega}{\sqrt{2\pi}} f(\mathbf{x}), &(13)\end{aligned}$$

and obtain the following relation on the measurements at the boundary $g_0(t) = p_0(\mathbf{x},t)$, $g_d(t) = p_d(\mathbf{x},t)$ for $\mathbf{x} \in \partial\Omega$

$$\hat{g}_0(K(\omega)) = \frac{K(\omega)}{\omega}\hat{g}_d(\omega), \tag{14}$$

with $K(\omega) = \sqrt{\omega^2 - i\omega\hat{d}(\omega)}$. Hence, the issue is to estimate g_0 from g_d using the relationship $g_d = \mathcal{T}g_0$, where $\mathcal{T}$ under the consideration of (14) is defined by

$$\mathcal{T}g_0(t) = \frac{1}{2\pi}\int_{\mathbb{R}} \frac{\omega}{K(\omega)} e^{i\omega t} \int_0^T g_0(t') e^{-iK(\omega)t'} dt' d\omega. \tag{15}$$

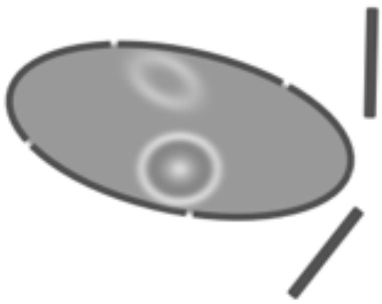

Figure 1: Schematic absorption map of a zebra fish.

We apply the strategy of [2] and solve the inverse problem by performing the following instructions:

1) Given the measurement g_d in a physical environment modelled by $\mathcal{L}_d p_d = 0$, estimate $g_0 = \mathcal{T}^{-1}g_d$.
2) Perform backprojection with respect to the adjusted projection data $\mathcal{T}^{-1}g_d$.

We carry out the inversion of the integral in (15) using an appropriate discretization and the Singular Value Decomposition (SVD) approach in the interest of succeeding the inversion of the ill-conditioned matrix representing $\mathcal{T}$.

III. Forward simulations and backprojection

We perform a numerical simulation of both plane wave and thermoelastic wave transport on a two-dimensional phantom, mimicking the zebra-fish case study of [4] and predict the sinogram information (forward problem). Moreover, we illustrate how errors, due to inappropriate projection approximation, can affect the reconstruction quality.

Figure 1 illustrates a simple musculoskeletal and digestive atlas of a zebra-fish axial cross-section, representing by different gray levels distinct regions of the EM energy absorption. If the absorption map can be considered as nearly acoustically homogeneous, we may simulate the photoacoustic forward and inverse problem i.e. with a cylindrical detector assuming a plane wave transport as in (8). The result is demonstrated in Figure 2. In case of a large number of projections, equally distributed around the object of interest, reconstruction artifacts are essentially insignificant. However, when diffusion alters the transport, as indicated by the thermoelastic wave equation (10), measurements approximate projections whose information content is not correctly interpreted, or rather the Radon transform is erroneously applied to the source. Figure 3 illustrates sinogram and the backprojection result when $K(\omega)$ is approximated by $\omega - 1/2cd\omega^3$ with constant $d = -0.02$. Interference artifacts, apparently weighted by the absorption coefficients, distort the image reconstruction. However, we are able to correct this impact by applying the inverse Radon transform on the transformed data $\mathcal{T}^{-1}g_d$. The reconstruction quality is expected to be as good as in Figure 2.

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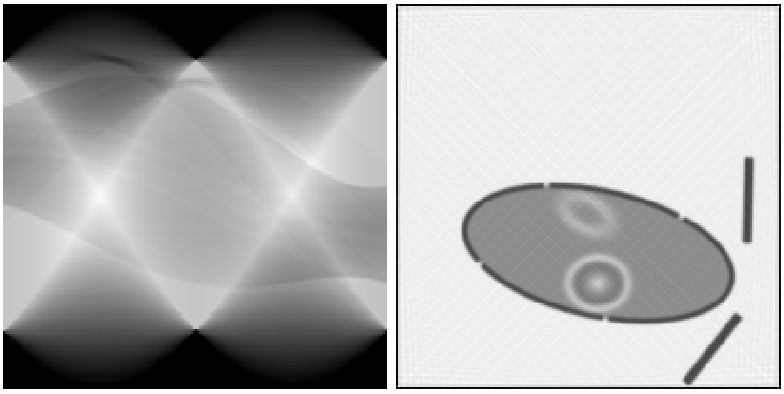

Figure 2: Projection approximation g_0 and the inverse solution $f_0 = \mathcal{R}^{-1}g_0 \approx f$ under a plane wave transport.

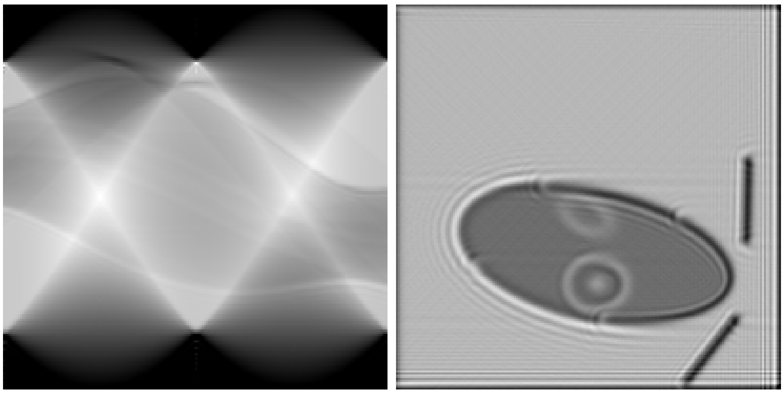

Figure 3: Projection approximation g_d and the inverse solution $f_d = \mathcal{R}^{-1}g_d$ under a diffusive transport.

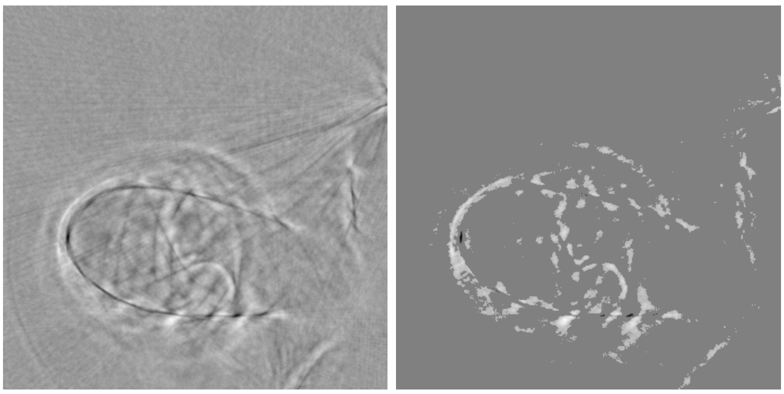

Figure 4: Backprojection $\mathcal{R}^{-1}g_d$ and its windowed image over the intervals $[0.0, 0.3]$ and $[0.7, 1.0]$.

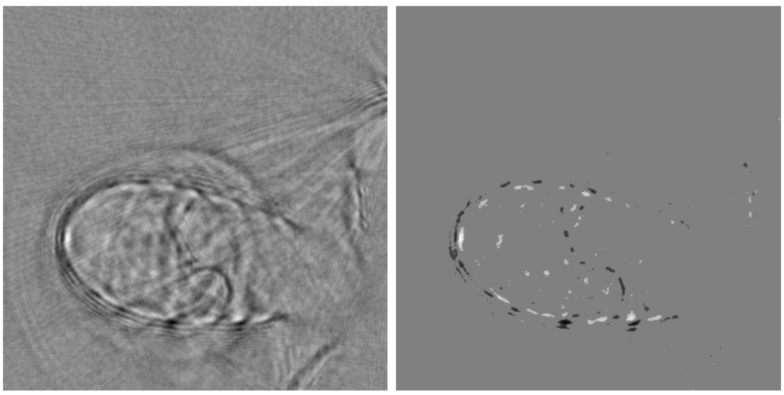

Figure 5: Backprojection $\mathcal{R}^{-1}(\mathcal{T}^{-1}g_d)$ and its windowed image over the intervals $[0.0, 0.3]$ and $[0.7, 1.0]$.

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IV. Adjusted biological section image reconstruction

The purpose of this section is to evaluate how an adjusted projection approximation with the diffusion approach simulated previously will alter the reconstruction result. Hence, we compare the registered sinogram of a real zebra-fish axial cross-section and its $\mathcal{T}$-Transformation and their normalized backprojection images in Figure 4 and Figure 5. Since both reconstructions do not reveal the essential differences at a first sight, we also visualize a windowed image and filter out normalized signal values of the interval $[0.3, 0.7]$ that are finally mapped on the neutral value 0.5.

Visual results

We observe that by applying the projection adjustment the well-defined body shape and the zebra-lines of the fish become more fitted and clear. Nonetheless, it is noticeable that poor absorption signals, e.g. from the pelvic fins or the digestive region, suffer the signal processing and disappear up to some extent.

V. Conclusions

We conclude from our results that a projection adjustment provide profitable advantages for small animal tomographic experiments, when the scientific study involves a (rough) segmentation problem. The particular benefit of the present adjustment technique is that the diffusion transformation is only applied once on the measurement, hence it does not imply extra computational reconstruction efforts.

We have demonstrated that the proposed photoacoustic transport model extension is a valid approach to improve tomographic image reconstruction in the presence of acoustic heterogeneities. Indeed, the present case study is a first application approach that apparently does not present a significant thermoelastic wave transport. That is why the constant diffusion parameter d in (14) has to be held small, and consequently backprojection results in Figure 4 and Figure 5 look very similar. Probably this adjustment strategy may interfere even better on photoacoustic tomography of more diffusive media. Our future expectation is to apply the model to some larger volumes and thus proceed refining the methodology. It remains an open problem to complete the transport equation in order to improve image reconstruction results.

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