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# Optimization of the CBSMAP Queueing Model

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***Abstract*—The present paper is devoted to the research of controlled queueing models at control of CBSMAP-flow, Controlled Batch Semi-Markov Arrival Process (Kashtanov, Kondrashova 2012). The control is based on the theory of controlled semi-markov processes and used for the system optimization. The control is carried out using the choice of the next batch type.**

***Index Terms*—controlled queuing models, optimization, controlled process**

## I. Introduction

The functioning of different systems can be described using queueing models. Application of control is used to increase the efficiency of the system functioning. In the present paper the process of system functioning is investigated using control by the arrival flow.

The Controlled Batch Semi-Markov Arrival Process is a generalization of the BMAP-flow [9]-[10]. BMAP-flow is good for modeling of data-flows in telecommunication networks.

Define CBSMAP-flow. After holding in the state comes to an end, the Controlled Semi-Markov process jumps to the other state and the batch of queries of CBSBMAP-flow will be generated.

## II. Controlled Semi-Markov process

Controlled Semi-Markov process $X(t) = \{\xi(t), u(t)\}$ is defined using homogeneous three-dimensional markov chain (Kashtanov 2010)

$$(\xi_n, \theta_n, u_n),\ \ n \geq 0,\ \ \xi_n \in E,\ \ \theta_n \in R^+ = [0, \infty),\ \ u_n \in U,$$

which is defined by transition probabilities of a special type.

$$\begin{aligned}&P\{\xi_{n+1} = j, \theta_{n+1} < t, u_{n+1} \in B / \xi_n = i, \theta_n = \tau, u_n = u\} = \\ &= P\{\xi_{n+1} = j, \theta_{n+1} < t, u_{n+1} \in B / \xi_n = i\}, \\ &i, j \in E,\ \ t, \tau \in R^+,\ \ u \in U,\ \ B \in A \\ &p_i = P\{\xi_0 = i, \theta_0 < \infty, u_0 \in U\}\ .\end{aligned} \tag{1}$$

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In the further we shall use the following designations:

$$\begin{aligned}&P\{\xi_{n+1} = j, \theta_{n+1} < t, u_{n+1} \in B / \xi_n = i\} = \\ &= \tilde{Q}_{ij}(t, B).\end{aligned} \tag{2}$$

For each state $i$ the set of controls $U_i$ and σ-algebra $A_i$ of subsets of this set $U_i$ is given.

At $t \to \infty$ and $B = U_i$ we obtain the transition probability

$$p_{ij} = \tilde{Q}_{ij}(\infty, U_i) = P\{\xi_{n+1} = j / \xi_n = i\} \tag{3}$$

for embedded Markov chain.

Use definition $\tilde{Q}_{ij}(t, B)$

$$\begin{aligned}&\tilde{Q}_{ij}(\infty, B) = lim_{t\to\infty}\tilde{Q}_{ij}(t, B) = \\ &= P\{\xi_{n+1} = j, u_{n+1} \in B / \xi_n = i\}.\end{aligned} \tag{4}$$

Then

$$G_i(B) = \sum_{j \in E} \tilde{Q}_{ij}(\infty, B) = P\{u_{n+1} \in B / \xi_n = i\} \tag{5}$$

and

$$\tilde{Q}_{ij}(t, B) = \int_B Q_{ij}(t, u) G_i(du), \tag{6}$$

where

$$Q_{ij}(t, u) = P\{\xi_{n+1} = j, \theta_{n+1} < t / \xi_n = i, u_{n+1} = u\}. \tag{7}$$

Thus, homogeneous Controlled Semi-Markov process can be set by family of matrixes, set of probability measures and initial distribution of probabilities $p_i$, $i,j \in E$, $t \in R^+$, $u \in U_i$, $B \in \mathbf{A}_i$.

Family of matrixes $\{Q_{ij}(t, u)\}$ is a Semi-Markov kernel of controlled Semi-Markov process, and family of probability measures $\vec{G} = \{G_1(B), G_2(B), ..., G_N(B)\}$ is family of controlling measures.

The counting process $\nu(t)$ is defined in following way

$$\nu(t) = \sup\{n : \sum_{k \leq n} \theta_k \leq t\},\ \ \theta_0 = 0.$$

The Controlled Semi-Markov process is defined as

$$X(t) = \{\xi(t),\ \ u(t)\},$$

where $\xi(t) = \xi_{\nu(t)}$, $u(t) = u_{\nu(t)+1}$.

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Process $\xi(t)$ coincides with a standard Semi-Markov process. The second component of controlled Semi-Markov process $u(t)$ defines a trajectory of accepted decisions.

It is possible to define one more way to give controlled Semi-Markov process. It is necessary to set:

· Markov homogeneous control strategy $\vec{G} = \{G_1(B), G_2(B), ..., G_N(B)\}$,

· characteristics of controlled Markov chain - initial distribution

$p_i = P\{\xi_0 = i\} \geq 0, \ i \in E, \ \sum_{i \in E} p_i = 1$ and a matrix of transition probabilities $p_{ij}(u) = P\{\xi_{n+1} = j / \xi_n = i, u_{n+1} = u\}$ ;

· conditional distributions of intervals $F_{ij}(t,u) = P\{\theta_{n+1} < t / \xi_{n+1} = j, \xi_n = i, u_{n+1} = u\}$.

## III. CBSMAP MODEL

### A. Assumptions

Consider, that the final batch of customers of k-th type arrives at the moment of CSMP (Controlled Semi-Markov Process) transition in state $k$ in queue model, the number of queries in batch $v_k$ is defined by generating function

$$\Phi_k(z) = \sum_{m=0}^{M_k} z^m p^{(k)}(m), \tag{8}$$

where $p^{(k)}(m)$ is a probability ( the number of *k*-th type customers in batch is *m*,) $M_k$ – a maximal number of customers in batch of *k*-th type.

Formulate the important assumptions at which the further researches will be carried out.

1. Customers of the same type arrive in the system (subsystem), each of which functions irrespective of other subsystems. Designate as system(k) the subsystem which is carrying out service of queries of k-th type.

Notice, that the process of service in each system is realized irrespective of other system states, however functioning of the systems is coordinated with the general arrival flow.

2. Between the next moments of the change of the CSMP states, customers do not arrive in the system, only the process of service in subsystems is carried out. Process of service in *k*-th subsystem is characterized by number $v^{(k)}(t)$ of the customers which are being in the subsystem during the moment *t*.

Note, that there are several types of admission discipline.

Three admission disciplines are known:

- partial admission, when only a part of the batch corresponding to the number of free places in the buffer is allowed to join the system;
- complete admission, when the whole batch is allowed to enter the system, if there is at least one free place in the buffer;
- complete rejection, when the whole batch is rejected.

### B. Description of the model. Algorithm

Describe the model. The given system consists of *N* subsystems.

The subsystem of *k*-th type $k = \overline{1, N}$, in designations of Kendall's notation can be described as follows:

CBSMAP/$M_k$/$n_k$/$N_k$, where

- CBSMAP means, that the arrival flow is controlled flow, defined earlier;
- Symbol $\mathbf{M_k}$ means, that service duration of customer in a subsystem is exponentionally distributed with parameter $\mu_k$;

·-Symbols $\mathbf{n_k}$ and $\mathbf{N_k}$ define the quantity of the service buffers and the number of places in the queue $n_k$ and $N_k$ accordingly.

In classification of queueing systems, the system can be considered as a Controlled Semi-Markov system as its evolution is defined with Controlled Semi-Markov Process.

For construction of CSMP, describing the evolution of the system, it is necessary to carry the following algorithm:

· Define Markov moments,

· Define the states of Semi-Markov Process;

· Define control set and control strategy;

· Define Semi-Markov kernel and a matrix of transition probabilities of embedded Markov chain;

· Construct income functional on the trajectories of CSMP;

· Define optimum strategy of control.

In the given model the Markov moments are the moments of arrivals of any type customers in system. In case of *k*-th type customers arrival, the given customers are taken on service to a subsystem of *k*-th type, and in other subsystems the batch of zero quantity "arrives".

The system states are defined using a vector $(i, l_1, l_2, ..., l_N)$ , where $i$ - a state of an arrival flow (at Markov moment the batch of *i*-th type customers arrives), $l_k$ - quantity of queries in a subsystem of *k*-th type, $M_k + N_k + n_k > l_k \geq 0, \ k \neq i, \ M_i + N_i + n_i > l_i > 0,$ $n_k$ and $N_k$ - accordingly quantity of service channels and quantity of places in the queue in System *(k)*, $i \in E = \{1, 2, ..., N\}$. The quantity of customers in a subsystem is final and depends on admission discipline and structure of queueing model. Therefore $l_k \in E_k = \{1, 2, ..., M_k + N_k + n_k - 1\}$

and $(i, l_1, l_2, ..., l_N) \in E \times E_1 \times ... \times E_N$.

Enter the following designations:

$$(l_1, l_2, ..., l_N) = \vec{l} \ ;$$

$$E \times E_1 \times ... \times E_N = \tilde{E}.$$

The transition from state $(i, l_1, l_2, ..., l_N)$ to state $(j, l'_1, l'_2, ..., l'_j, ..., l'_N)$ with positive probability occurs if $l'_k \leq l_k, \ k \neq j$, that is true to all subsystems, except for



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a subsystem of $j$-th type there is only a service customers which can be presented as process of death process, according to the quantity of customers in these subsystems is not more than the quantity of customers in subsystems at previous Markov moment of batch arrival.

Note, that the system control is carried out using control of arrival-flow at the moments of SMP states change, at the Markov moments.

Remind, that control Markov strategy $\vec{G} = (G_{(i,\vec{l})}(u),\ (i,\vec{l}) \in \tilde{E})$, depending only on a current state of controlled process, is a set of the probability measures, given for each state $(i,\vec{l}) \in \tilde{E}$ on $\sigma$ - algebra of subsets of decisions' set $U_{(i,\vec{l})}$.

*C. Control set*

As it was marked above, the problem of control with arrival flow is investigated. The flow is described by Controlled Semi-Markov Process.

Construct a control set. At Markov moments (transition moments, state $(i,\vec{l}) \in \tilde{E}$), the type of queries is chosen. Then equality $U_{(i,\vec{l})} = E = \{1, 2, ..., N\}$ is fair, and the probability measure on discrete set $E = \{1, 2, ..., N\}$ is defined with a set of probabilities

$$G_{(i,\vec{l})}(j) = P\{u(t) = j / \xi(t) = (i,\vec{l})\} = p_{(i,\vec{l}),j},\quad p_{(i,\vec{l}),j} \ge 0,\ \sum_{j\in E} p_{(i,\vec{l}),j} = 1,\ E = \{1, 2, ..., N\}, \tag{9}$$

where the decision accepted at Markov moment $t$ is designated $u(t)$;

The Semi-Markov kernel is a probability of that the Semi-Markov process will pass in state $(j,\vec{l})$ (at the Markov moment the batch of $j$-th type arrives and there will be $\vec{l}$ queries in subsystems) and the time of this transition will not surpass $t$, provided that the process stays in state $(i,\vec{m})$ and in the state the decision $u \in U_{(i,\vec{m})}$ from the control set is accepted.

Designate the probability as $Q^{(k)}_{(i,\vec{m}),(j,\vec{l})}(t,u)$.

In case $U_{(i,\vec{m})} = E = \{1, 2, ..., N\}$, the type of queries which arrive to the system is chosen:

$$u \in U_{(i,\vec{m})} = E = \{1, 2, ..., N\},\ \ k \in E = \{1, 2, ..., N\},$$
$$(i,\vec{m}),(j,\vec{l}) \in \tilde{E},\ \ t \ge 0$$

$$Q^{(k)}_{(i,\vec{m}),(j,\vec{l})}(t,u) =$$
$$= \begin{cases} \int_0^t \sum_{s=0}^{m_j} p^{(j)}_{m_j,s}(x)\tilde{p}^{(j)}_{N_j+n_j-s}(l_j - s)\prod_{v\ne j} p^{(v)}_{m_v,l_v}(x)dF_{i,j}(x), u = j; \\ 0,\ \ u \ne j. \end{cases}$$

Describe the designations used in previous equality.

- The probability $p^{(j)}_{m_j,s}(x)$ is defined as follows

$$P\{v^{(k)}(t) = s / v^{(k)}(0) = m\} = \tag{10}$$
$$= \begin{cases} p^{(k)}_{ms}(t) \ge 0,\ \ m \ge s \ge 0; \\ p^{(k)}_{ms}(t) = 0,\ \ m < s. \end{cases}$$

probability of that in $k$-th subsystem during time $t$ (between the Markov moments) *(m-s)* queries are operated, provided that during the initial moment there were $m$ queries in System.

If System *(k)* is a system without queue, $N_k = 0$,

$$p^{(k)}_{ms}(t) = C^s_m(1 - e^{-\mu_k t})^{(m-s)} e^{-s\mu_k t},\ \ n_k \ge m \ge s \ge 0 \tag{11}$$

where $n_k$ - the number of channels, $\mu_k$ - parameter of exponential distribution of the service duration. For other combinations of parameters $n_k, m, s$ required probabilities are equal to zero.

If System *(k)* is a single-channel system with queue, $n_k=1$, $N_k>0$, then

$$p^{(k)}_{ms}(t) = \frac{(\mu_k t)^{m-s}}{(m-s)!} e^{-\mu_k t},\ \ N_k + 1 \ge m \ge s > 0,$$

$$p^{(k)}_{m0}(t) = 1 - \sum_{l=1}^{m} p^{(k)}_{ml}(t) = 1 - e^{-\mu_k t}\sum_{l=0}^{m-1}\frac{(\mu_k t)^l}{l!}, \tag{12}$$

$$N_k + 1 \ge m \ge s = 0.$$

For other combinations of parameters $n_k, m, s$ required probabilities are equal to zero.

The general case, if System *(k)* is a multichannel system with queue $n_k \ge 1, N_k \ge 0$, then

$$p^{(k)}_{ms}(t) =$$
$$= \begin{cases} C^s_m(1 - e^{-\mu_k t})^{(m-s)} e^{-s\mu_k t},\ \ n_k \ge m \ge s \ge 0, \\ \dfrac{(n_k\mu_k t)^{m-s}}{(m-s)!} e^{-n_k\mu_k t},\ \ m \ge s \ge n_k, \\ \int_0^t C^s_{n_k}(1 - e^{-\mu_k(t-x)})^{(n_k-s)} e^{-s\mu_k x} n_k\mu_k \dfrac{(n_k\mu_k x)^{m-n_k-1}}{(m-n_k-1)!} e^{-n_k\mu_k x} dx, \\ m > n_k > s \ge 0. \end{cases}$$

- Probabilities $\tilde{p}^{(j)}_m(l)$ - probability to accept $m$ queries from batch of $j$-th type at presence of empty seats depending on an admission discipline. It is defined using

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following equalities.

In case of partial admission

$$\tilde{p}_m^{(k)}(s) = P\{\min(m, v_k) = s\} = \\ = \begin{cases} p^{(k)}(s), & s = 1, 2, ..., m-1, \ m > 0; \\ \sum_{l=m}^{\infty} p^{(k)}(l), & s = m > 0; \\ 1, & s = m = 0. \end{cases} \quad (13)$$

In case of complete admission

$$\tilde{p}_m^{(k)}(s) = \begin{cases} p^{(k)}(s), & s > 0, \ m > 0; \\ 1, & s = m = 0. \end{cases} \quad (14)$$

In case of complete rejection

$$\tilde{p}_m^{(k)}(s) = \begin{cases} p^{(k)}(s), & 0 < s \le m; \\ \sum_{l=m+1}^{\infty} p^{(k)}(l), & s = 0, \ m > 0; \\ 1, & s = m = 0. \end{cases} \quad (15)$$

- $F_{i,j}(x)$ - probability of that the following batch of customers will arrive in system before the moment $x$, provided that it is a batch of $j$-th type and the previous batch was a batch of $i$-th type.

The Semi-Markov kernel then

$$Q_{(i,\vec{m}),(j,\vec{l})}^{(k)}(t) = \int_{U_{(i,\vec{m})}} Q_{(i,\vec{m}),(j,\vec{l})}^{(k)}(t,u) G_{(i,\vec{m})}(du) =$$

$$= p_{(i,\vec{m}),j} \int_0^t \sum_{s=0}^{m_j} p_{m_j,s}^{(j)}(x) \tilde{p}_{N_j+n_j-s}^{(j)}(l_j - s) \prod_{v \ne j} p_{m_v,l_v}^{(v)}(x) dF_{i,j}(x).$$

At $t \to \infty$ a matrix of transition probabilities of embedded Markov chain

$$p_{(i,\vec{m}),(j,\vec{l})}^{(k)} = \lim_{t \to \infty} Q_{(i,\vec{m}),(j,\vec{l})}^{(k)}(t) =$$

$$= \int_{U_{(i,\vec{m})}} Q_{(i,\vec{m}),(j,\vec{l})}^{(k)}(\infty,u) G_{(i,\vec{m})}(du) =$$

$$= p_{(i,\vec{m}),j} \int_0^\infty \sum_{s=0}^{m_j} p_{m_j,s}^{(j)}(x) \tilde{p}_{N_j+n_j-s}^{(j)}(l_j - s) \prod_{v \ne j} p_{m_v,l_v}^{(v)}(x) dF_{i,j}(x).$$

### D. *Income functional*

Further it is necessary to define functions $R_{(i,\vec{m})(j,\vec{l})}^{(k)}(x,u)$ - a conditional mathematical expectation of the saved up income in System *(k)* provided that process stays in state $(i,\vec{m})$ , through time $t$ it will pass in state $(j,\vec{l})$ and the decision $u$ is made.

The conditional mathematical expectation of the saved up income $R_{(i,\vec{m})(j,\vec{l})}^{(k)}(x,u)$ depends on the incomes and the charges of system work. Enter the constants describing these incomes and charges:

$c_1^{(k)}$ ·- the income received for service of one customer;

$c_2^{(k)}$ - a payment for a time unit during the working of one device during service;

$c_3^{(k)}$ ·- a payment for a time unit of idle time of one device;

$c_4^{(k)}$ - a payment for a time unit for staying in queue for one query;

$c_5^{(k)}$ - a payment for one lost query of $k$-th type.

Then

$$R_{(i,\vec{m})(j,\vec{l})}^{(k)}(x,u) = c_1^{(k)} C_1(x, m_k, l_k, u) + \\ + \sum_{s=2}^{4} C_s(x, n_k, m_k, l_k, u) + c_5^{(k)} C_5(x, m_k, l_k, u), \quad (16)$$

where

$$C_1(x, m_k, l_k, u) = M(\zeta_k / v^{(k)}(0) = m_k, \xi^{(k)}(x) = l_k, u(0) = u),$$

$$C_2(x, n_k, m_k, l_k, u) =$$

$$= c_2^{(k)} M[\int_0^x \min(n_k, v^{(k)}(t)) dt / v^{(k)}(0) = m_k, \xi^{(k)}(x) = l_k, u(0) = u],$$

$$C_3(x, n_k, m_k, l_k, u) =$$

$$= c_3^{(k)} M[\int_0^x (n_k - \min(n_k, v^{(k)}(t)) dt / v^{(k)}(0) = m_k, \xi^{(k)}(x) = l_k, u(0) = u],$$

$$C_4(x, n_k, m_k, l_k, u) =$$

$$= c_4^{(k)} M[\int_0^x \max(0, v^{(k)}(t) - n_k) dt / v^{(k)}(0) = m_k, \xi^{(k)}(x) = l_k, u(0) = u],$$

$$C_5(x, m_k, l_k, u) =$$

$$= M(\eta / \xi^{(k)}(0) = v^{(k)}(0) = m_k, \xi^{(k)}(x) = l_k, u(0) = u).$$

Notice, that $\xi^{(k)}(x)$ - the number of customers at the Markov moment $x$, $v^{(k)}(t)$ - Markov process of destruction on the period $t \in (0, x)$.

Designate $S_{(j,\vec{l})}^{(k)}(t)$ - the mathematical expectation of the saved up income in subsystem $k$ during time $t > 0$,


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provided that the process starts from the state $(j,\vec{l})$.

For the functional $S^{(k)}_{(j,\vec{l})}(t)$ the following equality is fair [3]

$$S^{(k)} = \frac{\sum\limits_{(i,\vec{l})\in\tilde{E}} s^{(k)}_{(i,\vec{l})}\pi^{(k)}_{(i,\vec{l})}}{\sum\limits_{(i,\vec{l})\in\tilde{E}} m^{(k)}_{(i,\vec{l})}\pi^{(k)}_{(i,\vec{l})}}, \quad (17)$$

where

$$m^{(k)}_{(i,\vec{m})} = \int\limits_0^\infty [1 - \sum_{(j,\vec{l})\in\tilde{E}} Q^{(k)}_{(i,\vec{m})(j,\vec{l})}(t)]dt \quad (18)$$

mathematical expectation for the time of continuous staying of the process $\xi(t)$ in state $(i,\vec{m})$;

$$s^{(k)}_{(i,\vec{m})} =$$

$$= \int\limits_{u\in U_{(i,\vec{m})}} \sum_{(j,\vec{l})} [\int\limits_o^\infty R^{(k)}_{(i,\vec{m})(j,\vec{l})}(x,u)dQ^{(k)}_{(i,\vec{m})(j,\vec{l})}(x,u)]G_{(i,\vec{m})}(du) \quad (19)$$

mathematical expectation of the saved up income during the system *(k)* working during continuous staying of the process $\xi(t)$ in state $(i,\vec{m})$.

*THEOREM. The income functional* $S = \sum\limits_{k=1}^{N} S^{(k)}$ *for System as a whole is a fractional-linear functional concerning the distributions* $\vec{G} = \{G_{(i,\vec{m})}(u),\ (i,\vec{m})\in\tilde{E}\}$ *defining the Markov homogeneous strategy.*

The final stage of the research is a construction of the optimum control strategy. For solving of the problem we use the known fact [1]: if a fractional-linear functional has an extremum (a maximum or a minimum), this extremum is reached in a class of the determined strategy (fixed determined probability measure $P(\zeta < x) = \begin{cases} 0, & x \le c, \\ 1, & x > c. \end{cases}$

Describe the set of the determined strategies.

To control of the customer type strategy is defined by equality (9). Fixed determined measure in state $(i,\vec{l})$ is defined by equality

$$\begin{aligned} G_{(i,\vec{l})}(j) &= P\{u(t) = j \,/\, \xi(t) = (i,\vec{l})\} = p_{(i,\vec{l}),j} = 1, \\ G_{(i,\vec{l})}(n) &= P\{u(t) = n \,/\, \xi(t) = (i,\vec{l})\} = p_{(i,\vec{l}),n} = 0. \end{aligned} \quad (20)$$

Thus, we receive the algorithm for searching of optimum strategy:

· For the fixed strategy (20) the matrix of transition probabilities (3) is calculated;

· For this matrix the system of the algebraic equations is solved and the stationary distribution of embedded Markov chain at chosen fixed strategy is determined;

· At chosen fixed strategy characteristics (18) and (19) are calculated;

· The income functional (17) is calculated for the chosen strategy;

· Using all fixed strategies and the values of the income functional for these strategies, we define the maximal income and optimum strategy.

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