Comparative Study of Oversampling the Lossy Acoustic Signal Using Estimate Wavelet Transform and Cubic Spline Interpolation

Archit Yajnik

Abstract— **This paper exhibits the necessary condition of oversampling technique using Estimate Wavelet Transform [EWT] introduced in [2] and demonstrates the comparative study of EWT with Cubic spline interpolation. Both the techniques are applied to approximate the loss of the discontinuous (lossy) single channel audio file. It is observed that estimate wavelet transform using Daubechies D4 wavelet gives a better approximation of the lossy samples than cubic spline. However, the approximated values differ from the original sample values in the case of EWT while it is not the case in cubic spline.**

***Index Terms*— Discrete Wavelet Transform (DWT), Multiresolution Analysis, Estimate Wavelet Transform (EWT), Cubic spline, Oversampling**

I. INTRODUCTION

FOURIER analysis, being the most prevalent method of multiresolution analysis, has however a fundamental drawback: we lack the information when the event in frequency domain exactly happened. Due to its invariant feature of time and frequency localization, wavelet theory is applicable in various fields like data compression, the solution of integral and differential equations etc. Wavelet-based methods have been proved to yield efficient and fast algorithms. The discrete wavelet transform (DWT) is a powerful technique for audio coding because the DWT coefficients provide a compact, non-redundant representation of the signal [11] and [12].

Discrete Wavelet Transform and Multi Resolution Analysis using Wavelet Filters are now very widely used in the areas of feature extraction [3], [4], [5] and [6]. This article presents a typical application of oversampling a digital signal to double its original signal using wavelets as introduced in [1, 2]. The Estimate Wavelet Transform introduced in [2] takes Discrete finite length signals with finitely many discontinuities and yields double length approximation and hence fills the gap generated by discontinuities. The present article is organized as follows. After introducing oversampling technique and discrete wavelet transform in the first section, we present the necessary condition of existence of higher resolution presented in [1] in second section. In section three, a real life application of approximating the missing samples from the single channel acoustic (.wav) signal using EWT is described. The comparison of results obtained by the cubic spline interpolation technique and estimate wavelet transform is shown in section four followed by summary in section five.

II. NECESSARY CONDITION OF EXISTENCE OF HIGHER RESOLUTION OBTAINED IN [1]

Reconstruction of a discrete signal [13] which demonstrates the analysis phase (discrete wavelet transform) and synthesis phase (inverse discrete wavelet transform) for a given function $f \in L^2(R)$. Consider the resolution level of the function is (j+1), so we denote this function by y_{j+1}. For each $j \in z$, define sequences $x_j = (x_j(k))_{k\in z}$ and $y_j = (y_j(k))_{k\in z}$ by $x_j = D(y_{j+1} * \tilde{v})$ and $y_j = D(y_{j+1} * \tilde{u})$. Where D is the downsampling operator on $l^2(z)$. $u = (u_j(k))_{k\in z}$ and $v = (v_j(k))_{k\in z}$ are scaling and wavelet sequence respectively. And $\tilde{u}$ and $\tilde{v}$ are the dual sequences of u (approximation coefficients) and v (detailed coefficients) defined as $\tilde{u}(n) = u(N-n)$ and $\tilde{v}(n) = v(N-n)$ where N is the length of the signal.

The reconstruction of y_{j+1} using one analysis phase and one synthesis phase can be given by

$$y_{j+1} = U(y_j) * u + U(x_j) * v$$

Where U is the upsampling operator on $l^2(z)$.

Extending this result by adding one more level of synthesis phase for the purpose of getting approximated double length signal is shown in fig 1.

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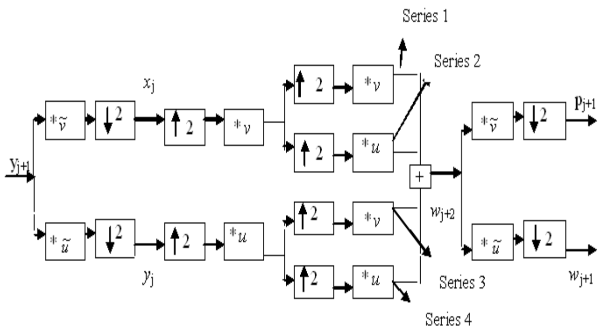

Fig. 1. Block diagram of Oversampling Technique using DWT

$$w_{j+2} = U(U(y_j)*u)*u + U(U(x_j)*v)*u + U(U(y_j)*u)*v + U(U(x_j)*v)*v$$

The output w_j+2 contains four parts viz. series 1, series 2, series 3 and series 4. Series 1 is made up of one dual High pass components and two High pass components so we call it as HHH. Similarly series 2, series 3 and series 4 are known as HHL, LLH and LLL respectively. These computations are implemented in Java.

The necessary condition for the w_j+2 be the higher resolution y_j+2 of y_j+1 can be derived as shown in the following Lemma.

Lemma Let $\{V_j\}_{j\in Z}$ be a multiresolution analysis. Suppose $f \in L^2(\Re)$ is a signal and for each $j \in Z$ (the set of integers), sequences $y_{j+1} = (y_{j+1}(k))_{k\in z}$ and $y_{j+2} = (y_{j+2}(k))_{k\in z}$ form (j+1)th and (j+2)th level resolutions of the signal f respectively.

Let $x_j = D(y_{j+1} * \tilde{v}),\ y_j = D(y_{j+1} * \tilde{u})$, then higher resolution of f can be presented by

$$U(U(y_j)*u)*u + U(U(x_j)*v)*u + U(U(y_j)*u)*v + U(U(x_j)*v)*v = y_{j+2}$$

(1)

Moreover, the necessary condition for $y_{j+2} = (y_{j+2}(k))_{k\in z}$ be the higher resolution of $y_{j+1} = (y_{j+1}(k))_{k\in z}$ is

$$x_{j+1} = y_{j+1}.$$

U is an upsampling operator and * denotes convolution

Proof:

We know that

$$x_j = D(y_{j+1} * \tilde{v}) = \langle f, \psi_{j,k} \rangle$$

where $\psi_{j,k} = 2^{j/2}\psi(2^j - k)$

$$= \sum_{m\in Z} v(m-2k)\varphi_{j+1,m}$$

$$y_j = D(y_{j+1} * \tilde{u}) = \langle f, \varphi_{j,k} \rangle$$

where $\varphi_{j,k} = 2^{j/2}\varphi(2^j - k)$

$$y_j = \sum_{m\in Z} u(m-2k)\varphi_{j+1,m} \text{ and}$$

$$y_{j+1}(m) = \langle f, \varphi_{j+1,m} \rangle = \sum_{k\in Z} \overline{\tilde{u}(2k-m)} \langle f, \varphi_{j,k} \rangle + \sum_{k\in Z} \overline{\tilde{v}(2k-m)} \langle f, \psi_{j,k} \rangle$$
$$= \sum_{k\in Z} u(m-2k) y_j(k) + \sum_{k\in Z} v(m-2k) x_j(k) \qquad (2)$$

where $\varphi_{j+1,m} = \sum_{k\in Z} \tilde{u}(2k-m)\varphi_{j,k} + \sum_{k\in Z} \tilde{v}(2k-m)\psi_{j,k}$

Now,

$$U(U(x_j)*v)*u = U\left[\sum_{k\in Z} v(m-2k) U(x_j)(2k) * u\right]$$

$$= U\left[\sum_{k\in Z} v(m-2k) x_j(k) * u\right]$$

$$= \sum_{l\in Z} u(m-2l)\left\{U\left(\sum_{k\in Z} v(m-2k) x_j(k)\right)(2l)\right\}$$

$$= \sum_{l\in Z} u(m-2l)\left\{\sum_{k\in Z} v(m-2k) x_j(k)\right\}(l)$$

$$= \sum_{l\in Z} u(m-2l)\left\{\sum_{k\in Z} v(m-2k)\langle f, \psi_{j,k}\rangle\right\}(l)$$

(3)

Similarly,

$$U(U(x_j)*v)*v$$
$$= \sum_{l\in Z} v(m-2l)\left\{\sum_{k\in Z} v(m-2k)\langle f, \psi_{j,k}\rangle\right\}(l)$$

(4)

$$U(U(y_j)*u)*u$$
$$= \sum_{l\in Z} u(m-2l)\left\{\sum_{k\in Z} u(m-2k)\langle f, \varphi_{j,k}\rangle\right\}(l)$$

(5)

and

$$U(U(y_j)*u)*v$$
$$= \sum_{l\in Z} v(m-2l)\left\{\sum_{k\in Z} u(m-2k)\langle f, \varphi_{j,k}\rangle\right\}(l)$$

(6)

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Substituting the value of the components of equation (1) from (3), (4), (5) and (6), we obtain

$$U\big(U(y_j)*u\big)*u+U\big(U(x_j)*v\big)*u+$$
$$U\big(U(y_j)*u\big)*v+U\big(U(x_j)*v\big)*v$$
$$=\sum_{l\in Z}u(m-2l)\sum_{k\in Z}v(m-2k)\langle f,\psi_{j,k}\rangle(l)$$
$$+\sum_{l\in Z}u(m-2l)\sum_{k\in Z}u(m-2k)\langle f,\varphi_{j,k}\rangle(l)$$
$$\sum_{l\in Z}v(m-2l)$$
$$=\left\{\sum_{k\in Z}v(m-2k)\langle f,\psi_{j,k}\rangle+\sum_{k\in Z}u(m-2k)\langle f,\varphi_{j,k}\rangle\right\}(l)$$
$$=\sum_{l\in Z}u(m-2l)y_{j+1}(l)+\sum_{l\in Z}v(m-2l)y_{j+1}(l)$$
$$=\sum_{l\in Z}u(m-2l)\langle f,\varphi_{j+1,l}\rangle+\sum_{l\in Z}v(m-2l)\langle f,\varphi_{j+1,l}\rangle$$
$$=y_{j+2}\ \text{(from (2))}$$

only if $y_{j+1}=x_{j+1}$

III. Oversampling of an Acoustic Signal

This paper presents consequences of Estimate wavelet transform introduced in [2]. It is applied to approximate the distorted single channel audio file (.wav). A single channel audio file 'Sunday.wav' is sampled with the rate 8192 Hertz using "wavread" command of matlab. Out of total 73,113 samples, the first 50,000 samples are collected for the experiment. The range of the sampled data is [-0.8,0.8]. Randomly selected 50 samples are removed to generate discontinuity and remaining 49950 samples are as shown in figure 3 and figure 2 respectively. The discrete signal depicted in figure 2 is highly nonlinear in nature. Let y_{j+1} and o_{j+1} be the lossy signal of the size 49950 and omitted samples respectively.

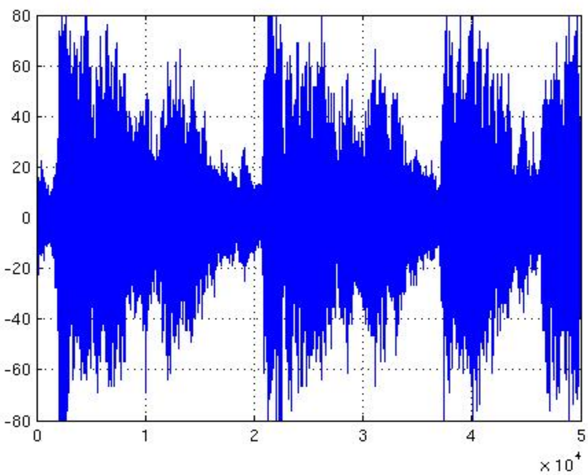

Fig.2. Original Lossy Acoustic signal (y_j+1)

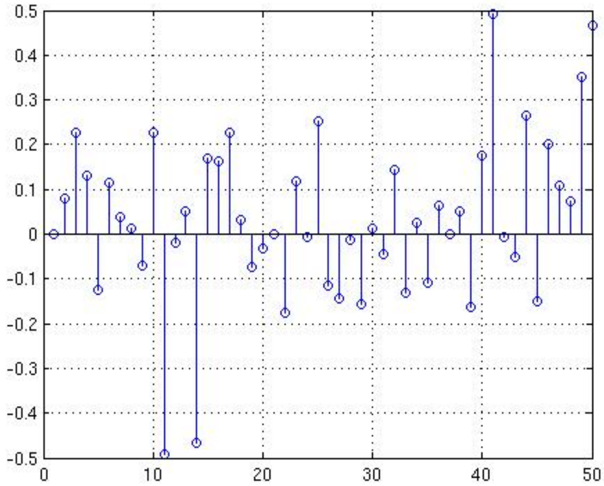

Fig. 3. Omitted original samples (o_j+1)

Using HHH and LLH components by taking algebraic addition of series 1 and series 3 of Figure 1, we get

$$r_{j+2}=\mu[U(U(x_j)*v)*v+U(U(y_j)*u)*v]$$
(7)

where x_j and y_j are detail and wavelet coefficients and μ $(1\le\mu\le\sqrt{2})$ is a scaling parameter.

The double length signal r_j+2 is obtained after applying equation (7) on the input signal x_j to approximate the discontinuities arose in the original signal. Here the scaling parameter μ is considered to be $\sqrt{2}$ .The length of the resultant signal r_{j+2} is 99900. In order to visualize the graph, the resultant signal is further divided into two parts by segregating even and odd samples which are of the length 49950 as shown in fig. 4 and 5. These figures captures the almost similar graphs to the graph of original lossy signal y_{j+1} in fig 2.

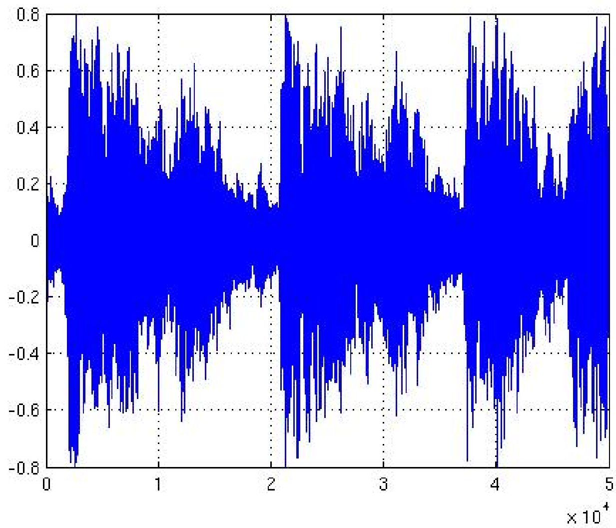

Fig. 4. Even samples of Approximated signal

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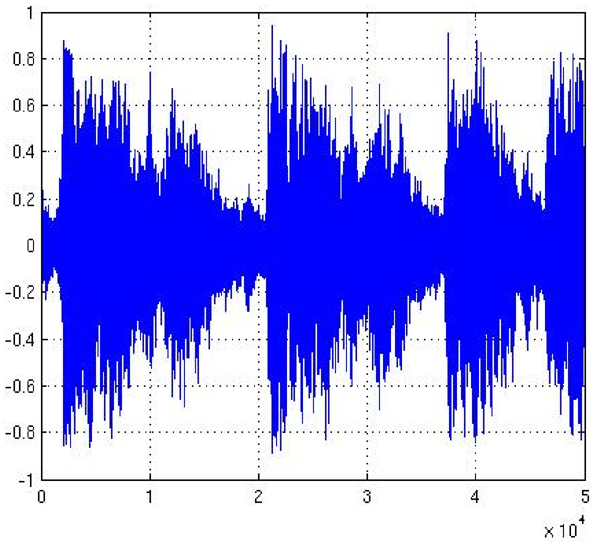

Fig. 5. Odd samples of Approximated signal

IV. Comparative Study of EWT and Cubic Spline

Now the question is of finding the approximating the discontinuities (fig. 3) arose in the original signal from r_j+2. Let n be the position of discontinuity in the original signal. The approximated value is expected to be available around 2n position of r_j+2. The values are chosen manually in the interval of [2n-1, 2n+1] in the either odd or even samples. Let a_j+1 be the approximated values which are depicted in the fig. 6 shown below

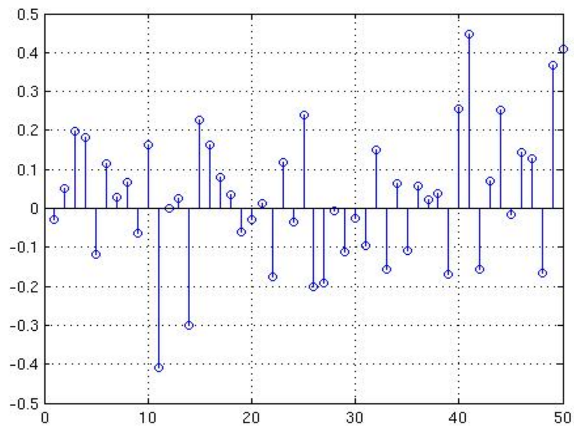

Fig. 6. Approximated samples using manual EWT (a_j+1)

In order to avoid manual identification of the approximated values, the concept of mean value is implemented i.e. suppose n is the position of discontinuity in the original signal. Let the mean of (n-1)th and (n+1)th items be α (say). Now finding the infimum absolute difference between α and the value lies at the position J = [2n-2, 2n+5] of the signal r_{j+2}. i.e.

$$\sigma = \inf_i |\alpha - \beta_i|$$

Where β_i is the sample value of r_{j+2} placed at i^{th} position. Let b_{j+1} be the approximated values obtained by mean method which are depicted in the fig. 7 shown below

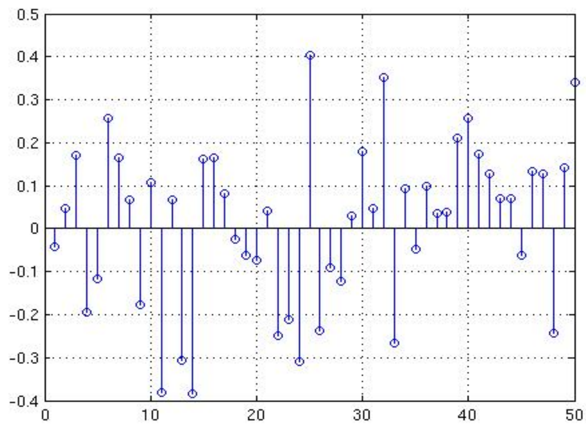

Fig. 7. Approximated samples using mean value EWT

TABLE I
Sum Squared Error

Sr. No	Techniques	Sum Squared Error
1	SSE of o_{j+1} and a_{j+1} (Manual EWT)	0.222158
2	SSE of o_{j+1} and b_{j+1} (Mean Value EWT)	1.253229
3	SSE of o_{j+1} and c_{j+1} (Cubic Spline)	4.160614

(b_j+1)]

The result is compared with cubic spline interpolation technique. Let c_{j+1} be the approximated values using cubic spline method which are depicted in the figure 8 shown below

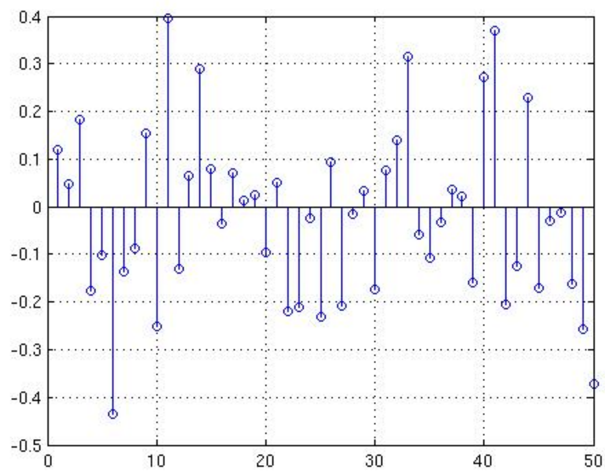

Fig. 8. Approximated samples using Cubic Spline (c_j+1)

Table 1 gives the error analysis of the all the three techniques viz. manual EWT (fig. 6), Mean value EWT (fig. 7) and Cubic Spline (fig. 8).

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V. Conclusion

The lemma presented in section 2 shows the necessary condition is $y_{j+1} = x_{j+1}$ of obtaining higher resolution w_{j+2} of the original signal y_{j+1} at level j+1 of multiresolution analysis of Discrete Wavelet Transform.

This paper demonstrates the technique of approximating finite number of discontinuities arise in the acoustic single channel .wav file using Estimate Wavelet Transform (algebraic sum of HHH and LLH of figure 1)). Odd and even samples of r_{j+2} captures significant information of the original signal as shown in figures 4 and 5. The discontinuous samples (o_{j+1}) are approximated using three methods viz. 1. Manual EWT (fig. 6) 2. Mean value EWT (fig. 7) 3. Cubic spline interpolation (fig. 8). It is observed from the Table 1 that even thought the sum squared error using manual EWT method is less as compared to other two techniques, the Mean value EWT method is more feasible than the manual one.

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