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A New Kind of Complexity

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***Abstract*—A new class of functions is presented. The structure of the algorithm, particularly the selection criteria (branching), is used to define the fundamental property of the new class. The most interesting property of the new functions is that instances are easy to compute but if input to the function is vague the description of a function is exponentially complex. This property puts a new light on randomness especially on the random oracle model with a couple of practical examples of random oracle implementation. Consequently, there is a new interesting viewpoint on computational complexity in general.**

I. Introduction

THE structured program theorem, also known as Böhm-Jacopini theorem is one of the premises for this paper. The theorem shows an ability of the algorithm or a program implementing that algorithm to compute any computable function combining only the three algorithmic structures. These structures are:

- The tasks in the program are done one after another (sequential order of execution)
- The program can branch to a different path of execution depending on some statement evaluation result (selection criteria)
- Repeating some task until an evaluation of some statement is satisfied (iteration)

The branching or selection criteria structure and its usage defines the new proposed class of functions. It is assumed that structured program theorem holds and that usage of the three structures is Turing complete. More precisely, it is assumed that further reduction from three structures to two structures (excluding branching) is impossible. In other words the selection criteria cannot be effectively replaced with a combination of the other two structures.

The second premise is the analysis of the branching structure in software metrics done by McCabe [1]. The main result of McCabe's work is the notion of Cyclomatic Complexity (CC). The flow chart of the above mentioned structures is used to count the individual execution paths that the program can take. The CC is mainly used as software testing metric. It evaluates a requirement of how many testing cases are needed for a piece of software. In the majority of cases the relation between branching and individual paths is exponential, meaning that if the number of branching in a program increases then the number of individual paths which the program can execute doubles for every added branching.

The combination of the two above notions can lead to an extraordinary case. The program can be written with a non restricted number of branching (n) implying an exponential growth of the number of paths that the program can take through execution (2^n). Additionally, the theory of algorithms demands that the formal description of an algorithm shall include every possible case it can take through execution.

> "For some such computational process, the algorithm must be rigorously defined: specified in the way it applies in all possible circumstances that could arise. *That is, any conditional steps must be systematically dealt with, case-by-case; the criteria for each case must be clear (and computable)* [emphasis added]." [2]

That means that an algorithm with a high CC cannot be practically described because the number of execution paths increases exponentially. On the other hand, the instances of such an algorithm can be easily computed because the increase of the number of branching in the program incurs only polynomial cost. This extraordinary case needs more thorough clarifications:

- It is not clear what the irreducible number of branching means. There is still a possibility that individual execution paths of an algorithm are actually identical transformations.
- Although CC shows exponential dependency between the number of branching and the number of execution paths, a possibility remains that the relationship between the selection criteria and the paths doubling numbers can be reduced to an acceptable level. Indeed, there is a suggestion to avoid high CC software metric: the first is to rewrite a program in question with reduced use of branching, and the second is to split the program in more manageable pieces [4].

These concerns and other relevant discussions are explored through 3n+1 problem (section II) and Wolfram's rule 30 (section III). Section IV summarises the new function features and speculates on the impacts on randomness and P/NP classes.

II. Programming take on 3n+1 problem

A. 3n+1 problem

The 3n+1 problem is ideal for exploring the relationship between the algorithm's selection criteria and the CC. One reason for this is that the 3n+1 problem is extensively studied and a lot of details about the problem are well established. Another reason is that the selection criteria are an essential part of the problem description. The problem is very simple to state: take any positive integer, if the integer is even divide that integer with 2, if the integer is odd multiply the integer by 3 and add 1. Repeat the procedure until the result is 1. The problem is to decide if all positive integers reach 1 [3]. It is possible to skip the evaluation if the integer is even after $3n+1$ operation and proceed with the operation $n/2$ because 3 times odd integer plus one is even. That is also an example of how CC of an algorithm can be reduced. The optimised version will be used throughout the rest of the paper.

The 3n+1 problem flow chart is shown in figure 1.

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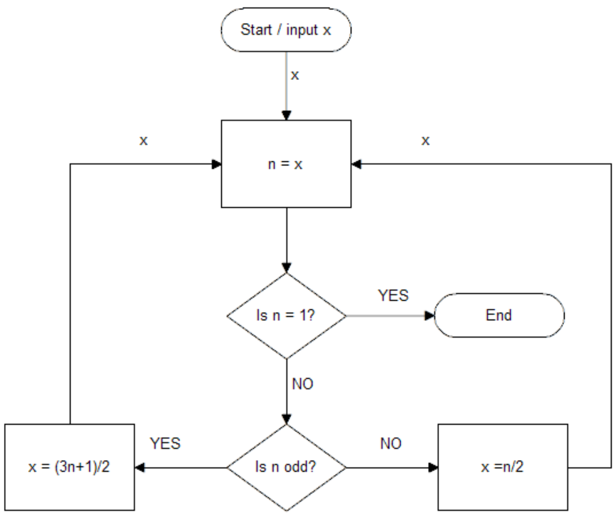

Fig. 1. 3n+1 flow chart with optimised (3n+1)/2 step

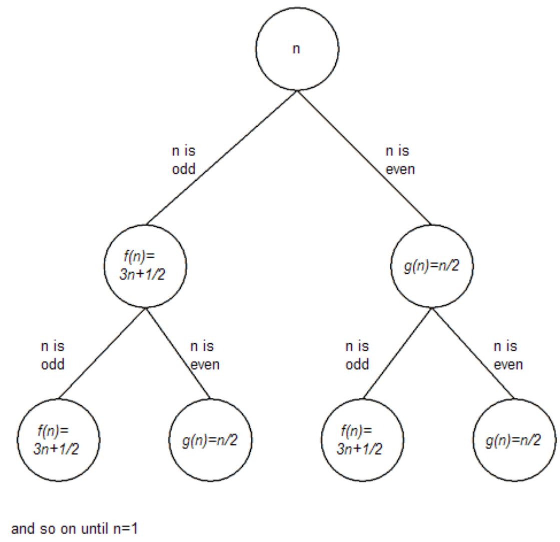

Fig. 2. Cyclomatic Complexity; two steps in 3n+1 algorithm (binary tree resemblance)

B. 3n+1 and Cyclomatic Complexity

In this section the CC of 3n+1 algorithm is explored. As mentioned in section I CC is software metric. That software metric measures how many paths the program can take through the execution. It uses graph theory to count individual execution paths. The formula for CC is as follows: $\nu(G) = e - n + 2$ where e and n are amounts of edges and nodes contained in the algorithm flow graph. ν is cyclomatic number and G means that the complexity is the function of algorithm flow graph [4]. Applying this formula to a particular algorithm is not a straight forward exercise. One of simpler ways is counting the binary decision predicates p . The formula for this approach is $\nu(G) = p + 1$. The figure 2 shows the 3n+1 algorithm doing two steps. Using a simpler method the three binary branching can be identified, therefore $\nu(G) = 3 + 1$ and indeed there are four individual paths the 3n+1 algorithm can take in two steps. It is evident from figure 2 that every 3n+1 step will double CC. That means after doing several 3x+1 steps the system starts to be very complex from the software testing perspective.

C. 3n+1 Preliminaries

A few details about 3n+1 problem are well known and mentioned here [3], some are listed for further discussion:

- The experimental data confirms that numbers up to $\approx 2^{60}$are reaching one[5].
- The lower bound of how many natural numbers reach one is shown by [6]. For any sequence of natural numbers in the interval $[1.x]$ the number of naturals reaching one in corresponding interval is $> x^{0.84}$.
- A parity sequence for each natural number as an input is unique and that is true even if not all natural numbers are reaching one. The parity sequence is formed by putting 1 or 0 in the sequence, depending on what operation is performed in the particular step. In other words if the branching in figure 1 “is n odd?” is NO put 0 in the parity sequence and if it is YES put 1.

D. 3n+1 as composite function

The composite function nature of 3n+1 problem comes from the parity sequence. For example if $n = 13$ the parity sequence for that input will be $1, 0, 0, 1, 0, 0, 0$. In the same manner a composite function $h(n)$ can be composed, for example $h(13) = f \circ g \circ g \circ f \circ g \circ g \circ g = 1$ where $f(n) = (3n + 1)/2$ and $g(n) = n/2$. It is obvious that the parity sequence pattern and the composite function pattern are identical. That should be expected because the parity sequence is the description of how a natural number is transformed under the 3n+1 rule.

If the natural numbers and their corresponding parity sequences have the bijective relationship (and that appears to be true even the number 1 is not periodicity revolving point [3]), then the natural numbers and their corresponding composite function are bijective as well. That is based on the fact that the pattern of the parity sequence is identical to the pattern of the composite function for the same natural number. *Therefore every natural number has a unique composite function $h(n)$ to map a natural number to a number where a period occurs under rule* 3n+1.

E. 3n+1 as encoding system

A parity sequence or $h(n)$ pattern can be considered as binary encoding for every natural number reaching 1 under 3n+1 rule. For example for the number 13 its binary representation is 1101. In 3n+1 binary world encoding for 13 is FG string $fggfggg$. The decoding is done by applying the rule 3n+1 backwards without the need for evaluating “if odd or even”. Just start from one and read the FG string backwards. If the character is g apply function $g(n) = 2n$ and go to next character. If the character is f apply function $f(n) = (2n - 1)/3$ and go to the next character. When all characters are read the final number is the decoded number. From traditional binary and parity encodings two languages can be defined:

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1) The binary language $L_{0,1}$ is written by $\{0, 1\}$ alphabet. The members of $L_{0,1}$ set are the binary encodings of all natural numbers reaching 1 under rule 3n+1 (n'). The ratio size of the set is $\geq n^{0.84}$ [6].
2) The parity language $L_{f,g}$ is written by $\{f, g\}$ alphabet. The members of $L_{f,g}$ set are the parity sequences of n'. The size of $L_{f,g}$ is the same as the size of $L_{0,1}$.

Although the amounts of the words forming languages $L_{0,1}$ and $L_{f,g}$ are equal the words explaining the same object differ in length between languages. For example encodings for number 13 are:

1) In $L_{0,1}$language the encoding is 1101; size of the word is 4.
2) In $L_{f,g}$language the encoding is $fggfggg$; size of the word is 7.

It is obvious that encodings from language $L_{f,g}$ can be compressed, but compression can not go below complexity of the language $L_{0,1}$. This means that *the complexity of language $L_{f,g}$ (3n+1 encoding) is greater or equal to complexity of language $L_{0,1}$ (optimal binary encoding).*

A significant implication is that the 3n+1 function description depends on input in an unusual way. The input is not just an ordinary variable but it is determinant of how a particular transformation (from input towards one) is composed. If the if else structure is used in 3n+1 rule then the composite function description for all natural numbers has at least sub exponential growth of $n^{0.84}$.

F. 3n+1 as a random function?

Random oracle is an abstraction used to model security protocols and schemes. Basically, random oracle is an imaginary machine which upon an input to oracle, randomly draws a function from a set of all functions possible and with that function an output is calculated and returned. A simple model can be used as an example: On input 0 flip fair coin and record the resulting tail/head occurrences as a truly random binary string; continue with same procedure for inputs $1, 00, 01, 10, 11, 000 \ldots$ (see table I).

TABLE I
MAPPING USING RANDOM FUNCTION

binary input	corresponding string
0	truly random string 1
1	truly random string 2
00	truly random string 3
01	truly random string 4
...	...

The table I is then used in proving various security systems (see [7] for details). It is apparent that the table I is not practical by means of storage and access to intended entry. In practice random oracle is replaced with cryptographically secure hash with undefined security consequences. The work of [8] argues that random oracle modelling is essentially unsound; a practical implementation of replacing a random oracle in proven secure scheme results in an insecure scheme.

An interesting property defined in [8] is a notion of correlation intractability. The correlation intractability is the resistance to put some relation between inputs and outputs on some mapping. It is easy to see that random oracle is resistant to correlation (table I) because of flipping fair coin. For potential replacement, and that is single functions or function assemblies, correlation intractability property can not be guaranteed. The reasoning behind is that mapping description is shorter than allowed input description used by adversary, therefore the correlation between input and output must exist and that can not be expected from efficient and fully described function or function assembly to behave randomly [8].

Despite that 3n+1 shall apply for random oracle replacement. One line of argument can go along the fact that 3n+1 is perceived as a hard problem [10]p4 and p17.

One obvious advantage of replacing table I with table II is that entries in 3n+1 parity table can be produced *deterministically*. Finding any pattern or structure in table II may open a way to attack the 3n+1 problem. A similar argument is made with hardness of integer factoring and consequent factoring use in asymmetric encryption.

The second line of replacing random oracle with parity sequences is complexity of the 3n+1 in terms of CC and composite function model. If 3n+1 is considered as composite function, the form without specifying input looks like formula 1 where $f\clubsuit g$ means depending on input use function f or g. That can not be considered as *a fully described function*. Only with an input the formula can make sense (and can be executed).

$$(f\clubsuit g) \circ (f\clubsuit g) \circ (f\clubsuit g) \ldots \tag{1}$$

The argumentation can also go along the line input and function description equality. As is shown in subsection II-E input language and composite function (parity) language for 3n+1 are of equal complexity. The configuration where input description and function description are of the same length, is actually listed as a possible case where random function can be replaced (see restricted correlation intractability section [8]). Although that case is considered as inefficient, as is table I for example (function description is actually input/output description). However table II is practical because entries can be calculated as is needed (full knowledge of all mappings are not necessary).

Here is how the 3n+1 implementation of the hash function (random function replacement) may look: Let the input n be a word with at least 256 bits in length. Treat n as an unsigned integer. Process n by the algorithm figure1. Form the binary sequence (parity) by recording 1 when "yes" and 0 when "no" is answered to the question "is n odd?". Stop when parity is 128 bit long. The game is to find n' in the way to produce identical first 128 bits in parity sequence as n does (a collision). The search for collision is needed for a specific input, because the powers of two $(32, 64, 128 \ldots)$ inputs will produce parities of zeros (collisions are trivial, see entry 8 in table II for example). Because there is only formula 1 and target parity for someone who wants to find the match for that parity, the task is impossible excluding exhaustive search.

G. 3n+1 and reductions

It was tried before to show that the 3n+1 problem is intractable. One example is here [12]. The main argument of that work goes on showing that the 3n+1 solution has to

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TABLE II
3N+1 PARITY MAPPING

n	corresponding parity
...	...
7	111010001000
8	000
9	10111010001000
...	...

be infinitely complex, using Solomon-Kolgomorov-Chaitin (SKC) complexity as an argument [13]. It relies on the fact that every 3n+1 transformation is unique and if we were to represent all of them, the only remaining option would be to list them all and consequently that option is obviously infeasible. It is similar reasoning to the one from section II-E. The problem with either reasoning is the possibility that 3n+1 inquiries might be calculated by some algorithm other than algorithm shown in figure 1 and furthermore that the other algorithm can be fundamentally different. It is impossible to know how that algorithm may look anyhow a couple of important properties can be defined:

- *low CC*; Only algorithms without using branching structure can be considered as candidates.
- *efficiency*; There are algorithms with low CC see figure 3 for example. The execution time of that algorithm depends on the oracle proposing the fg string (as shown in subsection II-E). One option for getting the answer from that algorithm is that the oracle goes through an exhaustive search to match fg string with output 1 (if 3n+1 conjecture holds). To be efficient it is required from the candidate algorithm to produce a matching fg string by evaluating input n in P time.

If both above requirements are met by the candidate algorithm, then the apparent CC of 3n+1 can be reduced in P time. The algorithm on presented input n can predict the branching fg string without using branching structure. Consequently selection programming structure can be replaced by a combination of sequence and iteration without significant cost (P time). In that case the structural programming theorem [11] needs revision.

III. About Wolfram's rule 30

A. Rule 30 complexity

The Rule 30 is probably one of the most represented rules in the Wolfram's NKS book [17]. The definitions of rule 30 are listed below:

- Boolean form is [15] $p\ Xor\ (q\ Or\ r)$
- English description [16] :

  "Look at each cell and its right-hand neighbor. If both of these where white on the previous step, then take the new color of the cell to be whatever the previous color of its left-hand neighbor was. Otherwise, take the new color to be opposite of that".
- Visual description and example is shown in figure 4.

The main features of the Rule 30 are chaotic behaviour and randomness. Both features are accomplished by an apparently simple rule and with an input with only one black cell - see figure 4 and NKS book [17] pages 27-28.

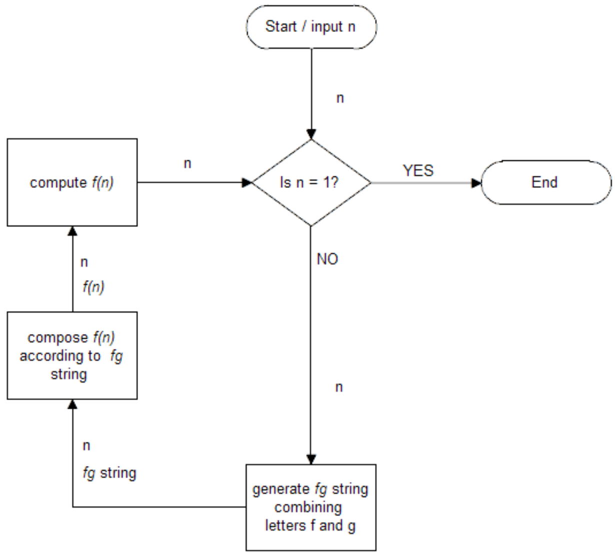

Fig. 3. 3n+1 flow chart with Do-While structure (looping)

That observation is mentioned numerous times and is not entirely correct on both accounts (simple rule, one black cell as input). Let us use figure 4 for example.

- The first row shows the input and it is *43 bits long* with 42 white cells and one black. Instead talking of only one black cell input, emphasis should be on low entropy of that input. Also it should be explained how entropy of the input is relevant to the rule 30 process, because the configuration with one black cell has the same probability of occurring as any other configuration.
- The English description of the rule already mentioned is actually the clue to chaotic / random behaviour. The description is as follows: if something is true do that else do something different. It is exactly the same structure already seen in the 3n+1 problem. Considering that, rule 30 can be considered as composite function in the same fashion as 3n+1. The difference between 3n+1 and rule 30 is that the rule 30 update of cell depends on outputs of neighbouring cells as well. Therefore it is trickier to calculate CC of rule 30 algorithm. A short-cut to estimating CC is to assume one branching per row evolution. Since rule 30 (figure 4) is iterated 21 times, the amount of possible execution paths for one cell is 2^{21}. From the software testing perspective anything over 2^{10} is practically non testable [4].

Consequently, it is not correct to brand rule 30 as a simple program while at the same time it has an inherently high level of CC.

B. Rule 30 function description

Although the rule 30 algorithm is fairly simple, its function description is certainly complex. The reason for this is that a particular input and particular number of iterations actually define which composite function is going to be executed at the time. Unlike the 3n+1 case where input alone determines the number of iterations and consequently CC, the rule 30 CC

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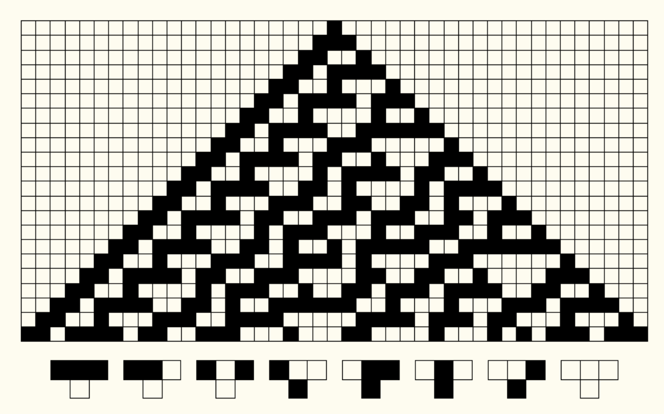

Fig. 4. rule 30, example of evolution and rules of transformation, as image from [14]

depends on input and the number of iterations. Quantifying rule 30 CC is shown below.

Let ln be the length of the input ($ln = 43$ figure 4) and li be the number of iterations ($li = 21$ figure 4). If $li \leq ln$ CC depends on li , and the number of paths $\nu(G)$ that the algorithm can take is $\nu(G) = 2^{li}$. In the case $li > ln$ the number of paths is $\nu(G) = 2^{ln}$. The reason for that is that the entropy of the number of paths is bounded by entropy of input.

This means that if input length is smaller than number of iteration, CC depends on input only as it is the case with 3n+1. Wolfram in his NKS [17] uses empirical methods to argue on some rule 30 attributes. For example empirical data shows that the period of rule 30 has an exponential growth in relation to input, which indicates that above assertion of exponential growth is true.

C. Rule 30 as hash function

Having the same algorithm structure as 3n+1, rule 30 is also a candidate for the hash function. There is a proposal which appeared on sci.crypt [18]

> "Let length of constant c be a desired length of a hash. Constant c can be arbitrary chosen. For example if 128 bits hash is required the constant c may easily be 128 zeros. The string s for hashing is then concatenated to the constant c to form a starting row r for rule 30; r = c + s. The row r is then evolved twice row length. For example if c = 128 bits in length and s = 128 bits in length then evolution is performed 512 times (column length is 512). Now the part (length of c) of last row serves as a hash. From above example the first 128 bits of 512th row is the hash h of the string s."

The two major points raised in the discussion are the efficiency of algorithm and the choice of c to be string of zeros. Even though the proposed hash is not practical (quadratic in nature) it is still in P. Constant c instead of zeros should employ some pseudo-random string such as π number sequence to avoid short cycle of rule 30.

IV. Conclusion

A. Summary

As is noted in the introduction, the whole discussion is about three notions:

1) Structured Programming Theorem; particularly treating branching as basic structure in programming.
2) Cyclomatic Complexity; exponential dependence between branching and number of execution paths a program can take. Basically every branching potentially doubles number of paths.
3) Formal description of an algorithm; requirement that every branching in algorithm shall be fully defined.

The first option is: one or more of above do not hold. The second option is: all above notions hold and there exists a program without any knowledge of output behaviour before input is presented. The discussion from this paper sees the second option as true. The main arguments for this are:

- The 3n+1 parity sequence can be used as encoding system, see subsection II-E. The argument is that 3n+1 encoding alias function description can not be simpler than standard binary encoding of an input.
- The 3n+1 algorithm description exponential growth can be reduced only if branching structure can be reduced to sequential and iteration programming structure in polynomial time. See subsection II-G.
- The random oracle framework provides the definition of correlation intractability and how that requirement can not be obtained by single function or function assembly (see subsection II-F). Contrary to that notion 3n+1 algorithm looks like: (select f or g) $\circ$ (select f or g) $\circ$ (select f or g) $\circ$ ... . It is apparent that function description without specific input is not present, and that the input actually defines function composition. Therefore there is the case when input description and function description have the same complexity. That case satisfies the correlation intractability requirement (subsection II-F).

Other arguments are various empirical findings, for example rule 30 is used as random number generator in Wolfram's Mathematica.

The common features of 3n+1 and rule 30 are:

- Composed from two distinctive functions f and g that are not commutative $f \circ g \neq g \circ f$.
- Cyclomatic Complexity raise with every branching step see subsection III-B
- Steps in program execution path are one of the function f or g
- Probability of executing f or g in next step is 0.5

B. Randomness and simple arithmetic?

As is discussed in subsection II-F the proposition is to exchange tables I and II without loosing any of the random oracle properties. The similarity between tables is: Both tables are impractical if used in tabular form. There is a storage problem (for example how to store 2^{128} entries and seek time costs). It is easy to see that random oracle table can not be compressed because the second column is by design true random. On the opposite side of tabular representation spectrum is binary encoding (table III). If input is given in the left column of the binary table it is easy to calculate corresponding entry in the right column and vice versa. This means that the tabular form is not needed (easily calculated/compressed) because it is easy to calculate entries both ways.

TABLE III
BINARY ENCODING

n	binary encoding
1	1
2	10
3	11
4	100
...	...

Now it is time to see how the hash function proposition from subsection II-F reflects on parity and binary encoding tables:

- Binary table is not affected if only part of the string in the right hand column is provided. For example, if the question is to find corresponding entries for binary string beginning with 10, just appending arbitrary suffix to 10 and decoding that string will find entry on the left hand side column..
- The parity table in the case when only a partial string in the right hand column is provided can not be calculated or compressed. The simple reason is that entries can be calculated only with complete input. Anything else faces the ambiguous prospect of $(f\clubsuit g) \circ (f\clubsuit g) \circ (f\clubsuit g)\dots$ where $f\clubsuit g$ means depending on input use function f or use function g.

The question is: can observation of entries in parity table (table II) provide any means of compressing that table? The answer is no, because the branching structure of algorithm prevents any type of Solomon-Kolgomorov-Chaitin (SKC) reductions, even though the table is deterministic in nature. In other words the data in the parity table ought to be random. Rule 30 sequences are in the same category. It is remarkable that randomness can be now interpreted as inability of reducing selection criteria programming structure. Translated to random oracle vocabulary that is notion of correlation intractability.

C. P and NP

The 3n+1 proposal for collision resistance (subsection II-F) can serve as P versus NP discussion as well. The game is to find input x (natural number) and with that x to produce the parity string p_x. Parity $p_x = s||a$ is the concatenation of given string s and arbitrary string a. Only one constraint is $l_x = 2l_s$, where l_x is the binary length of x and l_s is the binary length of the given string s.

For example the given string in C language notation is *char* $s = "DoesPequalsNp?"$ has the binary length $l_x = 14 * 8$. The task is to find natural number x with binary length $l_x = 2 * 14 * 8$ and with sequence *char* $p_x = "DoesPequalsNp?..."$.

First of all, nothing guaranties that any of the natural numbers 224 bit long will produce required parity sequence.

Secondly, because matching parity is not fully defined calculating x from s is impossible. The reasons are:

- To compose the transformation and do the calculation full knowledge of input is needed, because only input defines function composition.
- Trying to observe the mapping of natural numbers to corresponding parities and hoping to find some pattern/reduction is futile because the selection criteria programming structure can not be reduced.

TABLE IV
THE COMPLEXITY OF 3N+1 FUNCTION COMPOSITION GROWS EXPONENTIALLY; EXAMPLE OF FUNCTION 3 STEPS POSSIBLE OUTCOMES

$(f\clubsuit g) \circ (f\clubsuit g) \circ (f\clubsuit g)$
$f \circ f \circ f$
$f \circ f \circ g$
$f \circ g \circ f$
$f \circ g \circ g$
$g \circ f \circ f$
$g \circ f \circ g$
$g \circ g \circ f$
$g \circ g \circ g$

The game has all the main ingredients of $P \neq NP$. If complete input (either natural number or parity) is provided it is easy to do 3n+1 encoding (subsection II-E). If input is not fully defined the only technique for finding natural number/parity pair is exhaustive search. That exhaustive search is exponential in nature (table IV). On the other hand if match is found it is easy to verify that because complete input is now known. A full version of work, including the C code can be found here [19].

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