On the Modeling of Wireless Communication Networks

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***Abstract*— In the given paper, we're analyzing and modeling the reliability of complex wireless communication systems using the exponential distribution. We assume that the data is transmitted via unreliable environment. The transmitted message consists of n information blocks. The check of accuracy of getting information is made immediately after receiving the next block and the time spent on check is negligibly small, check is full. In the case of information distortion block transmission is repeated until it won't be correctly received. It takes place with a help of control bits. It is assumed that there is a time redundancy.**

***Index Terms*—exponential distribution, modeling, time redundancy, wireless communication networks.**

I. Introduction

CAN you predict the way a network will perform? Can you easily, exactly, correctly? Indeed, what metrics should you use to evaluate performance? These are the kinds of questions a network analyst asks himself all the time.

The fact is, there are a number of ways in which you can go about doing network system performance analysis.

In order of increasing ugliness, they are as follows:

1) Conduct a mathematical analysis which yields explicit performance expressions.
2) Conduct a mathematical analysis which yields an algorithmic or numerical evaluation procedure.
3) Write and run a simulation.
4) Build the system and then measure its performance [1]!

Over the years many continuous random variable probability distributions have been developed. One of the most widely used probability distributions in engineering, particularly in reliability is exponential distribution. It is relatively easy to handle in conducting analysis. The distribution probability density function (pdf) is defined by

$$f(t) = \lambda e^{-\lambda t}, t \geq 0,\ \lambda > 0. \qquad (1)$$

where λ is the distribution parameter.

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Cumulative distribution function is expressed by

$$F(t) = \int_{-\infty}^{t} f(y)\, d(y) \qquad (2)$$

where

t = time,

$F(t)$ = cumulative distribution function,

$f(y)$ = probability density function.

By substituting (2) into (1), we get the following expression for the exponential distribution cumulative distribution function [2]:

$$F(t) = \int_0^t \lambda e^{-\lambda y} dy = 1 - e^{-\lambda t} \qquad (3)$$

Most communication systems are sensitive to random errors and synchronization failures. The Physical Layer analysis of any Computer Communication network is very important. This is because many different problems concerning network execution and utilization are caused by errors and failures on the physical layer [3].

Managing performance of networks involves optimizing the way networks function in the effort to maximize capacity, minimize latency and offer high reliability regardless of bandwidth available and occurrence of failures. Network performance management consists of tasks like measuring, modeling, planning and optimizing networks to ensure that they carry traffic with the speed, capacity and reliability that is expected by the applications using the network or required in a particular scenario.

Networks are of different types and can be categorized based on several factors. However, the factors that affect the performance of the different networks are more or less the same [4].

II. Computational Complexity

Critical fault-tolerant systems frequently must use redundancy if they are required to meet extremely high reliability levels. Redundancy management (RM) is the process by which a redundant system selects among its redundant components so as to provide fault tolerance in the event a redundant component should fail. RM consists of the following tasks: detection (recognizing that a fault has occurred among the redundant components); isolation (identifying, given a fault has occurred, which of the components has failed; and reconfiguration (correctly altering the system's behavior so as to prevent a failed component from adversely affecting the system's ongoing performance) [6].

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Redundant systems are more reliable than single strand or series systems. Each additional level of redundancy reduces the likelihood of system failure [6].

Despite the importance of correctly modeling these complex multichannel systems, there is a paucity of literature addressing the topic; this is especially true of the reliability assessment of redundant systems that use voting-based selectors that may be subject to imperfect fault coverage. All redundant systems must have some means of selection among their redundant inputs, a task that has been termed *redundancy management* (in the aerospace vernacular, at least). Redundancy management can seldom, if ever, be done with perfect certainty, and therefore, redundant systems are subject to imperfect fault coverage. Imperfect fault coverage has a significant adverse impact on the reliability of redundant systems (as compared with systems that have perfect fault coverage) and, as a result, cannot properly be ignored in the assessment of complex multichannel system reliability [6].

Reliability modeling of large, complex systems is an inherently difficult task; the fundamental reliability problem is well known as being NP-complete. Furthermore, correct techniques for modeling redundant systems subject to imperfect fault coverage are not widely known, particularly in the case of systems that use voting as a critical means of redundancy management.

In my previous work two mathematical models of wireless networks have been analyzed with special Erlang distribution and exponential distribution. There were demonstrated that the Erlang family provides more flexibility in modeling that exponential family, which only has one parameter. In practical situations, the Erlang family provides more flexibility in fitting a distribution to real data that the exponential family provides. The Erlang distribution is also useful in queueing analysis because of its relationship to the exponential distribution [4].

III. Model

The following notations are introduced in the paper:

n- number of information blocks in message.

$\Phi_j(t)$ – the distribution function of the message transmission, time probability which consists of n information blocks if transmission is started from j-th block ($j=\overline{1,n}$).

$F_j(u)$-the distribution function of transmitted block length.

r-1 – quantity of information block transmission repetition on unreliable environment before repair.

α – the traffic rate of errors origin.

G(u) – distribution function of recovery.

Under creation of this model we used the following assumptions:

i. The transmitted message on unreliable environment consists of n information blocks with constant length, distributed by law $F_j(u) = 1(t-\tau_j), j=\overline{1,n},$ where τ_j – j-th block transmission time;

ii. The checking of accuracy of getting information is made immediately after receiving the next block and the time spent on check is negligibly small, the check is full.

Under assumptions i and ii in the case of information distortion block transmission is repeated until it won't be correctly received. It takes place with a help of control bits.

The given model is described by the following equation:

$$\Phi_j(t) = \int_0^t dF_j(u) e^{-\alpha_j u} \Phi_{j+1}(t-u) + \int_0^t dF_j(u)\left(1-e^{-\alpha_j u}\right) \times \int_0^{t-u} dG(\upsilon)\Phi_j(t-u-\upsilon) \qquad j=\overline{1,n} \tag{1}$$

Let explain the equation (1) relatively $\Phi_j(t)$ –completion of message transmission ("task solution") under above-mentioned conditions for a given time. For that we'll present a random event A – the completion of two incompatible A_1 and A_2 events. A_1 event consists in, that:

1) Transmission of *j*-th ($j=\overline{1,n}$) will be completed for a time u (probability of this event equals to $dF_j(u)$). 2) for a time u shouldn't take place branching of QS to the next phase on failures ($\exp(-\alpha_j u)$); 3) for a time t-u the transmission of the whole remained part of message strating from *j*+1 –th block when QS is in *l* state on failures (probability of this event $\Phi_{j+1}(t-u)$ is completed).

The boundary condition has the form:

$$\Phi_{n+1}(t) = 1 \tag{2}$$

Using the Laplace transform to solve the equations: (1) and (2), we obtain:

$$\overline{\Phi}_j(s) = \overline{f}_j\left(s+\alpha_j\right)\overline{\Phi}_{j+1}(s) + \left[\overline{f}_j(s) - \overline{f}_j\left(s+\alpha_j\right)\right]\overline{\Phi}_j(s); \tag{3}$$

$$\overline{\Phi}_{n+1}(s) = \frac{1}{s}; \tag{4}$$

We denote:

$$\overline{f}_j(s) = \int_0^\infty e^{-st} dF_j(t); \tag{5}$$

$$\overline{\Phi}_j^{(i)}(s) = \int_0^\infty e^{-st}\Phi_j^{(i)}(t)dt \;; \tag{6}$$

$$\overline{g}(s) = \int_0^\infty e^{-st} dG(t),\; j=\overline{1,n}; \qquad i=\overline{1,r-2} \tag{7}$$

For j=n, we have:

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$$\Phi_n(S) = \frac{\overline{f}(s+\alpha)}{S} + \left[\overline{f}(S) - \overline{f}(s+\alpha)\right] g(S)\overline{\Phi}n(S) \tag{8}$$

We rewrite the last equation:

$$\overline{\Phi}n(S) = \frac{\overline{f}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}} \tag{9}$$

For j=n-1

$$\overline{\Phi}_{n-1}(S) = \overline{f}(s+\alpha)\Phi_n(S) + \left[\overline{f}(s) - \overline{f}(s+\alpha)\right] g(S)\overline{\Phi}_{n-1}(S) \tag{10}$$

or

$$\overline{\Phi}_{n-1}(S) = \frac{\overline{f^2}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}^2} \tag{11}$$

for j=n-2

$$\overline{\Phi}_{n-2}(S) = \overline{f}(s+\alpha)\Phi_{n-1}(S) + \left[\overline{f}(s) - \overline{f}(s+\alpha)\right] g(S)\Phi_{n-2}(S) \tag{12}$$

or

$$\overline{\Phi}_{n-2}(S) = \frac{\overline{f^3}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}^3} \tag{13}$$

$$\overline{\Phi}_{n-j}(S) = \frac{\overline{f^{j+1}}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}^{j+1}} \tag{14}$$

For j=n-1

$$\overline{\Phi}_1(S) = \frac{\overline{f^n}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}^n} \tag{15}$$

The conditional mean is calculated by the following equation:

$$T_m = s\overline{\Phi}_1(s)\Big|'_{s=0} = \frac{\overline{f^n}(s+\alpha)}{S\left\{1-\left[\overline{f}(S)-\overline{f}(s+\alpha)\right]g(S)\right\}^n}\Bigg|'_{S=0} \tag{16}$$

Solving the equation (16), we obtain:

$$T_m = n\frac{\tau_b + \tau_r\left[1-\overline{f}(\alpha)\right]}{f(\alpha)} \tag{17}$$

Case study:

Let assume that the message is transmitted with constant length – T_m, then

$$\tau_b = \frac{T_m}{n} + \tau_c \tag{18}$$

$$\overline{f}(\alpha) = e^{-\tau_b\alpha} \tag{19}$$

where n – a number of blocks; τ_c – time of check (parity) bits transfer, accompanying each block plus time of check in digital computer (DC) (the latter may be neglected); *i* – quantity of operations. Under $i \le r$ (*r*=2,3,…) the channel is transferred for repair; *i* – quantity of repetitions. Under $i \le r$ (r=2,3,...) the channel is transferred for repair; τ_r – mean time of channel recovery, it is obvious, that

$$\tau_c = \tau_b\frac{\tilde{n}_c}{m_{bc}} + \tau_d \tag{20}$$

$$\alpha = \frac{1}{kC_{ch}} \tag{21}$$

$\tilde{n}_C$ - a number of bits in transmission process, in which one error arises averagely, and C_c is speed of transfer (channel performance)

$$T_m = N_m C_c \tag{21}$$

N_m.- number of bits in message;
C_c- transfer time of one bit

Naturally, $n_{ch.\ sum}$ depends on the length of the block.
It is obvious, that T_m depends on n as wells as on r. To determine their optimal values we solve the following equations with respect to r and n:

$$\frac{\partial T_m(r,n)}{\partial r} = 0 \text{ and } \frac{\partial T_m(r,n)}{\partial n} = 0 \tag{22}$$

We assume that T_m depends continuously upon r and n.
Solving (22), we obtain:

$$\frac{\partial T_m}{\partial r} = \{\{[\tau_b + (\tau_r - \tau_b)b(0)[n - A_1] + +D\}\beta L + na(0)\}/f(\alpha)b(0)a^n(0) \tag{23}$$

$$\frac{\partial T_m}{\partial n} = \{nf(\alpha)b(0)[T_m + n(T_c + b(0)D] \times \times a^n(0)L[Ra^n(0) - 1] - A_1T_m\}/ /n^2f^2(\alpha)b^2(0)a^{2n}(0) \tag{24}$$

where

$$\tau_r\overline{f(\alpha)} - \tau_b = D \tag{25}$$

$$1 - (n+1)b(0) = \beta \tag{26}$$

$$1 - a^n(0) = A_1 \tag{27}$$

$$\ln\left[1 - \overline{f(\alpha)}\right] = L \tag{28}$$

$$f(\alpha)b(0) = R \tag{29}$$

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IV. Conclusion

This paper provides with mathematical model of wireless communication system. In the given model the intervals between adjacent failures are distributed according to an exponential law with the assumption that the message is transferred via unreliable environment. The transmitted information is represented in the form of blocks each of which is provided with check bits (parity bits). When a fault is occurred the transmission of information is repeated until the information block is received correctly. Probabilistic characteristic of information transmission time is obtained. Optima values mean time has been obtained.

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